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Non-Cooperative Climate Policies among Asymmetric Countries: Production- versus Consumption-based Carbon Taxes

Noha Elboghdadly* and Michael Finus†

Abstract

Non-cooperative production-based carbon taxes might be set inefficiently low due to the concern of governments about carbon leakage and the loss of competitiveness of their industries. In a strategic trade model, we study the effect of a gradual shift from bilateral production- to unilateral or bilateral consumption-based carbon taxes, considering various forms of border carbon adjustments (BCAs). We analyse the optimal response of two countries in a non-cooperative policy game. We show that if the environmentally more concerned government shifts unilaterally to a consumption-based policy, BCAs on imports create a new incentive for the optimal tax structure. Although profit-shifting and carbon leakage distortions are gradually reduced or even eliminated by combining carbon tariffs with export rebates, the optimal tax may still be below individual marginal damages in strategic setting. In contrast, a bilateral consumption-based tax, could be set equal to or even above individual marginal damages. In equilibrium, all forms of BCAs could allow both governments to set higher carbon taxes than under a bilateral production-based tax regime. However, BCA-regimes which add export rebates to import tariffs should be chosen carefully, as they may actually increase global emissions.

Keywords: Carbon Taxes, Border Carbon Adjustments, Carbon Leakage-shifting Effect, Profit-shifting Effect, Consumer Effect, Tariff and Export Rebate Income Effect
JEL-Classification: C72, F12, F18, H23,Q58.

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1 Introduction

Actions to mitigate climate change have mainly focused on emissions released within national boundaries of countries, referred to as the production-based approach. However, in the absence of a cooperative climate policy, this raises two main concerns. First, an environmental concern about the effectiveness of unilateral or sub-global actions: emission reductions by environmentally friendly countries are partly or completely offset by higher emissions in other countries, a phenomenon known as 'carbon leakage'. Second, a competitiveness concern about the loss of market shares of firms located in countries with stricter climate policies.

In order to address these concerns and to support more ambitious climate policies, some economists have argued in favour of taxing emissions based on consumption rather than production (Elliott et al., 2010; Helm et al., 2012; Steininger et al., 2014; Stiglitz, 2006).[1] Consumption-based climate policies can be implemented through trade instruments, such as border carbon adjustments (BCAs). That is, environmentally concerned countries complement their production-based carbon tax with carbon tariffs (BCAs on imports) and/or export rebates (BCAs on exports). Current discussions among economists and policy makers explore which climate policy design would be most effective in reducing global emissions. Contributing to this debate, our paper analyses different carbon tax regimes, with unilateral and bilateral trade measures. We are interested in the incentives to tax carbon, strategic policy spillovers of unilateral measures to other countries and equilibrium carbon taxes, by considering welfare effects on consumers, producers, environmental damages and tax and tariff revenues.

The choice of environmental policies has been widely studied in strategic trade-environment models. For instance, Barrett (1994), Conrad (1993) and Kennedy (1994) conclude that if only environmental policy instruments (e.g., taxes, permits, standards) are available to regulate emissions (i.e., production-based instruments), environmental policies may be set inefficiently lax, i.e., not only below global but also individual marginal damages.[2] This literature demonstrates that if firms engage in Cournot-competition, governments have an incentive to set lax policy levels in order to provide their firms with a strategic advantage over their rivals, which is known as the profit-shifting incentive (Barrett, 1994; Conrad, 1993; Kennedy, 1994). Moreover, in the case of a global pollutant, the effectiveness of environmental policies is undermined by carbon leakage, which is again a source for setting policy levels at low levels (Conrad, 1993; Duval and Hamilton, 2002; Kennedy, 1994). We complement this literature by studying whether a gradual shift from production-based to consumption-based policies, using various forms of BCAs, including carbon tariffs and different forms of export rebates, could restore the effectiveness of non-cooperative climate policies.

We solve a two-stage game in which two countries, perceiving global damages from greenhouse gases differently, choose their carbon taxes in the first stage, and then firms choose their output levels in the second stage. Our analysis starts with a bilateral production-based tax regime (PB- regime), under which both governments impose a carbon tax on their home firm only. BCAs are then introduced as unilateral measures,

[1] See also Jakob et al. (2014) for a survey of the literature that compares production with consumption-based approaches.

[2] These studies build on the strategic trade model due to Brander and Spencer (1985), extending their model by including consumers and environmental damages.

supplementing the production-based tax of the environmentally more concerned country. The three BCA-regimes which we consider are: (i) BCAs on imports, which fully adjust the difference between the two national tax levels (BI-regime), (ii) BCAs on imports and exports where the export rebate rate is chosen optimally (BIE-regime) and (iii) BCAs on imports with a full export rebate, sometimes referred to as full BCAs (BF-regime). Along this line, we move de facto from a unilateral partial to a full consumption-based tax. Finally, we consider a bilateral (full) consumption-based tax regime (CB-regime), under which both governments impose a carbon tax not on production but on consumption.

This paper is related to two strands of the literature on BCAs. The first strand is based on numerical analyses, focusing on quantifying the effects of BCAs, often by employing Computable General Equilibrium or Integrated Assessment Models of Climate Change (e.g., Böhringer et al., 2012 ; 2014 and 2018; Fischer and Fox, 2012) but also structural multi-sector, multi-country gravity frameworks (e.g., Larch and Wanner, 2017). Most of these papers conclude that BCA measures effectively mitigate carbon leakage and the competitive loss of firms. However, most of the reduction in total emissions are driven by carbon tariffs and not by export rebates. Due to the complicated nature of these models, these studies typically assume exogenous climate policy targets and countries on which BCA measure are imposed to be passive. Therefore, they cannot capture the strategic role of BCAs in an endogenous policy setting as we do.

The second strand of literature is based on theoretical models. One of the first paper goes back to Markusen (1975) who provides an economic rationale for using trade measures to internalise foreign emissions. He shows that a combination of a Pigouvian tax and import tariffs is optimal for the active and environmentally concerned country. See also, e.g., Hoel (1996) and Copeland (1996). Another insight in this literature is that full BCAs do not necessarily lead to less carbon leakage and total emissions. For instance, this could occur if the foreign country shifts its production to a more carbon intensive non-exporting sector (Jakob et al., 2013), or if the income effects associated with a consumption-based tax are strong (Eichner and Pethig, 2015). However, also most of these papers assume the foreign government to be passive; only few theoretical papers analyse BCAs in the context of bilateral endogenous policy choices. For instance, Sanctuary (2018) assumes perfect competition and shows that a country on which unilaterally BCAs on imports are imposed, may react by setting higher carbon taxes. This is confirmed, though in an imperfect competition model and based on numerical simulations, by Eyland and Zaccour (2012; 2014). This result is also in line with Helm et al. (2012), who conclude, albeit in a highly stylized political game without micro-foundation, that the country faced with carbon tariffs responds by setting a carbon tax on its exports. Our model comes to the same conclusion.

However, none of the above studies consider export rebates, which have a less obvious strategic effect than carbon tariffs. An exception is Hecht and Peters (2018), though they assume symmetric BCAs on imports and exports, which is usually not optimal. They show, in an imperfect competition model, that BCA-measures allow the home country to impose a higher carbon tax but incentivise the foreign country to lower its carbon taxes. The first part of their conclusion is confirmed by our model, for the second part we come to a more nuanced conclusion. In our model, the foreign country, faced with import tariffs and export rebates, may lower its tax compared to the situation when it only faces import tariffs. However, its tax rate may still be higher than without any BCA-measures. This crucially depends on the environmental damage evaluation in the two countries, and whether the home country can choose its export rate freely or whether this rate is fixed at the level of a full rebate.

The present paper provides a more general framework by considering more regimes, ranging from bilateral production-based carbon taxes to different forms of unilateral and bilateral consumption-based carbon taxes, which allows for more nuanced conclusions. In the tradition of Haufler et al. (2005) and Lockwood (2001), we derive the general optimal tax structure of the two asymmetric countries, by separating the effect of carbon taxes on consumers, producers, environmental damages, as well as tax and tariff revenues. In addition, we derive equilibrium carbon taxes and global emissions across all regimes considered in the paper. This requires to impose slightly more structure on the demand and damage cost function.

We show that for the country which unilaterally imposes BCAs, moving from a partial to full consumption-based tax, this addresses competitive and leakage concerns, either to some or full extent, reducing the pressure to adjust taxes downward for strategic reasons. Import tariffs by itself also induce the country on which imports are levied to increase its carbon tax, in order to protect its tax revenues. Hence, there are positive policy spillovers. In contrast, export rebates have a less positive and ambiguous impact. Not only the positive policy spillover effect may be reduced, but the country that provides rebates to its exporting sector, even though it reduces foreign carbon leakage, may increase its own emissions. Thus, there is a kind of negative or substitutional carbon leakage effect. Moreover, unilateral BCA-measures provide incentives for governments to reduce their taxes in order to compensate their consumers. Hence, the overall impact of BCAs on the optimal tax structure is not always straightforward, and, due to the strategic interaction among governments, also the impact on equilibrium taxes is less straightforward as commonly assumed. Nevertheless, our example shows that in a non-cooperative policy setting, unilateral BCA-measures reduce global emissions in equilibrium, but a bilateral consumption-based tax does so even more. Adding export rebates to import tariffs always increase equilibrium global emissions if export rebates are chosen optimally.

In what follows, we present the model in Section 2. In Section 3, we derive the general structure of the optimal carbon tax in the social optimum and in Section 4 we do so for the non-cooperative policy regimes. In Section 5, we use an example in order to rank equilibrium carbon taxes and global emissions across different regimes. Section 6 concludes.

2 Model

2.1 Preliminaries

We consider a two-stage game. In the first stage, governments simultaneously choose non-cooperatively their climate policy levels to regulate emissions associated with the production of good x. For concreteness, we assume that emissions are greenhouse gases and the policy instrument is a carbon tax. We consider the choice of carbon taxes under different policy regimes, which represent different combinations of production and consumption-based taxes. In the second stage, firms simultaneously choose their output levels. The game is solved by backward induction.

We consider two countries, $i = 1, 2$, with a representative consumer in each country. There is a representative firm k, $k = 1, 2$, in each country, producing a homogeneous good x, which generates greenhouse gas emissions. Firm 1 is located in country 1 and firm 2 is located in country 2, where firms compete in outputs, i.e., in a Cournot-fashion. Each firm supplies the home and the foreign market. There is also a clean numeraire good y produced under perfect competition.

Consumers have identical preferences, represented by the following quasi-linear utility function:

$$U_i\left(X_i, Y_i\right) = u_i\left(X_i\right) + Y_i \tag{1}$$

where $u_i\left(X_i\right)$ represents the utility from consuming the carbon-intensive good with the first and second derivative $u_i^{'} > 0$ and $u_i^{''} < 0$. Total consumption of x in country i is $X_i = x_{1i} + x_{2i}$, where x_{1i} and x_{2i} are the outputs supplied by firm 1 and 2 to market i, respectively. Hence, total production of x is $X = X_1 + X_2$. The utility of consuming the numeraire good y is represented by Y_i.

Each consumer owns a fixed amount of Labor (L), where one unit of L supplies one unit of good y; hence, the price of labor equals one. Consumers also receive profits (Π) obtained by the domestic firm k. Furthermore, carbon tax revenues (TR) are distributed back to the consumers in a lump-sum fashion. Hence, the representative consumer in country i faces the following budget constraint:

$$L + \Pi_k + TR_i = p_i X_i + Y_i\,. \tag{2}$$

Maximization of utility in (1), subject to the budget constraint in (2), delivers the following inverse demand function in market i:

$$p_i\left(X_i\right) = u_i^{'}\left(X_i\right), \tag{3}$$

with $p_i(X_i)$ the price of good x supplied to market i, where $p_i^{'}\left(X_i\right) < 0$.

For good x, we assume that the two firms employ an identical production technology with a linear production cost function, i.e., $C(x_{ki}) = cx_{ki}$ for $k = 1, 2$ and $i = 1, 2$. Markets are segmented. That is, firms take separate quantity decisions for the home and the foreign market. Profits obtained in market 1 and market 2 by the two firms are given by:

$$Market\,1 : \pi_{11} = (p_1(X_1) - c - t_{11})x_{11} \;\&\; \pi_{21} = (p_1(X_1) - c - t_{21})x_{21}\,, \tag{4}$$

$$Market\,2 : \pi_{12} = (p_2(X_2) - c - t_{12})x_{12} \;\&\; \pi_{22} = (p_2(X_2) - c - t_{22})x_{22} \tag{5}$$

where t_{11} (t_{21}) is the effective carbon tax which firm 1 (2) faces on its supply to market 1 and t_{12} (t_{22}) is the effective carbon tax which firm 1 (2) faces on its supply to market 2; total supply to market 1 is given by $X_1 = x_{11} + x_{21}$ and to market 2 by $X_2 = x_{12} + x_{22}$. We assume a constant emission-output ratio across firms, which, we normalise to 1 without loss of generality, such that a carbon emission tax is de facto an output tax.

2.2 Second Stage

In this stage, firms choose their profit-maximising output levels for each market. Firms maximise their profits in (4) and (5) simultaneously, which leads to the following first-order conditions in an interior equilibrium:

$$Market\,1 : \frac{\partial \pi_{11}}{\partial x_{11}} = p_1 + x_{11}p_1^{'} - c - t_{11} = 0 \;\&\; \frac{\partial \pi_{21}}{\partial x_{21}} = p_1 + x_{21}p_1^{'} - c - t_{21} = 0\,, \tag{6}$$

$$Market\,2 : \frac{\partial \pi_{12}}{\partial x_{12}} = p_2 + x_{12}p_2^{'} - c - t_{12} = 0 \;\&\; \frac{\partial \pi_{22}}{\partial x_{22}} = p_2 + x_{22}p_2^{'} - c - t_{22} = 0\,. \tag{7}$$

We assume that goods are strategic substitutes in market 1 and market 2. Sufficient conditions to ensure this are $\frac{\partial^2 \pi_{11}}{\partial x_{11}\partial x_{21}} = p_1^{'} + x_{11}p_1^{''} < 0$ and $\frac{\partial^2 \pi_{21}}{\partial x_{21}\partial x_{11}} = p_1^{'} + x_{21}p_1^{''} < 0$ in market 1, and $\frac{\partial^2 \pi_{12}}{\partial x_{12}\partial x_{22}} = p_2^{'} + x_{12}p_2^{''} < 0$ and $\frac{\partial^2 \pi_{22}}{\partial x_{22}\partial x_{12}} = p_2^{'} + x_{22}p_2^{''} < 0$ in market 2. That is, reaction functions of firms are negatively sloped: increasing the production level of one firm reduces the marginal revenues of its rival, which reacts by decreasing production. Assuming that the second-order conditions are satisfied, as derived in Appendix A.1, and solving the first-order conditions in (6) and (7) simultaneously, yield the profit-maximising output levels produced by firm 1 and 2 in both markets.

The profit-maximising output levels are function of the effective taxes, i.e., $x_{1i} = f_i(t_{1i}, t_{2i})$ and $x_{2i} = g_i(t_{1i}, t_{2i})$. Thus, the outcome of the second stage of the game is a Nash equilibrium in output levels in each of the two markets.[3]

2.3 First Stage

Given the equilibrium output levels of firms, governments simultaneously choose the level of their carbon tax t_i in the first stage by maximising their individual welfare function. We consider five non-cooperative policy regimes: a bilateral production-based tax (PB-regime) and a bilateral consumption-based tax (CB-regime) regime. Moreover, we consider three unilateral regimes under which country 1 imposes border carbon adjustments (BCAs) on country 2, supplementing its production-based tax with import tariffs and/or export rebates, while country 2 reacts by a production-based tax. These unilateral regimes are abbreviate as BI-regime, BIE-regime and BF-regime and are explained in more detail subsequently.

The welfare function of country 1 and country 2 are given by:

$$W_1 = CS_1 + PS_1 + TR_1 + L - D_1 + BCAI_1 - BCAE_1\,, \tag{8}$$

$$W_2 = CS_2 + PS_2 + TR_2 + L - D_2\,, \tag{9}$$

where CS_i is the consumer surplus derived from consuming good x in country i, with $CS_i = \int_0^{X_i} p_i(X_i)\,dX_i - p_i X_i$, which follows from (1), recalling that the total supply to market i is given by $X_i = x_{1i} + x_{2i}$. PS_i is the producer surplus, which is the total profit of the home firm k obtained in both markets, i.e., $PS_i = \Pi_k = \pi_{k1} + \pi_{k2}$. TR_i is the tax revenue of government i with $TR_i = t_i(x_{k1} + x_{k2})$ under all regimes, except under the CB-regime where tax revenues are given by $TR_i = t_i(x_{1i} + x_{2i})$. That is, the tax revenue is based on total production in country i under all regimes, but is based on total consumption in country i under the CB-regime. L is labor income.

D_i are individual damages from global greenhouse gas emissions released in the production of good x. Global damages from global emissions are $D(e)$, $e = e_1 + e_2$ where $e_1 = x_{11} + x_{12}$ and $e_2 = x_{22} + x_{21}$, as we normalise the emission-output ratio to 1. Hence, global emissions are equal to total production, which is equal to total consumption, i.e., $e = X$. Countries may perceive or evaluate global damages differently. Therefore, the damage function by country 1 and country 2 is given by:

$$D_1(e) = \gamma D(e)\,, D_2(e) = (1-\gamma) D(e) \;\; \gamma \in [0.5, 1]\,, \tag{10}$$

[3] The conditions derived in Appendix A.1 ensure the existence and uniqueness of a Nash equilibrium in output levels.

respectively, where $D^{'}(e) > 0$ and $D^{''}(e) \geq 0$. Because $\gamma \in [0.5, 1]$, country 1 is at least as concerned as country 2 about environmental damages and, usually, more whenever γ is strictly larger than 0.5.[4]

Finally, the last two terms in the welfare function in (8), $BCAI_1$ and $BCAE_1$, stand for the tariff revenues from a unilateral BCA-policy on imports and the expenses from a tax rebate on exports, respectively. These two terms are only relevant under the three unilateral BCA-regimes and are zero by assumption under the bilateral PB- and CB-regime. Given $\gamma \in [0.5, 1]$, we assume that it is country 1 which implements BCAs under the unilateral BCA-regimes.

In the following, we explain the difference between the five non-cooperative policy regimes and their impacts on effective carbon taxes, which are summarised in Table 1.

First, we consider that both governments impose a production-based carbon tax on their home firm (PB-regime). Hence, the effective tax which each firm faces is equal to the tax imposed on it in its home country, i.e., $t_{11} = t_{12} = t_1$ and $t_{21} = t_{22} = t_2$.

Second, we consider border carbon adjustments on imports (BI-regime). Country 1 imposes not only a production-based tax but also a carbon tariff on imports from country 2.[5] Hence, firm 1 faces the effective tax $t_{11} = t_{12} = t_1$ as under the PB-regime, and also firm 2 faces $t_{22} = t_2$ on its supply to country 2, but faces $t_{21} = t_2 + \omega(t_1 - t_2)$ on its exports to country 1 if $t_1 > t_2$, with ω the border tax adjustment parameter on imports (Eyland and Zaccour, 2012). We assume that country 1 imposes a carbon tariff which fully adjusts the difference between the two national tax levels, i.e., $\omega = 1$. On the one hand, any value of ω above 1 would violate the equal treatment rules under the WTO.[6] On the other hand, any value of ω below 1 would not be optimal for country 1.[7] This assumption implies that both firms supplying market 1 face the same effective carbon tax. Therefore, the term $BCAI_1$ in (8) is given by $BCAI_1 = (t_1 - t_2)\, x_{21}$ if $t_1 > t_2$, otherwise $BCAI_1 = 0$.

Third, we consider border carbon adjustments on imports and exports (BIE-regime). Under this regime, country 1 complements its production-based tax and carbon tariff with a rebate on exports to country 2. Thus, compared to the previous regime, only the effective tax firm 1 faces on its supply to country 2 will change, which is now given by $t_{12} = t_1(1 - \hat{\varphi})$ if $t_1 > t_2$, with φ the border tax adjustment parameter on exports, which may also be called the export rebate rate.[8] $\hat{\varphi}$ indicates that φ is chosen optimally. Unlike carbon tariffs, it is not always optimal for country 1 to provide a full export rebate to its firm. In the spirit of the WTO equal treatment rule, we impose a constraint such that the effective tax that firm 1 faces on its exports is at least as high as the tax that the foreign firm 2 faces. That is, we impose the condition $t_1(1 - \hat{\varphi}) \geq t_2$. This implies a constraint on the maximum optimal rebate rate, which will be explained in detail in Section 4. Therefore, the term $BCAE_1$ in (8) is given by $BCAE_1 = \hat{\varphi}\, t_1\, x_{12}$ if $t_1 > t_2$ and

[4]Eq. (10) allows us to distinguish between the level of damages and the distribution of damages. Similar qualitative conclusions would hold if, alternatively, we assumed $D_1 = d_1 D(e)$ and $D_2 = d_2 D(e)$.

[5]We use the terms carbon tariff and BCAs on imports interchangeably.

[6]The GATT allows WTO members to apply a border tax adjustment at a rate which is not higher than the rate applied to domestically produced "like" products.

[7]See Hecht and Peters (2018). In other words, if ω was chosen optimally by country 1, it would be chosen at a value larger than 1.

[8]Note that we cannot model BCAs on exports in the same way as on imports. That is, we cannot assume for instance $t_{12} = t_1 - \varphi(t_1 - t_2)$, as this would imply that country 2 imposes tax t_2 on the output of firm 1 for market 2, which is not the case.

$t_1(1-\hat{\varphi}) \geq t_2$.

Table 1: Effective Carbon Taxes under Non-Cooperative Policy Regimes

	Effective taxes	PB	BI	BIE	BF	CB
Market 1	t_{11}	t_1	t_1	t_1	t_1	t_1
	t_{21}	t_2	t_1	t_1	t_1	t_1
Market 2	t_{12}	t_1	t_1	$t_1(1-\hat{\varphi})$	0	t_2
	t_{22}	t_2	t_2	t_2	t_2	t_2

Fourth, we consider border carbon adjustments on imports with a full export rebate (BF-regime). That is, this regime is similar to the previous regime, except that we do not assume that φ is chosen optimally but assume $\varphi = 1$. Therefore, firm 1 faces the effective tax $t_{12} = 0$ on its supply to country 2. The BF-regime implies de facto a unilateral consumption-based tax imposed by country 1. Again, this effective tax is required to be at least as high as t_2.[9,10]

Finally, we consider that both governments impose a consumption-based carbon tax (CB-regime). Hence, both firms supplying country i face t_i, i.e., $t_{11} = t_{21} = t_1$ and $t_{12} = t_{22} = t_2$. It is interesting to note that this regime is equivalent to a regime in which each country imposes a production-based tax supplemented by a tariff on imports and a full export rebate. With reference to our border adjustment parameters introduced above, this implies $\omega_i = 1$ and $\varphi_i = 1 \; \forall \, i = 1, 2$. In other words, the CB-regime is equivalent to a bilateral BF-regime.

3 Optimal Climate Policy: Normative Benchmark

Before turning to the non-cooperative regimes, we briefly discuss the normative benchmark of the social optimum. We assume for expositional simplicity that governments choose a uniform tax t^S. Hence, the Nash equilibrium output levels in the second stage are given by $x_{1i}^S = f_i\left(t^S, t^S\right) = x_{2i}^S = g_i\left(t^S, t^S\right)$.
In the first stage, governments choose a uniform carbon tax t_S by maximising the aggregate welfare, $W = W_1 + W_2$.[11] Assuming that the second-order condition is satisfied, the socially optimal carbon tax, which is derived in Appendix A.2, can be written as follows:

$$\hat{t}^S = \underbrace{\frac{2p_i^{'} x_{ki} \frac{dX^S}{dt^S}}{\Lambda^S}}_{CSE\;(-)} + \underbrace{\frac{-p_i^{'} x_{ki} \frac{dX^S}{dt^S}}{\Lambda^S}}_{PSE(+)} + \underbrace{\frac{(\gamma + (1-\gamma))\; D^{'} \left(\frac{\partial e^S}{\partial t^S}\right)}{\Lambda^S}}_{EDE(+)}$$
$$\Longleftrightarrow \hat{t}^S = p_i^{'} x_{ki} + D^{'}, \tag{11}$$

[9] In our strategic context, this regime is compatible with our constraint $t_1(1-\varphi) \geq t_2$ if $t_2 \leq 0$, which is likely to be the case, as we show in Section 5.

[10] Note that in a setting with bilateral endogenous policy choices (i.e., t_2 is not necessarily zero), full export rebates and full export adjustments are generally not the same. This is evident by considering the case where country 2 chooses a subsidy, i.e., $t_2 < 0$, in which case the full export rebate is smaller than the full export adjustment. In order to keep the number of policy regimes at a reasonable level, we do not consider a regime with full export adjustment.

[11] Hence, $BCAI_1$ and $BCAE_1$ in (8) are zero in the cooperative solution.

where $D' = \frac{\partial D(e)}{\partial e} > 0$, $EDE = D'$, $\Lambda^S = \frac{dX^S}{dt^S} = \frac{2p_1'}{J_1} + \frac{2p_2'}{J_2} < 0$, and $\frac{dx_{1i}^S}{dt^S} = \frac{dx_{2i}^S}{dt^S} = \frac{p_i'}{J_i} < 0$ with $J_i > 0$ where J_i denotes the Jacobian matrix of the second-order conditions in the second stage. We may recall $p_i' < 0$ as the demand function is downward sloping and note that $\frac{\partial e^S}{\partial t^S} = \frac{dX^S}{dt^S}$, as total emissions are equal to total output.

In this but also in the following section, the structure of the optimal climate policy level is broken down into several effects: 1) the consumer surplus effect (CSE), which is related to distortions created by the underproduction associated with imperfect competition, 2) the producer surplus effect (PSE), which is related to the profits of firms[12], and 3) the environmental damage effect (EDE), stemming from the internalisation of damages from global emissions. Thus, we are dealing multiple externalities.

In (11), the CSE is negative. Hence, the CSE would call for a subsidy in order to raise production in both countries (Barnett, 1980). The PSE is positive. Hence, the PSE would call for a tax. That is, when governments choose their taxes cooperatively, imposing a positive tax will raise the collective net profits of the two firms by enforcing a monopolistic output. Finally, governments jointly internalise global damages from emissions. Thus, the EDE is positive and would call for a tax equal to global marginal damages. Although the CSE and the PSE have opposite signs, the CSE dominates. Therefore, the socially optimal carbon tax level is smaller than global marginal damages, $\hat{t}^S < D'$.[13] Whether the equilibrium tax will be positive or negative cannot be deduced at this level of generality, though it increases with the value attached to global marginal damages.

4 Non-cooperative Optimal Climate Policies

In this section, we analyse the impact of a gradual shift from a bilateral production-based to a bilateral consumption-based tax on the three effects introduced in the previous section. Moreover, we investigate whether new effects emerge.

4.1 Bilateral Production-based Tax (PB-regime)

We start our analysis by considering the bilateral production-based tax regime (PB-regime), which serves as a reference for the later regimes. Both governments impose a carbon tax on the production of their home firm. With reference to Table 1, solving the second stage delivers: $x_{1i}^{PB} = f_i(t_1, t_2)$ and $x_{2i}^{PB} = g_i(t_1, t_2)$ for $i = 1, 2$. We denote a country's welfare function under the PB-regime by W_i^{PB}. All details are provided in Appendix A.3.

Maximising W_i^{PB} with respect to the national tax t_i gives the optimal PB-tax structure

[12] The PSE is a net effect, which also considers the tax revenues of governments, i.e., $PS_i + TR_i$, which are net profits. This is because taxes paid by firms are equal to tax revenues.

[13] See also Kennedy (1994) and Duval and Hamilton (2002). In Conrad (1993), the cooperative tax level is larger than global marginal damages because he assumes consumption takes place in a third market and hence *CSE=0*.

of country 1 and country 2, respectively:[14]

$$\hat{t}_1^{PB} = \underbrace{\frac{p_1^{'}\left(x_{11}+x_{21}\right)\frac{p_1^{'}}{J_1}}{\Lambda_1^{PB}}}_{CSE_1\ (-)} + \underbrace{\frac{-p_1^{'}x_{11}\frac{dx_{21}^{PB}}{dt_1} - p_2^{'}x_{12}\frac{dx_{22}^{PB}}{dt_1}}{\Lambda_1^{PB}}}_{PSE_1(-,-)} + \underbrace{\frac{\gamma\, D^{'}\left(\frac{\partial e^{PB}}{\partial t_1}\right)}{\Lambda_1^{PB}}}_{EDE_1\ (+)} \tag{12}$$

where $\Lambda_1^{PB} = \frac{dx_{11}^{PB}}{dt_1} + \frac{dx_{12}^{PB}}{dt_1} < 0$, $\frac{\partial e^{PB}}{\partial t_1} = \frac{\partial e_1^{PB}}{\partial t_1} + \frac{\partial e_2^{PB}}{\partial t_1} < 0$, with $\underbrace{\frac{\partial e_1^{PB}}{\partial t_1}}_{HEE_1} < 0$, $\underbrace{\frac{\partial e_2^{PB}}{\partial t_1}}_{FEE_1} > 0$,

$EDE_1 = \gamma\, D^{'} + \underbrace{\frac{\gamma\, D^{'}\left(FEE_1\right)}{\Lambda_1^{PB}}}_{(-)}$ and

$$\frac{dx_{11}^{PB}}{dt_1} = \frac{2p_1^{'} + p_1^{''}x_{21}}{J_1} < 0\,, \quad \frac{dx_{12}^{PB}}{dt_1} = \frac{2p_2^{'} + p_2^{''}x_{22}}{J_2} < 0\,,$$
$$\frac{dx_{21}^{PB}}{dt_1} = -\frac{p_1^{'} + p_1^{''}x_{21}}{J_1} > 0\,, \quad \frac{dx_{22}^{PB}}{dt_1} = -\frac{p_2^{'} + p_2^{''}x_{22}}{J_2} > 0\,. \tag{13}$$

$$\hat{t}_2^{PB} = \underbrace{\frac{p_2^{'}\left(x_{22}+x_{12}\right)\frac{p_2^{'}}{J_2}}{\Lambda_2^{PB}}}_{CSE_2\ (-)} + \underbrace{\frac{-p_1^{'}x_{21}\frac{dx_{11}^{PB}}{dt_2} - p_2^{'}x_{22}\frac{dx_{12}^{PB}}{dt_2}}{\Lambda_2^{PB}}}_{PSE_2(-,-)} + \underbrace{\frac{(1-\gamma)\, D^{'}\left(\frac{\partial e^{PB}}{\partial t_2}\right)}{\Lambda_2^{PB}}}_{EDE_2\ (+)} \tag{14}$$

where $\Lambda_2^{PB} = \frac{dx_{22}^{PB}}{dt_2} + \frac{dx_{21}^{PB}}{dt_2} < 0$, $\frac{\partial e^{PB}}{\partial t_2} = \frac{\partial e_1^{PB}}{\partial t_2} + \frac{\partial e_2^{PB}}{\partial t_2} < 0$, with $\underbrace{\frac{\partial e_2^{PB}}{\partial t_2}}_{HEE_2} < 0$, $\underbrace{\frac{\partial e_1^{PB}}{\partial t_2}}_{FEE_2} > 0$,

$EDE_2 = (1-\gamma)D^{'} + \underbrace{\frac{(1-\gamma)D^{'}\left(FEE_2\right)}{\Lambda_2^{PB}}}_{(-)}$ and

$$\frac{dx_{11}^{PB}}{dt_2} = -\frac{p_1^{'} + p_1^{''}x_{11}}{J_1} > 0 \quad \frac{dx_{12}^{PB}}{dt_2} = -\frac{p_2^{'} + p_2^{''}x_{12}}{J_2} < 0\,,$$
$$\frac{dx_{21}^{PB}}{dt_2} = \frac{2p_1^{'} + p_1^{''}x_{11}}{J_1} < 0 \quad \frac{dx_{22}^{PB}}{dt_2} = \frac{2p_2^{'} + p_2^{''}x_{12}}{J_2} > 0\,. \tag{15}$$

recalling that $p_i^{'} < 0$ and $J_i > 0$.

Under the PB-regime, both carbon taxes have the same negative impact on the consumer surplus in each country, $\partial CS_i/\partial t_i = \partial CS_i/\partial t_j = -p_i^{'}\left(x_{1i}+x_{2i}\right)\left(p_i^{'}/J_i\right) < 0$. Thus, the CSE is negative and would call for a subsidy in order to support consumers in both countries. However, subsidies cause positive consumer price spillovers (Lockwood, 2001). That is, lowering taxes in one country benefits not only domestic but also foreign consumers. Since governments are assumed to behave non-cooperatively, they do not care about foreign consumers; this reduces the incentives of both governments to subsidise

[14] The optimal taxes display the various effects that play a role when choosing taxes. They are different from equilibrium taxes, which would require to solve (12) and (14) simultaneously, which is not possible at this level of generality. The same applies to all subsequent regimes.

their consumers compared to the social optimum. Hence, focusing exclusively on the CSE, leads to a subsidy below the socially optimal level.

Under the non-cooperative regimes, the PSE reflects the concern of governments about the competitiveness of their firms. From (13) and (15) it is evident that production of each firm decreases (increases) in the tax of the home (foreign) country. As a result, the incentives related to producers are different from the cooperative solution. Setting a higher carbon tax reduces the sales of the home firm at the expenses of an expansion of the sales of the foreign firm. Hence, higher carbon taxes decrease the net profits (profits net of tax payments) of the home firm. Therefore, in line with the analysis of Brander and Spencer (1985), each government has an incentive to give a subsidy to its firm in order to shift profits from the foreign to the home firm. This is known as the profit-shifting effect. The PSE in both countries comprises two terms that reflect the incentive to shift profits in market 1 (first term) and market 2 (second term). Under this regime, both terms are negative and hence would call for a subsidy.

Environmental damages and the problem of carbon leakage are captured by the EDE. Different from the social optimum, countries do not internalise global but only their individual damages under the non-cooperative regimes, i.e., a fraction of total damages. Both the home and the foreign PB-tax have the same effect on individual damages in both countries in that they reduce damages, i.e., $\partial D_i/\partial t_i = \partial D_i/\partial t_j = D_i' \left(p_1'/J_1 + p_2'/J_2 \right) < 0$. If we exclusively focus on the EDE, ignoring other market distortions, the effectiveness of countries' climate policy can be measured by the departure of the EDE from individual marginal damages.

The impact of a country's climate policy on global emissions comprises two effects: the "home emission effect" (HEE) and the "foreign emission effect" (FEE). The HEE captures the change of emissions released by the home firm, $\partial e_i/\partial t_i$, and the FEE captures the change of emissions released by the foreign firm, $\partial e_j/\partial t_i$. The latter effect is sometimes referred to as the carbon leakage or transboundary pollution effect (Duval and Hamilton, 2002; Kennedy, 1994). Under the PB-regime, the HEE is negative but the FEE is positive, though overall the EDE is positive. Hence, the HEE is stronger than the FEE in absolute terms. Thus, carbon leakage weakens the EDE. In total, the EDE calls for a tax smaller than individual marginal damages, i.e., $EDE_i < D_i' \; \forall \, i = 1, 2$.[15] Recalling that the CSE and PSE are negative, it is clear that taxes under the PB-regime are set below individual marginal damages, $\hat{t}_i^{PB} < D_i' \; \forall \, i = 1, 2$.

In equilibrium, each government chooses its tax, given the tax level of the other government. That is, Nash equilibrium carbon taxes are obtained by solving $\partial W_1/\partial t_1 = 0$ and $\partial W_2/\partial t_2 = 0$ simultaneously. Each country's carbon tax only affects the welfare of the other country indirectly through affecting production levels (Conrad, 1993).[16] Therefore, the equilibrium tax rate in each country is less than its individual marginal damages.

It is clear that since both countries face the same demand, the CSE is the same in both countries. If both firms were to produce the same quantities, the PSE would also be the same. Therefore, the difference in equilibrium taxes must be due to the EDE, which is higher in country 1 than in country 2. Therefore, in equilibrium, country 1 will have a higher tax than country 2, i.e., $t_1^{PB*} > t_2^{PB*}$ for all $\gamma > 0.5$. As a result, equilibrium

[15] If pollution is local, the FEE will vanish and the EDE is equal to the country's marginal damage, see for instance Kennedy (1994).

[16] This will not be the case under the next three unilateral BCA-regimes, where, as will be shown below, the carbon tax of country 2 could also affect the welfare of country 1 directly through carbon tariffs.

quantities produced by firm 1 are smaller than by firm 2 and, consequently, the producer surplus but also the net producer surplus (profits net of tax revenues) are smaller in country 1 than in country 2. Again, at this level of generality, nothing can be said whether equilibrium taxes will be positive or negative, except that, ceteris paribus, taxes in each country increase in the valuation of its individual marginal damages. Nevertheless, based on our discussion, we can state the following.

Proposition 1. *In our strategic trade model, the optimal (equilibrium) production-based carbon taxes are set below individual marginal damages,* $\hat{t}_i^{PB}(t_i^{PB*}) < D_i^{'}\ \forall\, i = 1, 2$ *with* $t_1^{PB*} \geq t_2^{PB*}$ *for all* $\gamma \geq 0.5$.

Given that both firms are assumed to be identical, different profits only stem from differences in effective taxes that firms face. Thus, firm 1 suffers from a competitive disadvantage, both in the home and the foreign market. Therefore, in the subsequent analysis, we consider that country 1 complements its PB-policy with BCAs to protect its home firm and to raise the effectiveness of its climate policy. These regimes can be considered as a gradual shift to a unilateral consumption-based carbon tax imposed by country 1.

4.2 Border Carbon Adjustments on Imports (BI-regime)

Under the carbon adjustment on imports regime (BI-regime), country 1 complements its production-based tax with tariffs on imports. Export rebates are not considered. From Table 1, the output levels in the second stage are given by $x_{11}^{BI} = f_1(t_1, t_1) = x_{21}^{BI} = g_1(t_1, t_1)$ for the supply to market 1, while $x_{12}^{BI} = f_2(t_1, t_2)$ and $x_{22}^{BI} = g_2(t_1, t_2)$ for the supply to market 2.

Maximising W_1^{BI} and W_2^{BI}, which are the welfare function of country 1 and country 2 under the BI-regime, with respect to own national taxes, leads to the optimal tax structure of each country (see Appendix A.4 for details):

$$\hat{t}_1^{BI} = \underbrace{\frac{p_1^{'}(x_{11} + x_{21})\frac{2p_1^{'}}{J_1}}{\Lambda_1^{BI}}}_{CSE_1\ (-)} + \underbrace{\frac{-p_1^{'}x_{11}\frac{dx_{21}^{BI}}{dt_1} - p_2^{'}x_{12}\frac{dx_{22}^{BI}}{dt_1}}{\Lambda_1^{BI}}}_{PSE_1(+,-)} + \underbrace{\frac{\gamma D^{'}\left(\frac{\partial e^{BI}}{\partial t_1}\right)}{\Lambda_1^{BI}}}_{EDE_1\ (+)} + \underbrace{\frac{-x_{21} + t_2\frac{dx_{21}^{BI}}{dt_1}}{\Lambda_1^{BI}}}_{BAIE_1(+,?)} \tag{16}$$

where $\Lambda_1^{BI} = \frac{dx_{11}^{BI}}{dt_1} + \frac{dx_{12}^{BI}}{dt_1} + \frac{dx_{21}^{BI}}{dt_1} < 0$, $\frac{\partial e^{BI}}{\partial t_1} = \frac{\partial e_1^{BI}}{\partial t_1} + \frac{\partial e_2^{BI}}{\partial t_1} < 0$, with $\underbrace{\frac{\partial e_1^{BI}}{\partial t_1}}_{HEE_1} < 0$, $\underbrace{\frac{\partial e_2^{BI}}{\partial t_1}}_{FEE_1} \gtreqless 0$,

$EDE_1 = \gamma D^{'} + \underbrace{\frac{\gamma D^{'}\left(\frac{dx_{22}^{BI}}{dt_1}\right)}{\Lambda_1^{BI}}}_{(-)}$ and the effect of t_1 on outputs is given by:

$$\frac{dx_{11}^{BI}}{dt_1} = \frac{dx_{21}^{BI}}{dt_1} = \frac{p_1^{'}}{J_1} < 0,$$
$$\frac{dx_{12}^{BI}}{dt_1} = \frac{2p_2^{'} + p_2^{''}x_{22}}{J_2} < 0, \frac{dx_{22}^{BI}}{dt_1} = -\frac{p_2^{'} + p_2^{''}x_{22}}{J_2} > 0\,. \tag{17}$$

$$\hat{t}_2^{BI} = \underbrace{\frac{p_2^{'}\left(x_{22}+x_{12}\right)\frac{p_2^{'}}{J_2}}{\Lambda_2^{BI}}}_{CSE_2\ (-)} + \underbrace{\frac{-p_2^{'}x_{22}\frac{dx_{12}^{BI}}{dt_2}}{\Lambda_2^{BI}}}_{PSE_2(-)} + \underbrace{\frac{(1-\gamma)\,D^{'}\left(\frac{\partial e^{BI}}{\partial t_2}\right)}{\Lambda_2^{BI}}}_{EDE_2\ (+)} + \underbrace{\frac{-x_{21}}{\Lambda_2^{BI}}}_{BAIE_2(+)} \tag{18}$$

where $\Lambda_2^{BI} = \frac{dx_{22}^{BI}}{dt_2} < 0$, $\frac{\partial e^{BI}}{\partial t_2} = \frac{\partial e_1^{BI}}{\partial t_2} + \frac{\partial e_2^{BI}}{\partial t_2} < 0$ with $\underbrace{\frac{\partial e_2^{BI}}{\partial t_2}}_{HEE_2} < 0$, $\underbrace{\frac{\partial e_1^{BI}}{\partial t_2}}_{FEE_2} > 0$, $EDE_2 = (1-\gamma)D^{'} + \underbrace{\frac{(1-\gamma)D^{'}\left(\frac{dx_{12}^{BI}}{dt_2}\right)}{\Lambda_2^{BI}}}_{(-)}$ and the effect of t_2 on outputs is given by:

$$\begin{aligned} &\frac{dx_{11}^{BI}}{dt_2} = \frac{dx_{21}^{BI}}{dt_2} = 0\,, \\ &\frac{dx_{12}^{BI}}{dt_2} = -\frac{p_2^{'} + p_2^{''}x_{12}}{J_2} > 0\,,\ \frac{dx_{22}^{BI}}{dt_2} = \frac{2p_2^{'} + p_2^{''}x_{12}}{J_2} < 0\,. \end{aligned} \tag{19}$$

The CSE has the same sign as in the previous regime in both countries. However, t_1 has now a larger effect while t_2 has no effect on the consumer surplus in country 1, $\partial CS_1/\partial t_1 = -p_1^{'}\left(x_{11}+x_{21}\right)\left(2p_1^{'}/J_1\right) < 0$ and $\partial CS_1/\partial t_2 = 0$. Therefore, country 1 has a stronger incentive to subsidise its consumers than under the PB-regime. For country 2, the consumer price spillovers to country 1 are zero. Hence, also country 2 has a stronger incentive to subsidise its consumers than under the PB-regime.

Consider now the PSE. We recall that one of the main objectives of BCAs is to protect profits in the light of asymmetric carbon taxes. Since the BI-regime assumes a unilateral BCA-policy imposed by country 1, we focus first on country 1, which, by assumption, sets a higher carbon tax than country 2. The sign of the PSE in country 1 is no longer negative but is ambiguous after introducing carbon tariffs. In market 1, the profit-shifting incentive is eliminated. Hence, the first term of the PSE in (16) is now positive. BCAs on imports level the playing field in market 1. However, the second term of the PSE is still negative, as firm 1 is not protected in market 2. Therefore, carbon tariffs reduce but do not eliminate the pressure on country 1 of adjusting taxes downward. The sign of the aggregate PSE depends on the strength of these two effects which work in opposite directions. For a linear demand curve, the aggregate PSE is positive and hence would call for a tax.[17]

For country 2, the PSE is negative but comprises now only one component. This is because its carbon tax affects only profits of its firm in its home market 2, while it has no effect on the profits obtained from exports to market 1 (as firm 2 de facto faces t_1 on its exports). Therefore, the profit-shifting incentive of country 2 in market 1 disappears and only that in market 2 remains.[18]

We now consider the EDE, which is positive in both countries. However, the details are different. With tariffs, the impact of country 1's tax t_1 on global emissions and

[17] See Appendix A.4 for details.

[18] It is not straightforward to confirm at this level of generality whether the PSE in country 2 will decrease compared to the PB-regime because the absolute value of the denominator of the PSE-effect in (18) also decreases compared to (14).

carbon leakage becomes larger while the impact of country 2's tax t_2 becomes smaller. Whereas under the PB-regime, we had a symmetric impact of taxes on damages, $\partial D_i/\partial t_i = \partial D_i/\partial t_j = D_i' \left(p_1'/J_1 + p_2'/J_2\right)$, now, under the BI-regime, we have $\partial D_i/\partial t_1 = D_i' \left(2p_1'/J_1 + p_2'/J_2\right) > D_i' \left(p_2'/J_2\right) = \partial D_i/\partial t_2$.

For both countries, the HEE remains negative. For country 2, also the FEE remains positive due to carbon leakage. However, the FEE for country 1 is now different. This FEE has now two terms with opposite sign: $dx_{21}^{BI}/dt_1 < 0$ and $dx_{22}^{BI}/dt_1 > 0$. The first term may be viewed as anti-leakage: firm 2's sale in country 1 is reduced. The second term is the familiar leakage effect known from above. Hence, irrespective of the sign of the overall FEE, carbon leakage can be better controlled by country 1 through import tariffs. Therefore, country 1 faces less pressure to reduce taxes to avoid leakage.

Nevertheless, overall, the EDE would still call for a tax lower than individual marginal damages in country 1. For country 2, an increase in its carbon tax raises only foreign emissions in market 2, while there is no effect on market 1. Hence, the FEE in country 2 stems only from $dx_{12}^{BI}/dt_2 > 0$. As a result, although carbon leakage is less sever under the BI-regime, also the EDE in country 2 would call for a tax below marginal damages. Thus, together, we have $EDE_i < D_i' \; \forall \; i = 1, 2$.

Finally, introducing BCAs on imports creates a new strategic incentive in both countries related to tariff revenues, which we call the border adjustment income effect (BAI-effect), abbreviated BAIE. Recall that the effective carbon tariff rate depends on the difference between the two national taxes. Through the tariff, country 1 de facto taxes foreign production, which is a new source of revenues. This provides an incentive for country 1 to increase taxes. Interesting, also country 2 has an incentive to tax exports of its firm in order to capture a larger part of its tax revenues. This constitutes a kind of a 'race to the top' in carbon taxes.

For country 1, the BAIE comprises two terms. The first term, $-x_{21}/\Lambda_1^{BI}$, is strictly positive whereas the sign of second term, $\left(t_2 dx_{21}^{BI}/dt_1\right)/\Lambda_1^{BI}$, depends on the tax level of country 2. If $t_2 \geq 0$, the sign of the second term is positive and hence the BAIE in country 1 is unambiguously positive. However, if $t_2 < 0$, the sign of the second term is negative. Thus, the overall BAIE could be negative if t_2 is negative and very small (i.e., large in absolute terms), implying a large subsidy in country 2. In this case, if country 1 imposes a positive tax, the difference between the two national tax levels is large, implying a large effective tariff rate, which may erode the BCA revenues in the sense of the Laffer curve. However, it is very likely that, overall, the BAIE is positive, simply because if tax difference becomes too large, no interior equilibrium in output levels exists. This is confirmed in Section 5.

For country 2, the BAIE is unambiguously positive and would call for taxing emissions. This is in line with the argument presented in Helm et al. (2012) who argue that the country facing a tariff reacts by taxing its exports.

Proposition 2. *In our strategic trade model, BCAs on imports create a new incentive for the country on which tariffs are levied on its exports to raise its tax. For the country which imposes tariffs, the pressure to lower taxes in order to shift profits and to countervail leakage effects is reduced.*

Although introducing BCAs changes some incentives towards a higher carbon tax in both countries, considering the PSE, EDE and BAIE, we cannot compare optimal taxes across regimes at this level of generality. For instance, unlike under the PB-regime, it is not straightforward to conclude whether the optimal carbon taxes are set below or above individual marginal damages under the unilateral BCA-measures because we also have effects with opposite signs, e.g., the CSE calls for a larger subsidy than under the PB-regime. Consequently, any prediction about equilibrium taxes are also not possible, except, trivially, $t_1^{BI*} > t_2^{BI*}$, as otherwise country 1 would not be allowed to impose tariffs on imports from country 2 by assumption.

4.3 Border Carbon Adjustments on Imports and Exports

Adding BCAs on imports eliminates the difference in profits of firms in market 1. However, firm 1 still faces a competitive disadvantage in market 2. Therefore, we consider that country 1 will complement its import tax with an export rebate. Adding export rebates implies that firm 1 faces effective tax $t_1(1-\varphi)$ on its supply to market 2 with φ the rebate rate. We recall that we impose a constraint such that $t_1(1-\varphi) \geq t_2$. We consider two forms of BCAs on exports. The BIE-regime assumes that country 1 chooses the export rebate parameter $\hat{\varphi}$ optimally, which can be positive or negative and can be smaller or larger than 1, as explained in detail in Appendix A.5. The BF-regime assumes a full export rebate. Hence, the rebate rate is fixed at $\varphi = 1$.

4.3.1 BCAs on Imports with Optimal Export Rebate (BIE-regime)

Under the BIE-regime, the output levels from the second stage are $x_{11}^{BIE} = f_1(t_1, t_1) = x_{21}^{BIE} = g_1(t_1, t_1)$ in market 1, and $x_{12}^{BIE} = f_2(t_1, t_2, \varphi)$ and $x_{22}^{BIE} = g_{22}(t_1, t_2, \varphi)$ in market 2 (see Table 1). Let W_1^{BIE} and W_2^{BIE} be the welfare function of country 1 and country 2, respectively, under this regime.

We derive the structure of the optimal export rebate rate by maximising W_1^{BIE} with respect to φ:

$$\hat{\varphi} = 1 + \underbrace{\frac{p_2' x_{12} \frac{dx_{22}^{BIE}}{d\varphi}}{t_1 \frac{dx_{12}^{BIE}}{d\varphi}}}_{PSE_1(+)} + \underbrace{\frac{-\gamma\, D' \left(\frac{dx_{12}^{BIE}}{d\varphi} + \frac{dx_{22}^{BIE}}{d\varphi}\right)}{t_1 \frac{dx_{12}^{BIE}}{d\varphi}}}_{EDE_1(-)} \tag{20}$$

where the effect of φ on market 2 is given by:

$$\begin{aligned}
\frac{dx_{12}^{BIE}}{d\varphi} &= \frac{(-t_1)\ \left(2p_2' + p_2'' x_{22}\right)}{J_2} > 0\,, \\
\frac{dx_{22}^{BIE}}{d\varphi} &= -\frac{(-t_1)\ \left(p_2' + p_2'' x_{22}\right)}{J_2} < 0\,, \\
\frac{dX_2^{BIE}}{d\varphi} &= \frac{dx_{12}^{BIE}}{d\varphi} + \frac{dx_{22}^{BIE}}{d\varphi} = \frac{-t_1 p_2'}{J_2} > 0\,.
\end{aligned} \tag{21}$$

For simplicity, the signs in (20) and (21) as well as the subsequent interpretation assume positive values of t_1. See Appendix A.5 for further details and also for the case of negative values of t_1.[19]

The optimal export rebate rate depends on two opposing effects: the PSE, which calls for a large rebate rate to shift profits to firm 1 in market 2 and the EDE, which calls for a small rebate rate in order to reduce production and hence global emissions. From (20) it is evident that the optimal export rebate rate can be smaller than, equal or larger than a

[19]In the example, which we consider in Section 5, it will turn out that in equilibrium $t_1^{BIE*} > 0$.

full rebate, i.e., $\hat{\varphi} \lesseqgtr 1$. The larger the EDE compared to the PSE, the smaller will be the optimal export rebate rate. A full rebate, i.e., $\hat{\varphi} = 1$, implying a unilateral consumption-based carbon tax, is only optimal if the PSE is equal to the EDE. Subsequently, we will first analyse the case of an optimal export rebate, ignoring the possibility of $\hat{\varphi} = 1$ for simplicity, as this case is covered under the next regime below. We recall that the optimal rebate rate can be larger than a full rebate if the climate policy level of country 2 is a subsidy.

Maximising W_1^{BIE} and W_2^{BIE} with respect to the national tax levels delivers the optimal tax structure of country 1 and country 2 (see details in Appendix A.5):

$$\hat{t}_1^{BIE} = \underbrace{\frac{p_1^{'}\left(x_{11}+x_{21}\right)\frac{2p_1^{'}}{J_1}}{\Lambda_1^{BIE}}}_{CSE_1\ (-)} + \underbrace{\frac{-p_1^{'}x_{11}\frac{dx_{21}^{BIE}}{dt_1} - p_2^{'}x_{12}\frac{dx_{22}^{BIE}}{dt_1}}{\Lambda_1^{BIE}}}_{PSE_1(+,?)} + \underbrace{\frac{\gamma\, D^{'}\left(\frac{\partial e^{BIE}}{\partial t_1}\right)}{\Lambda_1^{BIE}}}_{EDE_1\ (?)} + \underbrace{\frac{-x_{21}+t_2\frac{dx_{21}^{BIE}}{dt_1}}{\Lambda_1^{BIE}}}_{BAIE_1\ (+,?)} \tag{22}$$

where $\Lambda_1^{BIE} = \frac{dx_{11}^{BIE}}{dt_1} + \frac{dx_{21}^{BIE}}{dt_1} + (1-\varphi)\frac{dx_{12}^{BIE}}{dt_1} < 0$[20], $\frac{\partial e^{BIE}}{\partial t_1} = \frac{\partial e_1^{BIE}}{\partial t_1} + \frac{\partial e_2^{BIE}}{\partial t_1} \lesseqgtr 0$, with $\underbrace{\frac{\partial e_1^{BIE}}{\partial t_1}}_{HEE_1} \lesseqgtr 0$, $\underbrace{\frac{\partial e_2^{BIE}}{\partial t_1}}_{FEE_1} \gtreqless 0$, $EDE_1 = \gamma\, D^{'} + \underbrace{\frac{\gamma\, D^{'}\left(\varphi\frac{dx_{12}^{BIE}}{dt_1} + \frac{dx_{22}^{BIE}}{dt_1}\right)}{\Lambda_1^{BIE}}}_{(?)}$ and the effect of t_1 on outputs is:

$$\begin{aligned} \frac{dx_{11}^{BIE}}{dt_1} &= \frac{dx_{21}^{BIE}}{dt_1} = \frac{p_1^{'}}{J_1} < 0, \\ \frac{dx_{12}^{BIE}}{dt_1} &= \frac{(1-\varphi)\left(2p_2^{'}+p_2^{''}x_{22}\right)}{J_2}\begin{cases} < 0 & if\,\varphi < 1 \\ > 0 & if\,\varphi > 1 \end{cases}, \\ \frac{dx_{22}^{BIE}}{dt_1} &= -\frac{(1-\varphi)\left(p_2^{'}+p_2^{''}x_{22}\right)}{J_2}\begin{cases} > 0 & if\,\varphi < 1 \\ < 0 & if\,\varphi > 1 \end{cases}. \end{aligned} \tag{23}$$

$$\hat{t}_2^{BIE} = \underbrace{\frac{p_2^{'}\left(x_{22}+x_{12}\right)\frac{p_2^{'}}{J_2}}{\Lambda_2^{BIE}}}_{CSE_2(-)} + \underbrace{\frac{-p_2^{'}x_{22}\frac{dx_{12}^{BIE}}{dt_2}}{\Lambda_2^{BIE}}}_{PSE_2(-)} + \underbrace{\frac{(1-\gamma)\, D^{'}\left(\frac{\partial e^{BIE}}{\partial t_2}\right)}{\Lambda_2^{BIE}}}_{EDE_2\ (+)} + \underbrace{\frac{-x_{21}}{\Lambda_2^{BIE}}}_{BAIE_2(+)} \tag{24}$$

where $\Lambda_2^{BIE} = \frac{dx_{22}^{BIE}}{dt_2} < 0$, $\frac{\partial e^{BIE}}{\partial t_2} = \frac{\partial e_1^{BIE}}{\partial t_2} + \frac{\partial e_2^{BIE}}{\partial t_2} < 0$, with $\underbrace{\frac{\partial e_2^{BIE}}{\partial t_2}}_{HEE_2} < 0$, $\underbrace{\frac{\partial e_1^{BIE}}{\partial t_2}}_{FEE_2} > 0$,

$EDE_2 = (1-\gamma)D^{'} + \underbrace{\frac{(1-\gamma)D^{'}\left(\frac{dx_{12}^{BIE}}{dt_2}\right)}{\Lambda_2^{BIE}}}_{(-)}$ and the effect of t_2 on outputs is similar to the

[20]We have: $\Lambda_1^{BIE} = \frac{2p_1^{'}}{J_1} + \frac{2p_2^{'}+p_2^{''}x_{22}}{J_2}\left(1-\varphi\right)^2 < 0$.

previous regime, namely:

$$\frac{dx_{21}^{BIE}}{dt_2} = \frac{dx_{11}^{BIE}}{dt_2} = 0\,,$$
$$\frac{dx_{22}^{BIE}}{dt_2} = \frac{2p_2^{'} + p_2^{''}x_{12}}{J_2} < 0\,,\ \frac{dx_{12}^{BIE}}{dt_2} = -\frac{p_2^{'} + p_2^{''}x_{12}}{J_2} > 0\,. \tag{25}$$

Adding BCAs on exports affects the profit-shifting incentive of country 1 in market 2, which is the second term in the PSE in (22). On the one hand, if $\hat{\varphi} < 1$, the profit-shifting incentive in market 2 is negative and would call for a subsidy similar to the BI-regime. However, a higher t_1 will now induce a smaller increase in the market share of firm 2 in market 2. Consequently, the incentive to shift profits by means of a low tax t_1 is reduced compared to the BI-regime. On the other hand, if $\hat{\varphi} > 1$, then the second term in the PSE is positive. Hence, the entire PSE is positive, which would call for a tax. Thus, export rebates reduces the pressure on the incentive of country 1 to shift profits and may even eliminate it. However, given the effective tax on exports is $t_{12} = (1 - \hat{\varphi})t_1$, $\hat{\varphi} > 1$ implies that firm 1 is overcompensated in its endeavour of competing in the foreign market.[21] In other words, high export rebates combined with high taxes benefits exports at the expensive of home sales of firm 1.

The impact of complementing tariffs with an optimal export rebate on the EDE is not straightforward because export rebates raise emissions of the home firm which may lead to a reverse leakage effect. That is, emissions released in country 1 may increase and those in country 2 decrease, which in the extreme case could even increase overall emissions. More specifically, we can distinguish between two cases.

First, if $\hat{\varphi} < 1$, a higher carbon tax in country 1 leads to a reduction in total emissions of the home firm. Hence, as usual, the HEE is negative. If the rebate rate is sufficiently large, the positive effect of t_1 on x_{22} is small so that it is most likely to be offset by the negative effect of t_1 on x_{21}, such that the total emissions of firm 2 decrease in t_1. Hence, the FEE (or carbon leakage effect) is also negative. Thus, we have $\partial e_1^{BIE}/\partial t_1 < 0$ and $\partial e_2^{BIE}/\partial t_1 < 0$, and the EDE could call for a tax larger than individual marginal damages, i.e., $EDE_1 > D_1^{'}$. In other words, adding export rebates may lead to an over-internalisation of individual damages in country 1 in the absence of other effects.

Second, if $\hat{\varphi} > 1$, a higher carbon tax in country 1 always reduces total emissions of the foreign country, i.e., the FEE is always negative. However, if $\hat{\varphi}$ is sufficiently large, the HEE could become positive. That is, we observe a leakage-shifting effect from country 2 to country 1 and not vice versa. That is, total emissions released by firm 1 could increase in t_1 due to a high export rebate. Global emissions could even increase in t_1 such that the EDE is negative.

Taken together, the observations about the EDE support the argument that adding export rebates may not address the environmental problem in that they reduce global emissions, at least if export rebates are very high. We note that this may well be combined with a high tax in country 1, signalling a high environmental concern, but the effective tax burden on the firm located in country may be much lower, at least in the export sector.

The structure of the optimal climate policy of country 2 does not change after adding export rebates, which is evident by comparing (18) with (24). However, as will be shown

[21] Overcompensating firms is not uncommon. For example, Martin et al. (2014) show that the free allocation of emission permits under the European Union Emissions Trading Scheme (EU-ETS) resulted in a sizeable overcompensation of emission-intensive industries.

later, its equilibrium tax may well change compared to the previous regimes due to the change of the best response of country 1.

4.3.2 BCAs on Imports with Full Export Rebate (BF-regime)

The only difference to the previous BIE-regime is that under the BF-regime the export rebate is set to $\varphi = 1$, which is equivalent to a unilateral consumption-based carbon policy. The output levels in the second stage are given by $x_{11}^{BF} = f_1(t_1, t_1) = x_{21}^{BF} = g_1(t_1, t_1)$ in market 1 and $x_{12}^{BF} = f_2(0, t_2)$ and $x_{22}^{BF} = g_2(0, t_2)$ in market 2.

Maximising the welfare functions of country 1 and 2, W_1^{BF} and W_2^{BF}, with respect to own national taxes gives the structure of optimal taxes (see Appendix A.6 for details):

$$\hat{t}_1^{BF} = \underbrace{\frac{p_1^{'}(x_{11}+x_{21})\frac{2p_1^{'}}{J_1}}{\Lambda_1^{BF}}}_{CSE_1\,(-)} + \underbrace{\frac{-p_1^{'}x_{11}\frac{dx_{21}^{BF}}{dt_1}}{\Lambda_1^{BF}}}_{PSE_1(+)} + \underbrace{\frac{\gamma\, D^{'}\left(\frac{\partial e^{BF}}{\partial t_1}\right)}{\Lambda_1^{BF}}}_{EDE_1\,(+)} + \underbrace{\frac{-x_{21}+t_2\frac{dx_{21}^{BF}}{dt_1}}{\Lambda_1^{BF}}}_{BAIE_1\,(+,?)}$$
$$\Leftrightarrow -p_1^{''}x_{21}x_{11} + \frac{1}{2}t_2 + \gamma\, D^{'} \tag{26}$$

where $\Lambda_1^{BF} = \frac{dx_{11}^{BF}}{dt_1} + \frac{dx_{21}^{BF}}{dt_1} < 0$, $\frac{\partial e^{BF}}{\partial t_1} < 0$, with $\underbrace{\frac{\partial e_1^{BF}}{\partial t_1}}_{HEE_1} = \underbrace{\frac{\partial e_2^{BF}}{\partial t_1}}_{FEE_1} < 0$, $EDE_1 = \gamma\, D^{'}$ and the effect of t_1 on both markets is:

$$\frac{dx_{11}^{BF}}{dt_1} = \frac{dx_{21}^{BF}}{dt_1} = \frac{p_1^{'}}{J_1} < 0\,,$$
$$\frac{dx_{12}^{BF}}{dt_1} = \frac{dx_{22}^{BF}}{dt_1} = 0\,. \tag{27}$$

$$\hat{t}_2^{BF} = \underbrace{\frac{p_2^{'}(x_{22}+x_{12})\frac{p_2^{'}}{J_2}}{\Lambda_2^{BF}}}_{CSE_2\,(-)} + \underbrace{\frac{-p_2^{'}x_{22}\frac{dx_{12}^{BF}}{dt_2}}{\Lambda_2^{BF}}}_{PSE_2\,(-)} + \underbrace{\frac{(1-\gamma)\;D^{'}\left(\frac{\partial e^{BF}}{\partial t_2}\right)}{\Lambda_2^{BF}}}_{EDE_2\,(+)} + \underbrace{\frac{-x_{21}}{\Lambda_2^{BF}}}_{BAIE_2\,(+)} \tag{28}$$

where $\Lambda_2^{BF} = \frac{dx_{22}^{BF}}{dt_2} < 0$, $\frac{\partial e^{BF}}{\partial t_2} < 0$, with $\underbrace{\frac{de_2^{BF}}{\partial t_2}}_{HEE_2} < 0$ and $\underbrace{\frac{de_1^{BF}}{\partial t_2}}_{FEE_2} > 0$, $EDE_2 = (1-\gamma)D^{'} + \underbrace{\frac{(1-\gamma)D^{'}\left(\frac{dx_{12}^{BF}}{dt_2}\right)}{\Lambda_2^{BF}}}_{(-)}$ and the effect of t_2 on outputs is similar to the previous two BCA-regimes, namely:

$$\frac{dx_{11}^{BF}}{dt_2} = \frac{dx_{21}^{BF}}{dt_2} = 0\,,$$
$$\frac{dx_{12}^{BF}}{dt_2} = -\frac{p_2^{'}+p_2^{''}x_{12}}{J_2} > 0\,,\ \frac{dx_{22}^{BF}}{dt_2} = \frac{2p_2^{'}+p_2^{''}x_{12}}{J_2} < 0\,. \tag{29}$$

Under the BF-regime, the tax of each country affects only the consumers in the home country. Hence, t_1 has no effect on consumers in country 2, $\partial CS_2/\partial t_1 = 0$, which raises the incentive of country 1 to subsidise its consumers. The PSE in country 1 comprises now only one term, which would call for a tax to raise the net profits of the home firm obtained in market 1, while the effect of t_1 on market 2 vanishes. Therefore, the profit-shifting incentive has disappeared. In addition, country 1 completely controls the emissions released in the process of supplying its home market 1, without being offset by larger emissions released in the process of supplying market 2, either by its home or the foreign firm. Therefore, $\partial e_1^{BF}/\partial t_1 = \partial e_2^{BF}/\partial t_1 = p_1^{'}/J_1 < 0$. That is, the HEE and the FEE are equal and negative. As a result, under this regime, the EDE in country 1 would call for a tax equal to its marginal damages, $EDE_1 = D_1^{'}$.

Considering all effects, the answer to the question whether the optimal carbon tax of country 1 is equal to its individual marginal damages, depends on the demand curve and the tax level of country 2. It is clear from (26) that if the demand curve was linear ($p_1^{''} = 0$) and if country 2 was passive ($t_2 = 0$), full BCAs would completely restore the effectiveness of the carbon tax of country 1. That is, its optimal tax would be equal to its individual marginal damages. However, if country 2 subsidises its production, $t_2 < 0$, country 1 would choose a tax below its marginal damage.

Proposition 3. *In our strategic trade model, a production-based carbon tax supplemented by full BCAs is de facto a unilateral consumption-based tax, which corrects for both, the profit-shifting and carbon leakage distortions. The environmental damage effect calls for a tax equal to individual marginal damages in the environmentally more concerned country 1. The optimal tax level $\hat{t}_1^{BF}$ is positively correlated to the production-based carbon tax of the environmentally less concerned country 2, t_2.*

The optimal tax structure of country 2 is similar to the two previous BCA-regimes, implying that the structure of the optimal climate policy of country 2 is similar under the three BCA-regimes. However, the equilibrium tax level may nevertheless be different as the best response of country 1 under the three BCA-regimes is different. This will become apparent in Section 5.

4.4 Bilateral Consumption-based Tax (CB-regime)

Finally, under the CB-regime, both countries impose a carbon tax on consumption, irrespective of the location of production. Therefore, the equilibrium output levels obtained from the second stage are $x_{11}^{CB} = f_1(t_1, t_1) = x_{21}^{CB} = g_1(t_1, t_1)$ for the supply of market 1 and $x_{12}^{CB} = f_2(t_2, t_2) = x_{22}^{CB} = g_2(t_2, t_2)$ for the supply of market 2. Let W_1^{CB} and W_2^{CB} denote the welfare function of country 1 and country 2 under the CB-regime.

Maximising W_1^{CB} and W_2^{CB} with respect to the own national tax leads to following optimal tax structure (see Appendix A.7.):

$$\hat{t}_1^{CB} = \underbrace{\frac{p_1^{'}(x_{11} + x_{21})\frac{2p_1^{'}}{J_1}}{\Lambda_1^{CB}}}_{CSE_1\,(-)} + \underbrace{\frac{-p_1^{'}x_{11}\frac{dx_{21}^{CB}}{dt_1} - x_{21}}{\Lambda_1^{CB}}}_{PSE_1\,(+)} + \underbrace{\frac{\gamma\, D^{'}\left(\frac{\partial e^{CB}}{\partial t_1}\right)}{\Lambda_1^{CB}}}_{EDE_1\,(+)}$$

$$\Leftrightarrow -\, p_1^{''}x_{21}x_{11} + \gamma\, D^{'} \tag{30}$$

where $\Lambda_1^{CB} = \frac{dx_{11}^{CB}}{dt_1} + \frac{dx_{21}^{CB}}{dt_1} < 0$, $\frac{\partial e^{CB}}{\partial t_1} < 0$, with $\underbrace{\frac{\partial e_1^{CB}}{\partial t_1}}_{HEE_1} = \underbrace{\frac{\partial e_2^{CB}}{\partial t_1}}_{FEE_1} < 0$, $EDE_1 = \gamma\, D^{'}$ and the effect of t_1 on outputs is the same as under the BF-regime, namely:

$$\begin{aligned} \frac{\partial x_{11}^{CB}}{\partial t_1} &= \frac{\partial x_{21}^{CB}}{\partial t_1} = \frac{p_1^{'}}{J_1} < 0\,, \\ \frac{dx_{12}^{CB}}{dt_1} &= \frac{dx_{22}^{CB}}{dt_1} = 0\,. \end{aligned} \tag{31}$$

$$\begin{aligned} \hat{t}_2^{CB} &= \underbrace{\frac{p_2^{'}\left(x_{22}+x_{12}\right)\frac{2p_2^{'}}{J_2}}{\Lambda_2^{CB}}}_{CSE_2\ (-)} + \underbrace{\frac{-p_2^{'}x_{22}\frac{dx_{12}^{CB}}{dt_2}-x_{12}}{\Lambda_2^{CB}}}_{PSE_2\ (+)} + \underbrace{\frac{(1-\gamma)\ D^{'}\left(\frac{\partial e^{CB}}{\partial t_2}\right)}{\Lambda_2^{CB}}}_{EDE_2\ (+)} \\ &\Leftrightarrow -p_2^{''}x_{12}x_{22} + (1-\gamma)\ D^{'} \end{aligned} \tag{32}$$

where $\Lambda_2^{CB} = \frac{dx_{22}}{dt_2} + \frac{dx_{12}}{dt_2} < 0$, $\frac{\partial e^{CB}}{\partial t_2} < 0$, with $\underbrace{\frac{\partial e_2^{CB}}{\partial t_2}}_{HEE_2} = \underbrace{\frac{\partial e_1^{CB}}{\partial t_2}}_{FEE_2} < 0$, $EDE_2 = (1-\gamma)D^{'}$ and the effect of t_2 on market 2 changes, while on market 1 it remains as under the previous BCA-regimes.

$$\begin{aligned} \frac{dx_{11}^{CB}}{dt_2} &= \frac{dx_{21}^{CB}}{dt_2} = 0\,, \\ \frac{\partial x_{22}^{CB}}{\partial t_2} &= \frac{\partial x_{12}^{CB}}{\partial t_2} = \frac{p_2^{'}}{J_2} < 0\,. \end{aligned} \tag{33}$$

Under the CB-regime, there are no consumer price spillovers. Hence, the CSE would call for a subsidy which efficiently internalises the distortions stemming from underproduction (Haufler et al., 2005). That is, the CSE under the CB-regime and in the social optimum are the same (see (11), (30) and (32)).[22] In addition, the profit-shifting incentive in both countries disappears and the PSE is positive, calling for taxing producers. The first term is the positive effect of a higher tax on the net profits of the home firm supplying the home market. The second term is also positive and reflects the incentive of each country to shift tax revenues, recalling that the tax base under this regime is now $x_{1i} + x_{2i}$. Therefore, de facto, the competitiveness issue disappears, as both firms share each market equally and profits of firms are equalised.

Furthermore, the HEE and the FEE are also equalised, implying that the climate policy in each country is effective in fully internalising own damages. That is, the EDE calls for consumption-based taxes which are equal to individual marginal damages, $EDE_i = D_i^{'}$ $\forall\, i = 1, 2$.

While bilateral production-based (PB) carbon taxes are always set below individual marginal damages (see Proposition 1), the optimal level of bilateral consumption-based (CB) taxes depends on the opposing incentives regarding consumers and producers. The net effect of these incentives in turn depends on the demand function as shown in Proposition 1 in Haufler et al. (2005). That is, if the demand function is concave (convex), the incentive to tax producers (subsidise consumers) dominates, while the two incentives cancel out in the case of a linear demand curve.

[22] The CSE in (11) is $2p_i^{'}x_{ki}$, and the CSE_1 in (30) is $2p_1^{'}x_{k1}$ given $x_{11} = x_{21}$. Similarly, the CSE_2 in (32) is $2p_2^{'}x_{k2}$ given $x_{12} = x_{22}$.

Proposition 4. *In our strategic trade model, the optimal (equilibrium) consumption-based carbon taxes are set: i) above individual marginal damages if the demand function is concave,* $\hat{t}_i^{CB}(t_i^{CB*}) > D_i'$ *if* $p_i'' < 0$. *ii) below individual marginal damages if the demand function is convex:* $\hat{t}_i^{CB}(t_i^{CB*}) < D_i'$ *if* $p_i'' > 0$. *iii) equal to individual marginal damages if the demand function is linear:* $\hat{t}_i^{CB}(t_i^{CB*}) = D_i'$ *if* $p_i'' = 0$, *with* $t_1^{CB*} > t_2^{CB*}$ *for all* $\gamma > 0.5$.

Clearly, the difference between a unilateral consumption-based tax in (26) under the BF-regime and a bilateral consumption-based tax in (30) under the CB-regime for country 1 is due to the effect of the foreign climate policy level t_2. Both taxes are identical if country 2 is passive, i.e., $\hat{t}_1^{BF} = \hat{t}_1^{CB}$ if $\hat{t}_2^{BF} = 0$.

To sum up, we showed that a gradual shift from bilateral production-based carbon tax, along a unilateral consumption-based carbon tax to a bilateral consumption-based tax can partially or completely correct distortions affecting the choice of carbon taxes. On the one hand, country 1's incentive to choose a low tax is reduced as the profit-shifting and carbon leakage effect are reduced or even disappear. On the other hand, the incentives of governments to subsidise consumers increases along this line. Thus, it is not possible to compare equilibrium carbon taxes at this level of generality. Therefore, in the next section, we complete our analysis using specific functions in order to rank the Nash equilibrium taxes and their impact on global emissions.

5 Comparison of Equilibrium Climate Policies across Regimes

We assume a quadratic utility function in x: $u_i(X_i) = aX_i - \frac{1}{2}X_i^2$: Consequently, the inverse demand function for each country i is given by:

$$p_i = a - X_i\,, \quad \forall\, i = 1, 2\,, \tag{34}$$

where $a > 0$ is the chock-off price.

We also consider a strictly convex global damage function, which is given by:

$$D(e) = \frac{1}{2}de^2\,, \tag{35}$$

where $d > 0$ is a global damage parameter.

We provide all details of solving the two stages in Appendix B, including the derivation of the range of feasible parameter values for interior solutions under all regimes and the non-violation of WTO-rules under the three unilateral BCA-regimes.

The comparisons of equilibrium carbon taxes and global emissions depend only on two parameter values of the model: the global damage parameter d, and the degree of asymmetry of damages among countries related to parameter γ.

Proposition 5. Non-Cooperative Equilibrium Carbon Taxes and Global Emissions

Let the superscript BCA refer to the three BCA-regimes: BI, BIE and BF.

1. *Taxes are ranked as follows:*

i. Unilateral BCA-regimes: country 1: $t_1^{BI*} < t_1^{BIE*}, t_1^{BF*}$ *with* $t_1^{BIE*} \leq (>)\, t_1^{BF*}$ *if* $\varphi^* \leq (>)\, 1$*; country 2:* $t_2^{BIE*}, t_2^{BF*} < t_2^{BI*}$ *with* $t_2^{BIE*} \geq (<)\, t_2^{BF*}$ *if* $\varphi^* \leq (>)\, 1$.

ii. Unilateral BCA-regimes vs bilateral production-based tax regime: country 1: $t_1^{PB*} < t_1^{BCA*}$ *; country 2:* $t_2^{PB*} < t_2^{BCA*}$ *if the damage evaluation in country 2 is not too large.*

iii. Unilateral BCA-tax regimes vs bilateral consumption-based tax regime: country 1: $t_1^{BCA*} <, > t_1^{CB*}$*; country 2:* $t_2^{BCA*} < t_2^{CB*}$.

iv. Bilateral production-based vs bilateral consumtpion-based tax regime: $t_i^{PB*} < t_i^{CB*}\ \forall\, i = 1, 2$.

2. *Global emissions are ranked as follows:*

   i. $e^{CB*} < e^{BCA*} < e^{PB*}$, *ii.* $e^{BI*} < e^{BIE*}$, *iii.* $e^{BI*} \gtrless e^{BF*}$ *and*

   iv. $e^{BIE*} \leq (>)\, e^{BF*}$ *if* $\varphi^* \leq (>) 1$.

Proof. See Appendix B.8. □

From results 1(i) and (ii), it is evident that the equilibrium carbon tax of country 1 increases gradually from the bilateral production-based tax to the unilateral BCA-regimes.[23] As shown in Section 4, BCA measures reduce the rent shifting incentive and mitigate carbon leakages in country 1. These two forces reduce the pressure on country 1 to adjust its taxes downward. Furthermore, carbon tariffs are a new source of governmental income for country 1. Adding export rebates was shown to provide country 1 with more control over carbon leakage and the competitiveness of its firm, which explains that taxes under the BIE- and BF-regime are higher than under the BI-regime. The ranking of carbon taxes across the BIE- and BF-regime depends on whether the optimal equilibrium export rebate φ^* is smaller or larger than a full rebate $\varphi = 1$. We recall from section 4.3.1 that $\varphi^* > 1$ if the profit-shifting effect is stronger than environmental damage effect.

It is important to note that the ranking of equilibrium taxes of country 1 is only identical to effective taxes which firm 1 faces on its supply to country 1 under all regimes, i.e., $t_{11} = t_1^*$ as shown in Table 1. This is different for the tax firm 1 faces on its supply to country 2, t_{12}, under the BIE- and BF-regime, which depends on the rebate rate. This is immediately evident under the BF-regime with a full rebate, $\varphi = 1$, for which $t_{12}^{BF*} = 0$. However, also under the BIE-regime, we find $t_{12}^{BIE*} < t_1^{BIE*}$ as $t_1^{BIE*} > 0$ and $\varphi^* > 0$ (see Appendix B.4), given that we have $t_{12}^{BIE*} = t_1^{BIE*}(1 - \varphi^*)$. That is, the effective tax is lower than the equilibrium tax due to export rebates.

From results 1(i) and (ii) also the strategic impact of BCAs on country 2's carbon tax can be deduced. Again, it is important to keep in mind that the equilibrium taxes of country 2 are only the effective taxes firm 2 faces in market 2, but the effective taxes in market 1 are only the equilibrium taxes of country 2 under the PB-regime ($t_{21} = t_2^*$), but are those of country 1 under the three unilateral BCA-regimes ($t_{21} = t_1^*$). Two observations are important.

First, unilateral BCA-measures by country 1 may induce country 2 to choose higher equilibrium taxes, provided it gives not too much weight to environmental damages in its welfare function (result 1(ii)).[24] Thus, country 1 can export environmental standards

[23] For a qualitatively similar result, see Hecht and Peters (2018).

[24] See Appendix B.8 for details. Note that if we assumed a linear damage function, country 2 always imposes higher equilibrium carbon taxes under the BCA-regimes than under the PB-regime.

to country 2 if this country is not very sensitive to environmental concerns. Viewed differently, as $t_{21} = t_1^*$, and t_1^* increases gradually when complementing the production-based tax by import tariffs and export rebates, one could also say that country 1 partly shifts the cost of reducing emissions from the home to the foreign firm through these trade measures.

Second, when country 1 increases its taxes when adding export rebates to import tariffs, then country 2 reacts by matching higher equilibrium taxes t_1^* with lower taxes t_2^* (which are the effective taxes its firm faces in its home market, i.e., $t_{22} = t_2^*$). Hence, the ranking in result 1(i) for country 2 is just the reverse of country 1. Thus, import tariffs only are better suited to induce country 2 to implement a higher carbon tax, whereas adding export rebates weakens this incentive.

Interestingly, result 1 (iii) also shows that moving from a unilateral to a bilateral consumption-based tax implies even higher carbon taxes in country 2.

Result 1(iv) suggests that both countries implement higher carbon taxes under the CB-regime than under the PB-regime. Although the incentive to subsidise consumers is lower under the PB-regime than under the CB-regime, the absence of the profit-shifting effect and the full internalisation of individual damages work in the opposite direction and are in fact stronger. For country 2, equilibrium taxes under the CB-regime are even always higher than under the unilateral BCA-regimes. From this country's perspective, under the BCA-regime it can only passively react when setting its tax, whereas under the CB-regime, it is on equal playing field with country 1.

The second result about global emissions shows that different forms of unilateral BCA-regimes are effective in reducing global emissions compared to a bilateral production-based carbon tax regime. However, although complementing carbon tariffs with export rebates is more powerful in eliminating carbon leakage and competitive concerns from country 1's perspective, a regime with export rebates (the BIE- and the BF-regime) does not necessarily rank best in terms of reducing global emissions. Adding export rebates does not only raise emissions of firm 1 from exports, but also weakens the positive strategic effect induced by carbon tariffs on equilibrium taxes in country 2 (see result 1(i)). Thus, besides the leakage-shifting effect, a strategic effect may also contribute to undermining the effectiveness of export rebates regarding reducing global emissions. Clearly, export rebates, if chosen optimally by country 1, always lead to higher global emissions if added to import tariffs (BI versus BIE-regime). Interestingly, a bilateral consumption-based regime (CB-regime) is the most effective non-cooperative regime in reducing global emissions.

Proposition 6. Cooperative vs Non-Cooperative Equilibrium Carbon Taxes and Global Emissions

Let the superscript BCA refer to the three BCA-regimes: BI, BIE and BF.

i. Bilateral production-based carbon taxes (global emissions) are always smaller (higher) than the socially optimal carbon tax (global emissions): $t_i^{PB*} < t^{S*}$ $(e^{PB*} > t^{S*}(e^{S*})\ \forall\, i = 1, 2$.

ii. Unilateral or bilateral consumption-based carbon taxes could be higher than the socially optimal tax if countries are sufficiently asymmetric and perceive environmental damages to be generally low. In such cases, global emissions can be ranked as: $e^{CB*} < e^{BCA*} < e^{S*}$.

Proof. See Appendix B.9. □

Proposition 6 shows that the PB-taxes are always set below the socially optimal tax. Hence, global emissions under the PB-regime are strictly higher than in the social optimum. However, surprisingly, under the BCA- and the CB-regime, taxes in both countries may exceed the socially optimal tax level, though this is not generally the case. A sufficient condition for both countries to set higher taxes than in the social optimum is that the individual and global marginal damage functions are not very steep (i.e., the value of parameter d is small) and countries are sufficiently asymmetric (i.e., the value of parameter γ is sufficiently large).[25] In such particular cases, if governments behave non-cooperatively, the incentive to tax foreign production and/or to avoid carbon tariffs may lead to a 'race to the top' in non-cooperative taxes. As a result, all forms and degrees of consumption-based carbon taxes could lead to lower global emissions than in the social optimum.

6 Conclusions

Non-cooperative climate policies, which regulate emissions by imposing a price on carbon through a production-based tax, raise concerns about carbon leakage and a loss of competitiveness of domestic industries. In strategic trade models, it has been shown that these two concerns distort environmental policies to be inefficiently lax . In this paper, we analysed the effect of moving from a bilateral production-based carbon tax to a bilateral consumption-based tax on mitigating these distortions. In order to single out effects, we considered various intermediate forms of partial and full unilateral consumption-based taxes. We operationalized consumption-based taxes through border carbon adjustments (BCAs) in the form of import tariffs and export rebates.

We considered the benchmark scenario of a fully cooperative solution and five non-cooperative policy regimes. First, a cooperative and a non-cooperative bilateral production-based carbon tax, which is imposed by each government on its home firm. The cooperative solution was referred to as the social optimum, the non-cooperative solution corresponded to the PB-regime. Second, we assumed that country 1, which gives a higher weight to environmental damages in our model than country 1, uses border carbon adjustments (BCAs) by implementing carbon tariffs on imports (BI-regime). Third, we considered two regimes where import tariffs are supplemented by export rebates. In the BIE-regime, the export rebate rate was chosen optimally (and hence strategically) and in the BF-regime the export rebate rate was fixed and was equivalent to a full rebate. Finally, we considered a bilateral consumption-based carbon tax (CB-regime), which we argued is equivalent to a bilateral BF-regime.

We first derived the general optimal tax structure for both countries. We showed that BCA-measures, import tariffs and export rebates, reduce some of the distortionary effects associated with non-cooperative production-based carbon taxes. The incentive to choose low carbon taxes is reduced through BCAs, where 1) the profit-shifting incentives are reduced or eliminated. 2) Carbon tariffs create a new incentive for both governments to tax emissions. For government 1 this is true because this raises its tariff revenue and for government because this protects its tax revenues. 3) Carbon leakage from country 1's perspective becomes less sever, and may even become negative if import tariffs are supplemented with an optimal export rebate, such that country 1 increases emissions more than country 2 decreases its emissions. Only BCAs on imports with a full export rebate could restore the effectiveness of country 1's carbon tax by fully internalising its

[25] The exact values of d and γ are provided in Appendix B.9.

own damages. However, there are also two effects which point in the opposite direction. BCAs increase the incentive in both countries to subsidise their consumers. Moreover, due to the strategic interaction among countries, government 2 responds to export rebates by reducing its tax and even choosing a subsidy level which, in equilibrium, could lead country 1 to set its tax below individual marginal damages. Only a bilateral consumption-based tax (CB-regime) implied that carbon taxes could be set equal or even above individual marginal damages.

Second, assuming a specific demand and damage cost function, we ranked equilibrium carbon taxes and global emissions across different regimes. We found that all forms of consumption-based regimes might allow both countries to set their carbon tax at a higher level than under the PB-regime. However, there are two effects which question the effectiveness of export rebates for the internalisation of global damages. First, although adding export rebates to import tariffs supports a higher carbon tax in country 1, the reverse is true for country 2. Therefore, adding export rebates weakens the positive policy spillover effect of carbon tariffs on the climate policy level of the country which faces BCAs (country 2). Second, export rebates lead to a carbon leakage-shifting effect from country 1 to country 2, which could even raise global emissions. Certainly, if country 1 can choose its export rebate strategically, equilibrium global emissions will be higher than if country 1 only implements an import tariff. However, a bilateral non-cooperative consumption-based tax reduces global equilibrium emissions even more than any of the unilateral BCA-measures. Our results also showed that the PB-taxes always fall short of the socially optimal tax, while both governments could impose non-cooperative carbon taxes under all other regimes above those in the social optimum. However, we argued that this overshooting is only the case if countries are highly asymmetric in their perception of environmental damages and both governments perceive environmental damages to be rather low.

Despite the fact that a bilateral consumption-based carbon tax eliminates carbon leakages and generates the lowest non-cooperative global emission level, an agreement would be needed among countries to switch to this regime. However, in the light of asymmetric interests, this might face a coordination problem, similar to switching from a non-cooperative to cooperative carbon tax regime, which may face objections by the environmentally less concerned government. Therefore, it might be expected that a shift to unilateral consumption-based taxes in the form of BCA-measures is more likely to come about in the near future.

Several extensions could be considered in future work. First, we considered in this paper the effect of different forms of BCAs on non-cooperative carbon taxes. However, one could also examine the effect of these measures on enforcing cooperation among countries, either in the spirit of threat to sanction non-compliance (e.g., Anouliés, 2015) or in the spirit of supporting climate coalitions (e.g., Al Khourdajie and Finus, 2020; Helm and Schmidt, 2015) . Second, we assume the location of firms to be fixed. Hence, we considered the effect of BCAs on one channel of carbon leakage, which is the relocation of production through trade. However, firms may completely close down and relocate their production facilities abroad to countries with less strict climate policies. Therefore, a possible extension is to allow for the endogenous choice of the location of firms along the lines proposed by Markusen et al. (1995), Hoel (1997) or (Richter et al., 2019).

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Appendix

A General Functions

A.1 The Second Stage

The second-order conditions of profit maximisation in eqs. (6) and (7) in the text are given by:

$\frac{\partial^2 \pi_{11}}{\partial x_{11}^2} = 2p_1^{'} + x_{11}p_1^{''} < 0$ and $\frac{\partial^2 \pi_{21}}{\partial x_{21}^2} = 2p_1^{'} + x_{21}p_1^{''} < 0$ as well as

$\frac{\partial^2 \pi_{12}}{\partial x_{12}^2} = 2p_2^{'} + x_{12}p_2^{''} < 0$ and $\frac{\partial^2 \pi_{22}}{\partial x_{22}^2} = 2p_2^{'} + x_{22}p_2^{''} < 0\,,$

which we assume to hold. Together with the condition imposed in the paper for goods to be strategic substitutes, this implies:

$$J_1 = \begin{bmatrix} \frac{\partial^2 \pi_{11}}{\partial x_{11}^2} & \frac{\partial^2 \pi_{11}}{\partial x_{11}\partial x_{21}} \\ \frac{\partial^2 \pi_{21}}{\partial x_{21}\partial x_{11}} & \frac{\partial^2 \pi_{21}}{\partial x_{21}^2} \end{bmatrix} = p_1^{'}(3p_1^{'} + p_1^{''}(x_{11} + x_{21})) > 0 \text{ for market 1 and}$$

$$J_2 = \begin{bmatrix} \frac{\partial^2 \pi_{12}}{\partial x_{12}^2} & \frac{\partial^2 \pi_{12}}{\partial x_{12}\partial x_{22}} \\ \frac{\partial^2 \pi_{22}}{\partial x_{22}\partial x_{12}} & \frac{\partial^2 \pi_{22}}{\partial x_{22}^2} \end{bmatrix} = p_2^{'}(3p_2^{'} + p_2^{''}(x_{12} + x_{22})) > 0 \text{ for market 2.}$$

The above conditions ensure the existence and uniqueness of a Nash equilibrium (Eichberger, 1993; Friedman, 1986) in the second stage. If those conditions hold globally, they are also sufficient for the Routh-Hurwitz stability condition to be satisfied (Brander and Spencer, 1985).

A.2 Social Optimum

From the second stage, both firms face a uniform tax to supply each market, i.e., $x_{1i}^S = f_i\left(t^S, t^S\right) = x_{2i}^S = g_i\left(t^S, t^S\right)$ for $i = 1, 2$. The first-order conditions of profit-maximisation in eq. (6) in the text for market 1 can be written as:

$\frac{\partial \pi_{11}}{\partial x_{11}} = p_1 + x_{11}p_1^{'} - c - t^S = 0$ & $\frac{\partial \pi_{21}}{\partial x_{21}} = p_1 + x_{21}p_1^{'} - c - t^S = 0\,.$

In order to derive the effect of t^S on the equilibrium output of firms, we totally differentiate the first-order conditions:

$\frac{\partial^2 \pi_{11}}{\partial x_{11}^2}dx_{11} + \frac{\partial^2 \pi_{11}}{\partial x_{11}\partial x_{21}}dx_{21} + \frac{\partial^2 \pi_{11}}{\partial x_{11}\partial t^S}dt^S = 0$ and

$\frac{\partial^2 \pi_{21}}{\partial x_{21}\partial x_{11}}dx_{11} + \frac{\partial^2 \pi_{21}}{\partial x_{21}^2}dx_{21} + \frac{\partial^2 \pi_{21}}{\partial x_{21}\partial t^S}dt^S = 0\,.$

From Appendix A.1 and by substituting $\frac{\partial^2 \pi_{11}}{\partial x_{11}\partial t^S} = \frac{\partial^2 \pi_{21}}{\partial x_{21}\partial t^S} = -1$, we obtain:

$$\begin{bmatrix} 2p_1^{'} + x_{11}p_1^{''} & p_1^{'} + x_{11}p_1^{''} \\ p_1^{'} + x_{21}p_1^{''} & 2p_1^{'} + x_{21}p_1^{''} \end{bmatrix} \begin{bmatrix} dx_{11} \\ dx_{21} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} dt^S\,.$$

By applying Cramer's rule, the effects of t^S on x_{11} and x_{21} are given by:

$\frac{dx_{11}}{dt^S} = \begin{bmatrix} 1 & p_1^{'} + x_{11}p_1^{''} \\ 1 & 2p_1^{'} + x_{21}p_i^{''} \end{bmatrix} / J_1 = \frac{p_1^{'}}{J_1} < 0$, given $x_{11} = x_{21}$ and $J_1 > 0$,

$\frac{dx_{21}}{dt^S} = \begin{bmatrix} 2p_1^{'} + x_{11}p_1^{''} & 1 \\ p_1^{'} + x_{21}p_1^{''} & 1 \end{bmatrix} / J_1 = \frac{p_1^{'}}{J_1} < 0$, given $x_{21} = x_{11}$ and $J_1 > 0$.

Following the same steps for market 2, we obtain $\frac{dx_{12}}{dt^S} = \frac{dx_{22}}{dt^S} = \frac{p_2^{'}}{J_2} < 0$.

The aggregate welfare function is given by:

$$W = u_1(X_1) - p_1X_1 + u_2(X_2) - p_2X_2 + \Pi_1 + \Pi_2 \\ + t^S(x_{11} + x_{12}) + t^S(x_{22} + x_{21}) + 2L - D_1(e) - D_2(e) \,. \tag{A.1}$$

Differentiation of eq. (A.1) with respect to t^S, gives:

$\frac{\partial CS_i}{\partial t^S} = u_i^{'}\left(\frac{dx_{1i}}{dt^S} + \frac{dx_{2i}}{dt^S}\right) - \left[p_i\left(\frac{dx_{1i}}{dt^S} + \frac{dx_{2i}}{dt^S}\right) + \left(p_i^{'}\left(\frac{dx_{1i}}{dt^S} + \frac{dx_{2i}}{dt^S}\right)(x_{1i} + x_{2i})\right)\right]$. From eq. (1) in the text, we have $p_i = u_i^{'}$ and therefore $\frac{\partial CS_i}{\partial t^S} = -p_i^{'}\left(\frac{dx_{1i}}{dt^S} + \frac{dx_{2i}}{dt^S}\right)(x_{1i} + x_{2i}) < 0$.

$$\begin{aligned} \frac{\partial \Pi_k}{\partial t^S} &= \frac{\partial \pi_{k1}}{\partial t^S} + \frac{\partial \pi_{k2}}{\partial t^S} \\ &= \left(p_1 - c - t^S\right)\frac{dx_{k1}}{dt^S} + x_{k1}\left[p_1^{'}\left(\frac{dx_{k1}}{dt^S} + \frac{dx_{\ell 1}}{dt^S}\right) - 1\right] \\ &+ \left(p_2 - c - t^S\right)\frac{dx_{k2}}{dt^S} + x_{k2}\left[p_2^{'}\left(\frac{dx_{k2}}{dt^S} + \frac{dx_{\ell 2}}{dt^S}\right) - 1\right]. \end{aligned}$$

The national carbon tax has three effects on the profits of the home firm: a change in production, a change in prices and a change of tax payments. The effect of taxes on prices is through the domestic sales and imports. In the cooperative solution, an increase in the socially optimal tax reduces both, the domestic and the foreign production. From eqs. (6) and (7) in the text, $\left(p_1 - c - t^S\right) = -x_{k1}p_1^{'}$ and, similarly, $\left(p_2 - c - t^S\right) = -x_{k2}p_2^{'}$. A reduction in domestic production and an increase in prices through domestic sales cancels out, $\left(p_i - c - t^S\right)\frac{\partial x_{ki}}{\partial t^S} + x_{ki}p_i^{'}\frac{\partial x_{ki}}{\partial t^S} = 0$. Therefore, the net effect is an increase in the market price due to a reduction of imports, $x_{ki}p_i^{'}\frac{\partial x_{\ell i}}{\partial t^S} > 0$, and the tax payments x_{ki}. Therefore, we have: $\frac{\partial \Pi_k}{\partial t^S} = p_1^{'}x_{k1}\frac{dx_{\ell 1}}{dt^S} + p_2^{'}x_{k2}\frac{dx_{\ell 2}}{dt^S} - x_{k1} - x_{k2}$,

$\frac{\partial TR_i}{\partial t^S} = t^S\left(\frac{dx_{k1}}{dt^S} + \frac{dx_{k2}}{dt^S}\right) + x_{k1} + x_{k2}$,

$\frac{\partial D_1}{t^S} + \frac{\partial D_2}{t^S} = \gamma D'\left(\frac{\partial e}{\partial X}\frac{dX}{dt^S}\right) + (1-\gamma)D'\left(\frac{\partial e}{\partial X}\frac{dX}{dt^S}\right) = D'\frac{dX}{dt^S}$ where $e = X = X_1 + X_2$ and $\frac{\partial e}{\partial X} = 1$, given that we normalise the emission output coefficient to 1.

Since both firms produce the same output, each firm divides its production for both markets equally and since we assume symmetric utility functions, $\frac{dX}{dt^S} = \frac{dX_1}{dt^S} + \frac{dX_2}{dt^S} < 0$.

Therefore, the first-order condition derived from maximising eq. (A.1) with respect to t^S can be written as:

$$\frac{\partial W}{\partial t^S} = -2p_i^{'}x_{ki}\left(\frac{dX}{dt^S}\right) + p_i^{'}x_{ki}\left(\frac{dX}{dt^S}\right) - D^{'}\left(\frac{dX}{dt^S}\right) + t^S\left(\frac{dX}{dt^S}\right) = 0\,. \tag{A.2}$$

Simplifying: $\frac{\partial W}{\partial t^S} = -p_i^{'}x_{ki}\left(\frac{dX}{dt^S}\right) + \left(t^S - D^{'}\right)\left(\frac{dX}{dt^S}\right) = 0$, leads to eq. (11) in the text.

A.3 PB-regime

The first-order conditions of profit-maximisation in eqs. (6) and (7) in the text can be written as:

$\frac{\partial \pi_{11}}{\partial x_{11}} = p_1 + x_{11} p_1^{'} - c - t_1 = 0$ & $\frac{\partial \pi_{21}}{\partial x_{21}} = p_1 + x_{21} p_1^{'} - c - t_2 = 0$ and

$\frac{\partial \pi_{12}}{\partial x_{12}} = p_2 + x_{12} p_2^{'} - c - t_1 = 0$ & $\frac{\partial \pi_{22}}{\partial x_{22}} = p_2 + x_{22} p_2^{'} - c - t_2 = 0$.

The effect of t_1 on production levels follows from total differentiation of the above functions. In market 1, we have:

$\frac{\partial^2 \pi_{11}}{\partial x_{11}^2} dx_{11} + \frac{\partial^2 \pi_{11}}{\partial x_{11} \partial x_{21}} dx_{21} + \frac{\partial^2 \pi_{11}}{\partial x_{11} \partial t_1} dt_1 = 0$ and

$\frac{\partial^2 \pi_{21}}{\partial x_{21} \partial x_{11}} dx_{11} + \frac{\partial^2 \pi_{21}}{\partial x_{21}^2} dx_{21} + \frac{\partial^2 \pi_{21}}{\partial x_{21} \partial t_1} dt_1 = 0$.

From Appendix A.1 and by substituting $\frac{\partial^2 \pi_{11}}{\partial x_{11} \partial t_1} = -1$ and $\frac{\partial^2 \pi_{21}}{\partial x_{21} \partial t_1} = 0$, we obtain:

$$\begin{bmatrix} 2p_1^{'} + x_{11} p_1^{''} & p_1^{'} + x_{11} p_1^{''} \\ p_1^{'} + x_{21} p_1^{''} & 2p_1^{'} + x_{21} p_1^{''} \end{bmatrix} \begin{bmatrix} dx_{11} \\ dx_{21} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} dt_1 ,$$

$$\frac{dx_{11}^{PB}}{dt_1} = \begin{bmatrix} 1 & p_1^{'} + x_{11} p_1^{''} \\ 0 & 2p_1^{'} + x_{21} p_1^{''} \end{bmatrix} / J_1 = \frac{2p_1^{'} + x_{21} p_1^{''}}{J_1} < 0,$$

$$\frac{dx_{21}^{PB}}{dt_1} = \begin{bmatrix} 2p_1^{'} + x_{11} p_1^{''} & 1 \\ p_1^{'} + x_{21} p_1^{''} & 0 \end{bmatrix} / J_1 = -\frac{p_1^{'} + x_{21} p_1^{''}}{J_1} > 0 \text{ and } \frac{dx_{11}^{PB}}{dt_1} + \frac{dx_{21}^{PB}}{dt_1} = \frac{p_1^{'}}{J_1} \text{ where } J_1 > 0.$$

In market 2, we have:

$\frac{\partial^2 \pi_{12}}{\partial x_{12}^2} dx_{12} + \frac{\partial^2 \pi_{12}}{\partial x_{12} \partial x_{22}} dx_{22} + \frac{\partial^2 \pi_{12}}{\partial x_{12} \partial t_1} dt_1 = 0$,

$\frac{\partial^2 \pi_{22}}{\partial x_{22} \partial x_{12}} dx_{12} + \frac{\partial^2 \pi_{22}}{\partial x_{22}^2} dx_{22} + \frac{\partial^2 \pi_{22}}{\partial x_{22} \partial t_1} dt_1 = 0$.

From Appendix A.1 and by substituting $\frac{\partial^2 \pi_{12}}{\partial x_{12} \partial t_1} = -1$ and $\frac{\partial^2 \pi_{22}}{\partial x_{22} \partial t_1} = 0$, we obtain:

$$\begin{bmatrix} 2p_2^{'} + x_{12} p_2^{''} & p_2^{'} + x_{12} p_2^{''} \\ p_2^{'} + x_{22} p_2^{''} & 2p_2^{'} + x_{22} p_2^{''} \end{bmatrix} \begin{bmatrix} dx_{12} \\ dx_{22} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} dt_1 ,$$

$$\frac{dx_{12}^{PB}}{dt_1} = \begin{bmatrix} 1 & p_2^{'} + x_{12} p_2^{''} \\ 0 & 2p_2^{'} + x_{22} p_2^{''} \end{bmatrix} / J_2 = \frac{2p_2^{'} + x_{22} p_2^{''}}{J_2} < 0,$$

$$\frac{dx_{22}^{PB}}{dt_1} = \begin{bmatrix} 2p_2^{'} + x_{12} p_2^{''} & 1 \\ p_2^{'} + x_{22} p_2^{''} & 0 \end{bmatrix} / J_2 = -\frac{p_2^{'} + x_{22} p_2^{''}}{J_2} > 0 \text{ and } \frac{dx_{12}^{PB}}{dt_1} + \frac{dx_{22}^{PB}}{dt_1} = \frac{p_2^{'}}{J_2} \text{ where } J_2 > 0.$$

These are the effects of t_1 on both markets, as given in eq. (13) in the text. The effect of t_2 on market 1 and 2 in eq. (15) in the text is obtained in a similar way.

The welfare function of each country is given by:

$$W_i^{PB} = u_i (X_i) - p_i X_i + \Pi_k + t_i (x_{k1} + x_{k2}) + L - D_i (e) . \tag{A.3}$$

Maximising W_1^{PB} with respect to t_1 leads to the following first-order condition:

$$\frac{\partial W_1^{PB}}{\partial t_1} = -p_1^{'}\left(\frac{dx_{11}^{PB}}{dt_1}+\frac{dx_{21}^{PB}}{dt_1}\right)(x_{11}+x_{21}) + p_1^{'}x_{11}\frac{dx_{21}^{PB}}{dt_1} + p_2^{'}x_{12}\frac{dx_{22}^{PB}}{dt_1}$$
$$+ t_1\left(\frac{dx_{11}^{PB}}{dt_1}+\frac{dx_{12}^{PB}}{dt_1}\right) - \gamma D^{'}\left(\frac{dx_{11}^{PB}}{dt_1}+\frac{dx_{12}^{PB}}{dt_1}+\frac{dx_{22}^{PB}}{dt_1}+\frac{dx_{21}^{PB}}{dt_1}\right) = 0\,, \quad \text{(A.4)}$$

which can be written as:

$-p_1^{'}\left(\frac{dx_{11}^{PB}}{dt_1}+\frac{dx_{21}^{PB}}{dt_1}\right)(x_{11}+x_{21}) + p_1^{'}x_{11}\frac{dx_{21}^{PB}}{dt_1} + p_2^{'}x_{12} + \left(t_1 - \gamma D^{'}\right)\left(\frac{dx_{11}^{PB}}{dt_1}+\frac{dx_{12}^{PB}}{dt_1}\right) - \gamma D^{'}\left(\frac{dx_{22}^{PB}}{dt_1}+\frac{dx_{21}^{PB}}{dt_1}\right) = 0\,,$
where $\frac{\partial CS_1}{\partial t_1} = -p_1^{'}\left(\frac{dx_{11}^{PB}}{dt_1}+\frac{dx_{21}^{PB}}{dt_1}\right)(x_{11}+x_{21}) < 0$, $\frac{\partial \Pi_1}{\partial t_1} = p_1^{'}x_{11}\frac{dx_{21}^{PB}}{dt_1} + p_2^{'}x_{12}\frac{dx_{22}^{PB}}{dt_1} - x_{11} - x_{12} < 0$, $\frac{\partial TR_1}{\partial t_1} = t_1\left(\frac{dx_{11}^{PB}}{dt_1}+\frac{dx_{12}^{PB}}{dt_1}\right) + x_{11} + x_{12}$, and $\frac{\partial D_1}{\partial t_1} = \gamma D^{'}\left(\frac{\partial e_1}{\partial t_1}+\frac{\partial e_2}{\partial t_2}\right) = \gamma D^{'}\left(\frac{\partial e_1}{\partial x_{11}}\frac{dx_{11}^{PB}}{dt_1}+\frac{\partial e_1}{\partial x_{12}}\frac{dx_{12}^{PB}}{dt_1}+\frac{\partial e_2}{\partial x_{22}}\frac{dx_{22}^{PB}}{dt_1}+\frac{\partial e_2}{\partial x_{21}}\frac{dx_{21}^{PB}}{dt_1}\right) = \gamma D^{'}\left(\frac{dx_{11}^{PB}}{dt_1}+\frac{dx_{12}^{PB}}{dt_1}+\frac{dx_{22}^{PB}}{dt_1}+\frac{dx_{21}^{PB}}{dt_1}\right) < 0\,.$

Having $\left(t_1 - \gamma D^{'}\right)\left(\frac{dx_{11}^{PB}}{dt_1}+\frac{dx_{12}^{PB}}{dt_1}\right) - \gamma D^{'}\left(\frac{dx_{22}^{PB}}{dt_1}+\frac{dx_{21}^{PB}}{dt_1}\right)$ as stated above, the EDE_1 in eq. (12) can be written as $\gamma\, D^{'} + \frac{\gamma\, D^{'}\left(\frac{dx_{22}^{PB}}{dt_1}+\frac{dx_{21}^{PB}}{dt_1}\right)}{\Lambda_1^{PB}}$.

Similarly, for country 2, we obtain:

$$\frac{\partial W_2^{PB}}{\partial t_2} = -p_2^{'}\left(\frac{dx_{22}^{PB}}{dt_2}+\frac{dx_{12}^{PB}}{dt_2}\right)(x_{22}+x_{12}) + p_2^{'}x_{22}\frac{dx_{12}^{PB}}{dt_2} + p_1^{'}x_{21}\frac{dx_{11}^{PB}}{dt_2}$$
$$+ t_2\left(\frac{dx_{22}^{PB}}{dt_2}+\frac{dx_{21}^{PB}}{dt_2}\right) - (1-\gamma)\, D^{'}\left(\frac{dx_{22}^{PB}}{dt_2}+\frac{dx_{21}^{PB}}{dt_2}+\frac{dx_{11}^{PB}}{dt_2}+\frac{dx_{12}^{PB}}{dt_2}\right) = 0\,, \quad \text{(A.5)}$$

with similar components as derived above for $\frac{\partial W_1^{PB}}{\partial t_1}$. Again, from eq. (A.5), we have $\left(t_2 - (1-\gamma)\, D^{'}\right)\left(\frac{dx_{22}^{PB}}{dt_2}+\frac{dx_{21}^{PB}}{dt_2}\right) - (1-\gamma)\, D^{'}\left(\frac{dx_{11}^{PB}}{dt_2}+\frac{dx_{12}^{PB}}{dt_2}\right)$ and, hence, $EDE_2 = (1-\gamma)\ D^{'} + \frac{(1-\gamma)\, D^{'}\left(\frac{dx_{11}^{PB}}{dt_2}+\frac{dx_{12}^{PB}}{dt_2}\right)}{\Lambda_2^{PB}}$ where the term in brackets is the FEE_2.

Solving $\frac{\partial W_i^{PB}}{\partial t_i}$ for t_i gives the optimal PB-tax for country 1 and country 2 in eqs. (12) and (14) in the text, respectively.

A.4 BI-regime

The first-order conditions of profit-maximisation in eqs. (6) and (7) in the text can be written as:
$\frac{\partial \pi_{11}}{\partial x_{11}} = p_1 + x_{11}p_1^{'} - c - t_1 = 0$ & $\frac{\partial \pi_{21}}{\partial x_{21}} = p_1 + x_{21}p_1^{'} - c - t_1 = 0\,,$

$\frac{\partial \pi_{12}}{\partial x_{12}} = p_2 + x_{12}p_2^{'} - c - t_1 = 0$ & $\frac{\partial \pi_{22}}{\partial x_{22}} = p_2 + x_{22}p_2^{'} - c - t_2 = 0\,.$

The effect of t_1 on market 1 follows from total differentiation of the first-order conditions. In market 1, we have:

$\frac{\partial^2 \pi_{11}}{\partial x_{11}^2} dx_{11} + \frac{\partial^2 \pi_{11}}{\partial x_{11}\partial x_{21}} dx_{21} + \frac{\partial^2 \pi_{11}}{\partial x_{11}\partial t_1} dt_1 = 0$ and

$\frac{\partial^2 \pi_{21}}{\partial x_{21}\partial x_{11}} dx_{11} + \frac{\partial^2 \pi_{21}}{\partial x_{21}^2} dx_{21} + \frac{\partial^2 \pi_{21}}{\partial x_{21}\partial t_1} dt_1 = 0\,.$

From Appendix A.1 and by substituting $\frac{\partial^2 \pi_{11}}{\partial x_{11}\partial t_1} = \frac{\partial^2 \pi_{21}}{\partial x_{21}\partial t_1} = -1$, we obtain:

$\begin{bmatrix} 2p_1^{'} + x_{11}p_1^{''} & p_1^{'} + x_{11}p_1^{''} \\ p_1^{'} + x_{21}p_1^{''} & 2p_1^{'} + x_{21}p_1^{''} \end{bmatrix} \begin{bmatrix} dx_{11} \\ dx_{21} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} dt_1,$

$\frac{dx_{11}^{BI}}{dt_1} = \begin{bmatrix} 1 & p_1^{'} + x_{11}p_1^{''} \\ 1 & 2p_1^{'} + x_{21}p_1^{''} \end{bmatrix} / J_1 = \frac{p_1^{'} + p_1^{''}(x_{21} - x_{11})}{J_1} = \frac{p_1^{'}}{J_1} < 0$, given $x_{11} = x_{21}$ and because both firms are identical and face the same effective tax.

$\frac{dx_{21}^{BI}}{dt_1} = \begin{bmatrix} 2p_1^{'} + x_{11}p_1^{''} & 1 \\ p_1^{'} + x_{21}p_1^{''} & 1 \end{bmatrix} / J_1 = \frac{p_1^{'} + p_1^{''}(x_{11} - x_{21})}{J_1} = \frac{p_1^{'}}{J_1} < 0$ and $\frac{dx_{11}^{BI}}{dt_1} + \frac{dx_{21}^{BI}}{dt_1} = \frac{2p_1^{'}}{J_1}$.

The effect of t_2 on market 1 is obviously zero. The effect of t_1 and t_2 on market 2 is the same as under the PB-regime.

The welfare function of country 1 and 2 under this regime are given by:

$$W_1^{BI} = u_1\left(X_1\right) - p_1 X_1 + \Pi_1 + L + t_1\left(x_{11} + x_{12}\right) + (t_1 - t_2)x_{21} - D_1\left(e\right)\,, \qquad \text{(A.6)}$$

$$W_2^{BI} = u_2\left(X_2\right) - p_2 X_2 + \Pi_2 + L + t_2\left(x_{22} + x_{21}\right) - D_2\left(e\right)\,. \qquad \text{(A.7)}$$

The first-order condition from maximising Eq. A.6 is given by:

$$\begin{aligned} \frac{\partial W_1^{BI}}{\partial t_1} &= -p_1^{'}\left(\frac{dx_{11}^{BI}}{dt_1} + \frac{dx_{21}^{BI}}{dt_1}\right)(x_{11} + x_{21}) + p_1^{'}x_{11}\frac{dx_{21}^{BI}}{dt_1} + p_2^{'}x_{12}\frac{dx_{22}^{BI}}{dt_1} \\ &+ t_1\left(\frac{dx_{11}^{BI}}{dt_1} + \frac{dx_{12}^{BI}}{dt_1} + \frac{dx_{21}^{BI}}{dt_1}\right) + x_{21} - t_2\frac{dx_{21}^{BI}}{dt_1} - \gamma D^{'}\frac{\partial e}{\partial t_1} = 0\,. \qquad \text{(A.8)} \end{aligned}$$

Rearranging terms, we have:

$\frac{\partial W_1^{BI}}{\partial t_1} = -p_1^{'}\left(\frac{dx_{11}^{BI}}{dt_1} + \frac{dx_{21}^{BI}}{dt_1}\right)(x_{11} + x_{21}) + p_1^{'}x_{11}\frac{dx_{21}^{BI}}{dt_1} + p_2^{'}x_{12}\frac{dx_{22}^{BI}}{dt_1} + x_{21} - t_2\frac{dx_{21}^{BI}}{dt_1} + \left(t_1 - \gamma D^{'}\right)\left(\frac{dx_{11}^{BI}}{dt_1} + \frac{dx_{12}^{BI}}{dt_1} + \frac{dx_{21}^{BI}}{dt_1}\right) - \gamma D^{'}\left(\frac{dx_{22}^{BI}}{dt_1}\right) = 0$ and solving for t_1 gives the optimal carbon tax in eq. (16) in the text.

From the above equation, the EDE_1 under this regime can be written as $\gamma\, D^{'} + \frac{\gamma\, D^{'}\left(\frac{dx_{22}^{BI}}{dt_1}\right)}{\Lambda_1^{BI}}$. The new component under this regime for country 1 is the BCAI term where $\frac{\partial BCAI}{\partial t_1} = t_1\frac{dx_{21}^{BI}}{dt_1} + x_{21} - t_2\frac{dx_{21}^{BI}}{dt_1}$.

Under this regime, we have $\frac{\partial \Pi_1}{\partial t_1} = \underbrace{p_1^{'}x_{11}\frac{dx_{21}^{BI}}{dt_1}}_{+} + \underbrace{p_2^{'}x_{12}\frac{dx_{22}^{BI}}{dt_1}}_{-} - x_{11} - x_{12}$. If the inverse demand curve is linear, the PSE is given by $-\frac{p_1^{'}p_2^{'}(x_{11} - x_{12})}{2p_1^{'} + 2p_2^{'}} > 0$, noting that $x_{11} > x_{12}$ and $t_1 > t_2$ because t_1 has a larger negative effect on x_{12} than on x_{11}, i.e.,

$\frac{dx_{11}^{BI}}{dt_1} = \frac{p_1^{'}}{J_1} < \frac{2p_2^{'}}{J_2} = \frac{dx_{12}^{BI}}{dt_1}$.

The first-order condition derived from maximisation of eq. (A.7) is given by:

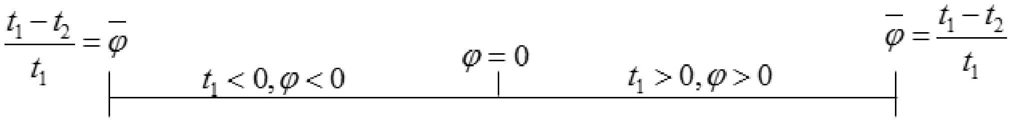
$$\frac{\partial W_2^{BI}}{\partial t_2} = -p_2^{'}\left(\frac{dx_{22}^{BI}}{dt_2} + \frac{dx_{12}^{BI}}{dt_2}\right)(x_{22} + x_{12}) + p_2^{'}x_{22}\frac{dx_{12}^{BI}}{dt_2} + t_2\left(\frac{dx_{22}^{BI}}{dt_2}\right) + x_{21} - (1-\gamma)\, D^{'}\frac{\partial e}{\partial t_2} = 0\,. \tag{A.9}$$

Rearranging terms, we have:

$\frac{\partial W_2^{BI}}{\partial t_2} = -p_2^{'}\left(\frac{dx_{22}^{BI}}{dt_2} + \frac{dx_{12}^{BI}}{dt_2}\right)(x_{22} + x_{12}) + p_2^{'}x_{22}\frac{dx_{12}^{BI}}{dt_2} + x_{21} + \left(t_2 - (1-\gamma)\, D^{'}\right)\left(\frac{dx_{22}^{BI}}{dt_2}\right) - (1-\gamma)\, D^{'}\left(\frac{dx_{12}^{BI}}{dt_2}\right) = 0$ and solving for t_2, gives the optimal tax in eq. (18) in the text where $\frac{dx_{11}^{BI}}{dt_2} = \frac{dx_{21}^{BI}}{dt_2} = 0$ and hence $\frac{\partial \pi_{21}}{\partial t_2} = 0$. Therefore, $\frac{\partial \Pi_2}{\partial t_2} = \frac{\partial \pi_{22}}{\partial t_2} = p_2^{'}x_{22}\frac{dx_{12}^{BI}}{dt_2} - x_{22}$. In addition, $\frac{\partial TR_2}{\partial t_2} = t_2\left(\frac{dx_{22}^{BI}}{dt_2}\right) + x_{22} + x_{21}$. Furthermore, from the above equation, it is clear that $EDE_2 = (1-\gamma)\; D^{'} + \frac{(1-\gamma)\, D^{'}\left(\frac{dx_{12}^{BI}}{dt_2}\right)}{\Lambda_2^{BI}}$.

A.5 BIE-regime

In models assuming imperfect competition, the equilibrium carbon tax can be positive or negative. Therefore, the feasible values of the rebate rate depends on the equilibrium policy in country 1 and 2. If $t_1 > 0$ and $\varphi > 0$, we have $0 < \varphi \leq \overline{\varphi} = \frac{t_1 - t_2}{t_1}$ where for the maximum allowable rebate rate we have $\overline{\varphi} \leq 1$ if $t_1 > t_2 \geq 0$, while $\overline{\varphi} > 1$ if $t_2 < 0$. However, if $0 > t_1 > t_2$, then $\varphi < 0$. In this case, the feasible values for φ are given by: $0 > \varphi \geq \overline{\varphi} = \frac{t_1 - t_2}{t_1}$. This can be illustrated as follows:



The first-order conditions of profit-maximisation in eqs. (6) and (7) in the text can be written as:

$\frac{\partial \pi_{11}}{\partial x_{11}} = p_1 + x_{11}p_1^{'} - c - t_1 = 0$ & $\frac{\partial \pi_{21}}{\partial x_{21}} = p_1 + x_{21}p_1^{'} - c - t_1 = 0\,,$

$\frac{\partial \pi_{12}}{\partial x_{12}} = p_2 + x_{12}p_2^{'} - c - t_1(1-\varphi) = 0$ & $\frac{\partial \pi_{22}}{\partial x_{22}} = p_2 + x_{22}p_2^{'} - c - t_2 = 0\,.$

The effect of t_1 on market 1 and the effect of t_2 on market 1 and 2 are the same as in the previous regime. The only change under this regime is the effect of t_1 on market 2:

$\frac{\partial^2 \pi_{12}}{\partial x_{12}^2}dx_{12} + \frac{\partial^2 \pi_{12}}{\partial x_{12}\partial x_{22}}dx_{22} + \frac{\partial^2 \pi_{12}}{\partial x_{12}\partial t_1}dt_1 = 0$ and

$\frac{\partial^2 \pi_{22}}{\partial x_{22}\partial x_{12}}dx_{12} + \frac{\partial^2 \pi_{22}}{\partial x_{22}^2}dx_{22} + \frac{\partial^2 \pi_{22}}{\partial x_{22}\partial t_1}dt_1 = 0\,.$

From Appendix A.1 and by substituting $\frac{\partial^2 \pi_{12}}{\partial x_{12}\partial t_1} = -(1-\varphi)$ and $\frac{\partial^2 \pi_{22}}{\partial x_{22}\partial t_1} = 0$, we obtain:

$$\begin{bmatrix} 2p_2' + x_{12}p_2'' & p_2' + x_{12}p_2'' \\ p_2' + x_{22}p_2'' & 2p_2' + x_{22}p_2'' \end{bmatrix} \begin{bmatrix} dx_{12} \\ dx_{22} \end{bmatrix} = \begin{bmatrix} (1-\varphi) \\ 0 \end{bmatrix} dt_1 \, ,$$

$$\frac{dx_{12}^{BIE}}{dt_1} = \begin{bmatrix} (1-\varphi) & p_2' + x_{12}p_2'' \\ 0 & 2p_2' + x_{22}p_2'' \end{bmatrix} / J_2 = \frac{(1-\varphi)\left(2p_2' + x_{22}p_2''\right)}{J_2} \text{ and}$$

$$\frac{dx_{22}^{BIE}}{dt_1} = \begin{bmatrix} 2p_2' + x_{12}p_2'' & (1-\varphi) \\ p_2' + x_{22}p_2'' & 0 \end{bmatrix} / J_2 = -\frac{(1-\varphi)\left(p_2' + x_{22}p_2''\right)}{J_2} \, .$$

In addition, the effects of the rebate rate on outputs are given by:

$$\frac{\partial^2 \pi_{12}}{\partial x_{12}^2} dx_{12} + \frac{\partial^2 \pi_{12}}{\partial x_{12} \partial x_{22}} dx_{22} + \frac{\partial^2 \pi_{12}}{\partial x_{12} \partial \varphi} d\varphi = 0 \text{ and}$$

$$\frac{\partial^2 \pi_{22}}{\partial x_{22} \partial x_{12}} dx_{12} + \frac{\partial^2 \pi_{22}}{\partial x_{22}^2} dx_{22} + \frac{\partial^2 \pi_{22}}{\partial x_{22} \partial \varphi} d\varphi = 0 \, .$$

From Appendix A.1 and by substituting $\frac{\partial^2 \pi_{12}}{\partial x_{12} \partial \varphi} = t_1$ and $\frac{\partial^2 \pi_{22}}{\partial x_{22} \partial \varphi} = 0$, we obtain:

$$\begin{bmatrix} 2p_2' + x_{12}p_2'' & p_2' + x_{12}p_2'' \\ p_2' + x_{22}p_2'' & 2p_2' + x_{22}p_2'' \end{bmatrix} \begin{bmatrix} dx_{12} \\ dx_{22} \end{bmatrix} = \begin{bmatrix} -t_1 \\ 0 \end{bmatrix} d\varphi \, ,$$

$$\frac{dx_{12}^{BIE}}{d\varphi} = \begin{bmatrix} -t_1 & p_2' + x_{12}p_2'' \\ 0 & 2p_2' + x_{22}p_2'' \end{bmatrix} / J_2 = \frac{(-t_1)\left(2p_2' + x_{22}p_2''\right)}{J_2} \begin{cases} > 0 & if \, t_1 > 0 \\ < 0 & if \, t_1 < 0 \end{cases} \text{ and}$$

$$\frac{dx_{22}^{BIE}}{d\varphi} = \begin{bmatrix} 2p_2' + x_{12}p_2'' & -t_1 \\ p_2' + x_{22}p_2'' & 0 \end{bmatrix} / J_2 = -\frac{(-t_1)\left(p_2' + x_{22}p_2''\right)}{J_2} \begin{cases} < 0 & if \, t_1 > 0 \\ > 0 & if \, t_1 < 0 \end{cases}$$

where $\frac{dX_2}{d\varphi} = \frac{dx_{12}^{BIE}}{d\varphi} + \frac{dx_{22}^{BIE}}{d\varphi} = \frac{-t_1 p_2'}{J_2} \begin{cases} > 0 & if \, t_1 > 0 \\ < 0 & if \, t_1 < 0 \end{cases}$.

Note that these effects can be explained in the case of a subsidy, i.e., $t_1 < 0$, as follows: if $t_1 < 0$ and $\varphi < 0$, then higher φ means a lower subsidy and the exports of firm 1 decrease. Hence, the sign of the PSE_1 and the EDE_1 in eq. (20) in the text will just be reversed if $t_1 < 0$.

The welfare function of country 1 and 2 are given by:

$$W_1^{BIE} = u_1(X_1) - p_1 X_1 + \Pi_1 + L + t_1(x_{11} + x_{12}) + (t_1 - t_2)x_{21} - \varphi t_1 x_{12} - D_1(e) \quad \text{(A.10)}$$

and

$$W_2^{BIE} = u_2(X_2) - p_2 X_2 + \Pi_2 + L + t_2(x_{22} + x_{21}) - D_2(e) \, . \quad \text{(A.11)}$$

First, we derive the optimal rebate rate by maximising eq. (A.10) with respect to φ:

$$\frac{\partial W_1^{BIE}}{\partial \varphi} = \frac{\partial \pi_{12}}{\partial \varphi} + \frac{\partial TR_1}{\partial \varphi} - \frac{\partial BCAE}{\partial \varphi} - \frac{\partial D_1}{\partial \varphi} = 0 \, \text{, which can be written as:}$$

$$\frac{\partial W_1^{BIE}}{\partial \varphi} = p_2' x_{12} \frac{dx_{22}^{BIE}}{d\varphi} + t_1 x_{12} + t_1 \frac{dx_{12}^{BIE}}{d\varphi} - \varphi t_1 \frac{dx_{12}^{BIE}}{d\varphi} - t_1 x_{12} - \gamma D' \left(\frac{dx_{12}^{BIE}}{d\varphi} + \frac{dx_{22}^{BIE}}{d\varphi} \right) = 0 \, .$$

Solving the above first-order condition for φ leads to the optimal export rebate rate derived in eq. (20) in the text, where $\frac{\partial \pi_{12}}{\partial \varphi} = (p_2 - c - t_1 + \varphi t_1)\left(\frac{dx_{12}}{d\varphi}\right) + x_{12}\left(p_2'\left(\frac{dx_{12}}{d\varphi} + \frac{dx_{22}}{d\varphi}\right) + t_1\right)$. From the first-order condition, we have $p_2 - c - t_1 + \varphi t_1 = -x_{12}p_2'$. Therefore, $\frac{\partial \pi_{12}}{\partial \varphi} = x_{12}p_2' \frac{dx_{22}}{d\varphi} + t_1 x_{12}$, $\frac{\partial TR_1}{\partial \varphi} = t_1 \frac{dx_{12}}{d\varphi}$ and $\frac{\partial BCAE}{\partial \varphi} = \varphi t_1 \frac{dx_{12}}{d\varphi} + t_1 x_{12}$.

We now derive the optimal taxes of countries.

The first-order condition of eq. (A.10) is given by:

$$\frac{\partial W_1^{BIE}}{\partial t_1} = -p_1^{'}\left(\frac{dx_{11}^{BIE}}{dt_1} + \frac{dx_{21}^{BIE}}{dt_1}\right)(x_{11}+x_{21}) + p_1^{'}x_{11}\frac{dx_{21}^{BIE}}{dt_1} + p_2^{'}x_{12}\frac{dx_{22}^{BIE}}{dt_1}$$
$$+ t_1\left(\frac{dx_{11}}{dt_1} + \frac{dx_{21}}{dt_1} + (1-\varphi)\frac{dx_{12}}{dt_1}\right) + x_{21} - t_2\frac{dx_{21}^{BIE}}{dt_1} - \gamma D^{'}\frac{\partial e}{\partial t_1} = 0\,. \quad \text{(A.12)}$$

Rearranging terms leads to:
$\frac{\partial W_1^{BIE}}{\partial t_1} = -p_1^{'}\left(\frac{dx_{11}^{BIE}}{dt_1} + \frac{dx_{21}^{BIE}}{dt_1}\right)(x_{11}+x_{21}) + p_1^{'}x_{11}\frac{dx_{21}^{BIE}}{dt_1} + p_2^{'}x_{12}\frac{dx_{22}^{BIE}}{dt_1} + x_{21} - t_2\frac{dx_{21}^{BIE}}{dt_1} + (t_1 - \gamma D^{'})\left(\frac{dx_{11}^{BIE}}{dt_1} + \frac{dx_{21}^{BIE}}{dt_1} + (1-\varphi)\frac{dx_{12}^{BIE}}{dt_1}\right) - \gamma D^{'}\left(\varphi\frac{dx_{12}^{BIE}}{dt_1} + \frac{dx_{22}^{BIE}}{dt_1}\right) = 0\,.$

Therefore, from the above equation, the EDE_1 can be written as $\gamma D' + \frac{\gamma D'\left(\varphi\frac{dx_{12}^{BIE}}{dt_1} + \frac{dx_{22}^{BIE}}{dt_1}\right)}{\Lambda_1^{BIE}}$. The new component in the welfare function of country 1 under this regime is the BCAE term: $\frac{\partial BCAE}{\partial t_1} = \varphi t_1\frac{dx_{12}}{dt_1} + \varphi x_{12}\,.$

Solving the above condition for t_1 leads to the optimal tax in eq. (22) in the text, where the effect of t_1 on the profits of firm 1 are: $\frac{\partial \Pi_1}{\partial t_1} = p_1^{'}x_{11}\frac{dx_{21}^{BIE}}{dt_1} + p_2^{'}x_{12}\frac{dx_{22}^{BIE}}{dt_1} - x_{11} - x_{12} + \varphi x_{12}$.

The differentiation of country 2's welfare function with respect to its tax is the same as in the previous regime in eq. (A.9). Hence, the formula of the optimal tax in eq. (24) is the same as in eq. (18) in the text.

A.6 BF-regime

The first-order conditions of profit-maximisation in eqs. (6) and (7) in the text can be written as:

$\frac{\partial \pi_{11}}{\partial x_{11}} = p_1 + x_{11}p_1^{'} - c - t_1 = 0$ & $\frac{\partial \pi_{21}}{\partial x_{21}} = p_1 + x_{21}p_1^{'} - c - t_1 = 0$ and

$\frac{\partial \pi_{12}}{\partial x_{12}} = p_2 + x_{12}p_2^{'} - c = 0$ & $\frac{\partial \pi_{22}}{\partial x_{22}} = p_2 + x_{22}p_2^{'} - c - t_2 = 0$.

The effect of t_1 on market 1 and the effect of t_2 on market 1 and market 2 are the same as in the previous regimes. However, the effect of t_1 on market 2 is now:

$\frac{\partial^2 \pi_{12}}{\partial x_{12}^2}dx_{12} + \frac{\partial^2 \pi_{12}}{\partial x_{12}\partial x_{22}}dx_{22} + \frac{\partial^2 \pi_{12}}{\partial x_{12}\partial t_1}dt_1 = 0$ and

$\frac{\partial^2 \pi_{22}}{\partial x_{22}\partial x_{12}}dx_{12} + \frac{\partial^2 \pi_{22}}{\partial x_{22}^2}dx_{22} + \frac{\partial^2 \pi_{22}}{\partial x_{22}\partial t_1}dt_1 = 0\,.$

From Appendix A.1 and by substituting $\frac{\partial^2 \pi_{12}}{\partial x_{12}\partial t_1} = 0$ and $\frac{\partial^2 \pi_{22}}{\partial x_{22}\partial t_1} = 0$, we obtain:

$$\begin{bmatrix} 2p_2^{'} + x_{12}p_2^{''} & p_2^{'} + x_{12}p_2^{''} \\ p_2^{'} + x_{22}p_2^{''} & 2p_2^{'} + x_{22}p_2^{''} \end{bmatrix}\begin{bmatrix} dx_{12} \\ dx_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} dt_1\,,$$

$\frac{dx_{12}^{BF}}{dt_1} = \begin{bmatrix} 0 & p_2^{'} + x_{12}p_2^{''} \\ 0 & 2p_2^{'} + x_{22}p_2^{''} \end{bmatrix} / J_2 = 0$ and

$\frac{dx_{22}^{BF}}{dt_1} = \begin{bmatrix} 2p_2^{'} + x_{12}p_2^{''} & 0 \\ p_2^{'} + x_{22}p_2^{''} & 0 \end{bmatrix} / J_2 = 0\,.$

The welfare function of country 1 and 2 are given by:

$$W_1^{BF} = u_1(X_1) - p_1 X_1 + \Pi_1 + L + t_1(x_{11} + x_{12}) + (t_1 - t_2)x_{21} - t_1 x_{12} - D_1(e) \quad \text{(A.13)}$$

and

$$W_2^{BF} = u_2(X_2) - p_2 X_2 + \Pi_2 + L + t_2(x_{22} + x_{21}) - D_2(e)\,. \quad \text{(A.14)}$$

Maximising eq. (A.13) with respect to t_1 leads to the following first-order condition:

$$\frac{\partial W_1^{BF}}{\partial t_1} = -p_1^{'}\left(\frac{dx_{11}^{BF}}{dt_1} + \frac{dx_{21}^{BF}}{dt_1}\right)(x_{11} + x_{21}) + p_1^{'} x_{11}\frac{dx_{21}^{BF}}{dt_1} + x_{21} - t_2\frac{dx_{21}}{dt_1} + \left(t_1 - \gamma D^{'}\right)\left(\frac{dx_{11}^{BF}}{dt_1} + \frac{dx_{21}^{BF}}{dt_1}\right) = 0\,. \quad \text{(A.15)}$$

Solving this first-order condition for t_1 gives the optimal carbon tax of country 1 in eq. (26) in the text where $\frac{\partial \pi_{12}}{\partial t_1} = 0$, $\frac{\partial \Pi_1}{\partial t_1} = \frac{\partial \pi_{11}}{\partial t_1} = p_1^{'} x_{11}\frac{dx_{21}^{BF}}{dt_1} - x_{11}$, $\frac{\partial TR_1}{\partial t_1} = t_1\left(\frac{dx_{11}^{BF}}{dt_1}\right) + x_{11} + x_{12}$ and $\frac{\partial BCAE}{\partial t_1} = x_{12}$, noting $\frac{\partial TR_1}{\partial t_1} - \frac{\partial BCAE}{\partial t_1} = t_1\left(\frac{dx_{11}}{dt_1}\right) + x_{11}$.

Simplifying the optimal tax structure gives:
$\hat{t}_1^{BF} = p_1^{'} x_{21} + \frac{p_1^{'} x_{11}\left(p_1^{'}/J_1\right)}{\left(2p_1^{'}/J_1\right)} - \frac{x_{21}}{\left(2p_1^{'}/J_1\right)} + \frac{t_2\left(p_1^{'}/J_1\right)}{\left(2p_1^{'}/J_1\right)} + \gamma D^{'}$, which leads to the simplified formula in eq. (26) in the text. Under this regime, $EDE_1 = \gamma D^{'}$ which follows directly from eq. (A.15).

The first-order condition of country 2 is the same as under the previous BCA-regimes and, hence, also the optimal tax structure.

A.7 CB-regime

The first-order conditions of profit-maximisation in eqs. (6) and (7) in the text can be written as:

$\frac{\partial \pi_{11}}{\partial x_{11}} = p_1 + x_{11}p_1^{'} - c - t_1 = 0$ & $\frac{\partial \pi_{21}}{\partial x_{21}} = p_1 + x_{21}p_1^{'} - c - t_1 = 0\,,$

$\frac{\partial \pi_{12}}{\partial x_{12}} = p_2 + x_{12}p_2^{'} - c - t_2 = 0$ & $\frac{\partial \pi_{22}}{\partial x_{22}} = p_2 + x_{22}p_2^{'} - c - t_2 = 0\,.$

The effect of t_1 on both markets are similar to the previous regime, thus the change is the effect of t_2 on market 2:

$\frac{\partial^2 \pi_{12}}{\partial x_{12}^2}dx_{12} + \frac{\partial^2 \pi_{12}}{\partial x_{12}\partial x_{22}}dx_{22} + \frac{\partial^2 \pi_{12}}{\partial x_{12}\partial t_2}dt_2 = 0$ and

$\frac{\partial^2 \pi_{22}}{\partial x_{22}\partial x_{12}}dx_{12} + \frac{\partial^2 \pi_{22}}{\partial x_{22}^2}dx_{22} + \frac{\partial^2 \pi_{22}}{\partial x_{22}\partial t_2}dt_2 = 0\,.$

From Appendix A.1 and by substituting $\frac{\partial^2 \pi_{12}}{\partial x_{12}\partial t_2} = -1$ and $\frac{\partial^2 \pi_{22}}{\partial x_{22}\partial t_2} = -1$, we obtain:

$$\begin{bmatrix} 2p_2^{'} + x_{12}p_2^{''} & p_2^{'} + x_{12}p_2^{''} \\ p_2^{'} + x_{22}p_2^{''} & 2p_2^{'} + x_{22}p_2^{''} \end{bmatrix}\begin{bmatrix} dx_{12} \\ dx_{22} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} dt_2\,,$$

$$\frac{dx_{12}^{CB}}{dt_2} = \begin{bmatrix} 1 & p_2^{'} + x_{12}p_2^{''} \\ 1 & 2p_2^{'} + x_{22}p_2^{''} \end{bmatrix} / J_2 = \frac{p_2^{'} + p_2^{''}(x_{22} - x_{12})}{J_2} = \frac{p_2^{'}}{J_2} < 0\,,$$

$\frac{dx_{22}^{CB}}{dt_2} = \left[\begin{array}{cc} 2p_2^{'} + x_{12}p_2^{''} & 1 \\ p_2^{'} + x_{22}p_2^{''} & 1 \end{array} \right] / J_2 = \frac{p_2^{'}+p_2^{''}(x_{12}-x_{22})}{J_2} = \frac{p_2^{'}}{J_2} < 0$, and $\frac{dx_{12}^{CB}}{dt_2} + \frac{dx_{22}^{CB}}{dt_2} = \frac{2p_2^{'}}{J_2} < 0$.

The welfare functions of country 1 and 2 under this regime are given by:

$$W_1^{CB} = u_1\left(X_1\right) - p_1X_1 + \Pi_1 + L + t_1\left(x_{11} + x_{21}\right) - D_1\left(e\right) , \tag{A.16}$$

$$W_2^{CB} = u_2\left(X_2\right) - p_2X_2 + \Pi_2 + L + t_2\left(x_{22} + x_{12}\right) - D_2\left(e\right) . \tag{A.17}$$

Maximising eq. (A.16) with respect to t_1 gives:

$$\begin{aligned} \frac{\partial W_1^{CB}}{\partial t_1} &= -p_1^{'}\left(\frac{dx_{11}}{dt_1} + \frac{dx_{21}}{dt_1}\right)\left(x_{11} + x_{21}\right) + p_1^{'}x_{11}\frac{dx_{21}}{dt_1} \\ &\quad + \left(t_1 - \gamma D^{'}\right)\left(\frac{dx_{11}}{dt_1} + \frac{dx_{21}}{dt_1}\right) + x_{21} = 0\,. \end{aligned} \tag{A.18}$$

Similarly, maximising eq. (A.17) with respect to t_2, delivers:

$$\begin{aligned} \frac{\partial W_2^{CB}}{\partial t_2} &= -p_2^{'}\left(\frac{dx_{12}}{dt_2} + \frac{dx_{22}}{dt_2}\right)\left(x_{12} + x_{22}\right) + p_2^{'}x_{22}\frac{dx_{12}}{dt_2} \\ &\quad + \left(t_2 - \left(1-\gamma\right)D^{'}\right)\left(\frac{dx_{12}}{dt_2} + \frac{dx_{22}}{dt_2}\right) + x_{12} = 0\,. \end{aligned} \tag{A.19}$$

Solving the above first-order condition for t_i, gives the optimal CB-tax of country 1 and country 2 in eqs. (30) and (32) in the text, respectively. The simplified formula can be obtained in the same manner as under the BF-regime as outlined in the previous appendix, even though the term $\frac{t_2\left(p_1^{'}/J_1\right)}{\left(2p_1^{'}/J_1\right)}$ disappears because country 2 does not tax its exports under the CB-regime.

B Specific Functions

From the linear demand function in eq. (34) in Section 5, we have $p_i^{'} = -1$ and $p_i^{''} = 0$. Hence, $J_i = 3$. For the damage function in eq. (35) in the text, we have $D' = de$.

The outcome of the second stage is given by:

$$x_{1i}^* = \frac{A - 2t_{1i} + t_{2i}}{3} \;\&\; x_{2i}^* = \frac{A - 2t_{2i} + t_{1i}}{3} \;\forall\; i = 1, 2\,, \tag{B.1}$$

with $A := a - c > 0$, which we interpret as a market size, or, as a proxy of the net benefits of production and consumption.
In the first stage, taking the above equilibrium output levels as a given, the individual welfare functions are given by:

$$\begin{aligned} W_1 = &\underbrace{\frac{(x_{11}+x_{21})^2}{2}}_{CS_1} + \underbrace{(x_{11})^2 + (x_{12})^2}_{PS_1} + \underbrace{t_1(x_{11}+x_{12})}_{TR_1} + L - \underbrace{\gamma d\frac{e^2}{2}}_{D_1} \\ &+ \underbrace{(t_1 - t_2)x_{21}}_{BCAI_1} - \underbrace{\varphi t_1 x_{12}}_{BCAE_1}\,, \end{aligned} \tag{B.2}$$

$$W_2 = \underbrace{\frac{(x_{12}+x_{22})^2}{2}}_{CS_2} + \underbrace{(x_{21})^2 + (x_{22})^2}_{PS_2} + \underbrace{t_2(x_{21}+x_{22})}_{TR_2} + L - \underbrace{(1-\gamma)\, d\frac{e^2}{2}}_{D_2}\,. \tag{B.3}$$

Note that under the CB-regime, $TR_i = t_i\,(x_{1i} + x_{2i})$.

Countries choose their tax levels cooperatively (in the social optimum) or non-cooperatively under the other regimes as explained in the text. The equilibrium tax levels are then inserted in the equilibrium output levels in eq. (B.1). We need to consider two parameter constraints. First, a non-negativity constraint (NN-constraint), which ensures positive production levels by both firms in both markets. This constraint implies that the damage parameter d cannot be too large, i.e., $d < \overline{d}\,(\gamma)$. Second, as we assume that BCA-measures are imposed by country 1, a BCA-constraint is needed to ensure that $t_1 > t_2$ and $t_1(1-\varphi) \geq t_2$. The BCA-constraint implies that the damage parameter d cannot be too small, i.e., $d > \underline{d}\,(\gamma)$. All constraints are summarised in Appendix B.7.

B.1 Social Optimum

Inserting the effective tax t^S into eq. (B.1) gives equilibrium outputs $x_{ki} = \frac{A-t^S}{3}$. Inserting equilibrium outputs into the aggregate welfare function $W^S = W_1 + W_2$ (see eq. (A.1)) implies the following first and second order conditions:

$\frac{\partial W^S}{\partial t^S} = \frac{16d(A-t^S)-8t^S-4A}{9} = 0$, and $\frac{\partial^2 W}{\partial t^{2S}} = -\frac{8}{9}\,(1+2d) < 0$.

Solving the first-order condition for t^S leads to:

$$t^{S*} = \frac{A\,(4d-1)}{2+4d}\,. \tag{B.4}$$

Consequently, $x_{ki}^{S*} = A/\,(2+4d)$. Hence, no NN-constraint needs to be imposed.

B.2 PB-regime

Using Table 1 in the text and inserting effective taxes into eq. (B.1), we obtain equilibrium outputs: $x_{11}^{PB} = x_{12}^{PB} = \frac{A-2t_1+t_2}{3}$ and $x_{22}^{PB} = x_{21}^{PB} = \frac{A-2t_2+t_1}{3}$. Inserting outputs into eq. (B.2) and eq. (B.3), the first-order conditions are given by:

$\frac{\partial W_1^{PB}}{\partial t_1} = \frac{1}{9}\left(8\gamma d\left(A - \frac{t_1}{2} - \frac{t_2}{2}\right) - 4A - 7t_1 - t_2\right) = 0$ and

$\frac{\partial W_2^{PB}}{\partial t_2} = \frac{1}{9}\left(8\,(1-\gamma)\,d\left(A - \frac{t_1}{2} - \frac{t_2}{2}\right) - 4A - 7t_2 - t_1\right) = 0.$

The second-order conditions are satisfied because: $\frac{\partial^2 W_1}{\partial {t_1}^2} = -\frac{7}{9} - \frac{4\gamma d}{9} < 0$ and $\frac{\partial^2 W_2}{\partial {t_2}^2} = -\frac{7}{9} - \frac{4(1-\gamma)d}{9} < 0$. Moreover, we have $\frac{\partial^2 W_1}{\partial t_1 \partial t_2} = -\frac{1}{9} - \frac{4\gamma d}{9} < 0$, $\frac{\partial^2 W_2}{\partial t_2 \partial t_1} = -\frac{1}{9} - \frac{4(1-\gamma)d}{9} < 0$ and, hence, $\frac{\partial^2 W_1}{\partial {t_1}^2}\frac{\partial^2 W_2}{\partial {t_2}^2} - \frac{\partial^2 W_1}{\partial t_1 \partial t_2}\frac{\partial^2 W_2}{\partial t_2 \partial t_1} = \frac{16+8d}{27} > 0$, which ensures a unique and stable equilibrium.

Solving the above two first-order conditions simultaneously, the equilibrium PB-taxes are given by:

$$t_1^{PB*} = \frac{A\,(4\gamma d - d - 1)}{d+2}\,, \tag{B.5}$$

$$t_2^{PB*} = \frac{A\,(3d - 4\gamma d - 1)}{d+2}\,. \tag{B.6}$$

Inserting these taxes into equilibrium outputs shows that the NN-constraint under this regime is given by $d < \frac{1}{2(2\gamma-1)}$ if $\gamma > 0.5$ and no constraint is needed if $\gamma = 0.5$.

B.3 BI-regime

Using Table (1) in the text and eq. (B.1), equilibrium outputs are given by $x_{11}^{BI} = x_{21}^{BI} = \frac{A-t_1}{3}$, $x_{12}^{BI} = \frac{A-2t_1+t_2}{3}$ and $x_{22}^{BI} = \frac{A-2t_2+t_1}{3}$. By inserting outputs into eq. (B.2) and eq. (B.3), the first-order conditions can be derived:

$$\frac{\partial W_1^{BI}}{\partial t_1} = \gamma d\left(\frac{4A-t_2}{3} - t_1\right) - A - \frac{10}{9}t_1 + \frac{2}{9}t_2 = 0 \text{ and}$$
$$\frac{\partial W_2^{BI}}{\partial t_2} = \frac{1}{3}\left((1-\gamma)\,d\left(\frac{4A-t_2}{3} - t_1\right) - t_1 - t_2\right) = 0,$$

where the second-order conditions hold because: $\frac{\partial^2 W_1}{\partial {t_1}^2} = -\frac{10}{9} - \gamma d < 0$ and $\frac{\partial^2 W_2}{\partial {t_2}^2} = -\frac{1}{3} - \frac{(1-\gamma)d}{9} < 0$. Because $\frac{\partial^2 W_1}{\partial t_1 \partial t_2} = \frac{2}{9} - \frac{\gamma d}{3}$ and $\frac{\partial^2 W_2}{\partial t_2 \partial t_1} = -\frac{1}{3} - \frac{(1-\gamma)d}{3} < 0$, we have $\frac{\partial^2 W_1}{\partial {t_1}^2}\frac{\partial^2 W_2}{\partial {t_2}^2} - \frac{\partial^2 W_1}{\partial t_1 \partial t_2}\frac{\partial^2 W_2}{\partial t_2 \partial t_1} = \frac{4}{9} + \frac{(2\gamma+16d)}{81} > 0$, which ensures a unique and stable equilibrium.

Solving the first-order conditions, equilibrium carbon taxes are given by:

$$t_1^{BI*} = \frac{A\,(29\gamma d + 7d - 3)}{36 + 2d\,(\gamma+8)}\,, \tag{B.7}$$

$$t_2^{BI*} = \frac{A\,(43d - 79\gamma d + 3)}{36 + 2d\,(\gamma+8)}\,. \tag{B.8}$$

It can be shown that the most restrictive NN-constraint requires $d < \frac{1}{3\gamma-1}$. Since the difference between the two national tax levels is ambiguous, we need to impose a constraint on the parameters such that $t_1^{BI*} > t_2^{BI*}$, which requires the BCA-constraint $d > \frac{1}{6(3\gamma-1)}$.

B.4 BIE-regime

Using Table (1) in the text and eq. (B.1), equilibrium output levels are: $x_{11}^{BIE} = x_{21}^{BIE} = \frac{A-t_1}{3}$, $x_{12}^{BIE} = \frac{A-2t_1(1-\varphi)+t_2}{3}$ and $x_{22}^{BIE} = \frac{A-2t_2+t_1(1-\varphi)}{3}$.
The first-order conditions are given by:

$$\frac{\partial W_1^{BIE}}{\partial t_1} = \frac{1}{3}4\gamma d\left(A - \frac{3}{4}t_1 - \frac{1}{4}t_2\right) - \frac{1}{9}\left(A + 10t_1 - 2t_2\right) + \frac{1}{9}\varphi^2\left(-\gamma d t_1 - 4t_1\right) + \frac{1}{9}\varphi\left(A + 8t_1 + t_2 - 4\gamma d(A - \frac{3}{2}t_1 - \frac{1}{4}t_2\right) = 0\,,$$

$$\frac{\partial W_1^{BIE}}{\partial \varphi} = \frac{1}{3}t_1\left(\frac{1}{3}\left(A - 2t_1\left(1-\varphi\right) + t_2\right)\right) + \frac{2}{3}t_1^2\left(1-\varphi\right) - \frac{1}{3}t_1\gamma d\left(\frac{4}{3}A - t_1 + \frac{1}{3}t_1\varphi - \frac{1}{3}t_2\right) = 0\,,$$

$$\frac{\partial W_2^{BIE}}{\partial t_2} = \frac{1}{3}\left((1-\gamma)\,d\left(\frac{4A-t_2}{3} - t_1 + \frac{1}{3}t_1\varphi\right) - t_1 - t_2\right) = 0.$$

Solving the first order conditions simultaneously, we have:

$$t_1^{BIE*} = \frac{17Ad\,(\gamma+1)}{36 + d\,(5\gamma+14)} > 0\,, \tag{B.9}$$

$$t_2^{BIE*} = \frac{68Ad\,(1/2-\gamma)}{36 + d\,(5\gamma+14)} \leq 0\,, \tag{B.10}$$

$$\varphi^* = \frac{d\,(29 - 37\gamma) + 9}{17d\,(\gamma + 1)} > 0\,, \tag{B.11}$$

noting that $\frac{\partial^2 W_1}{\partial {t_1}^2} = \frac{1}{9}\left(\varphi\,(6\gamma d + 8) + \varphi^2\,(-\gamma d - 4) - 10\right) - \gamma d < 0 \forall\, \varphi$ and $\gamma, d > 0$, $\frac{\partial^2 W_1}{\partial t_1 \partial t_2} = \frac{1}{9}\left(2 + \varphi + \gamma d\,(\varphi - 3)\right)$, $\frac{\partial^2 W_2}{\partial {t_2}^2} = -\frac{1}{3} - \frac{(1-\gamma)d}{9} < 0$, $\frac{\partial^2 W_2}{\partial t_2 \partial t_1} = -\frac{1}{3} + \frac{(1-\gamma)d(\varphi-3)}{9}$ and $\frac{\partial^2 W_1}{\partial \varphi^2} = -\frac{4}{9}t_1^2 - \frac{1}{9}\gamma d t_1^2 < 0\ \forall\, t_1 \neq 0$. Hence, $\frac{\partial^2 W_1}{\partial {t_1}^2}\frac{\partial^2 W_2}{\partial {t_2}^2} - \frac{\partial^2 W_1}{\partial t_1 \partial t_2}\frac{\partial^2 W_2}{\partial t_2 \partial t_1} > 0\ \forall\ 0 \leq \gamma \leq 1$ and $d > 0$. Inserting equilibrium taxes into outputs, it turns out that the most restrictive NN-constraint requires $d < \frac{6}{19\gamma - 8}$. We also need to impose a BCA-constraint such that $t_1^{BIE*}(1 - \varphi^*) \geq t_2^{BIE*}$ or, equivalently, $\varphi^* \leq \frac{t_1^{BIE*} - t_2^{BIE*}}{t_1^{BIE*}}$, which leads to $d \geq \frac{9}{2(61\gamma - 23)}$. $\varphi^* > 0$ follows directly for all $d > 0$ and $\gamma \leq \frac{29}{37} \simeq 0.78$, but is also true for $\gamma > 0.78$ if $d < \frac{9}{37\gamma - 29}$, which holds due to the NN-constraint. Thus, $\varphi^* > 0$ is always true.

B.5 BF-regime

Using Table (1) in the text and eq. (B.1): $x_{11}^{BF} = x_{21}^{BF} = \frac{A - t_1}{3}$, $x_{12}^{BF} = \frac{A + t_2}{3}$ and $x_{22}^{BF} = \frac{A - 2t_2}{3}$. Inserting these outputs into eqs. (B.2) and (B.3), the following first-order conditions can be derived:

$$\frac{\partial W_1^{BF}}{\partial t_1} = \frac{1}{3}\left(-2t_1 + t_2 + 2\gamma d\left(\frac{4A - 2t_1 - t_2}{3}\right)\right) = 0 \text{ and}$$
$$\frac{\partial W_2^{BF}}{\partial t_2} = \frac{1}{3}\left(-t_1 - t_2 + (1-\gamma)\,d\left(\frac{4A - 2t_1 - t_2}{3}\right)\right) = 0.$$

The second derivatives are: $\frac{\partial^2 W_1}{\partial {t_1}^2} = -\frac{2}{3} - \frac{4\gamma d}{9} < 0$, $\frac{\partial^2 W_1}{\partial t_1 \partial t_2} = \frac{1}{3} - \frac{2\gamma d}{9}$, $\frac{\partial^2 W_2}{\partial {t_2}^2} = -\frac{1}{3} - \frac{(1-\gamma)d}{9} < 0$ and $\frac{\partial^2 W_2}{\partial t_2 \partial t_1} = -\frac{1}{3} - \frac{2(1-\gamma)d}{9} < 0$. Hence, $\frac{\partial^2 W_1}{\partial {t_1}^2}\frac{\partial^2 W_2}{\partial {t_2}^2} - \frac{\partial^2 W_1}{\partial t_1 \partial t_2}\frac{\partial^2 W_2}{\partial t_2 \partial t_1} = \frac{1}{3} + \frac{d(4-2\gamma)}{27} > 0\ \forall\, \gamma \leq 2$.

Solving the first-order conditions, equilibrium taxes are given by:

$$t_1^{BF*} = \frac{-4Ad\,(\gamma + 1)}{d\,(2\gamma - 4) - 9} > 0\,, \tag{B.12}$$

$$t_2^{BF*} = \frac{8Ad\,(2\gamma - 1)}{d\,(2\gamma - 4) - 9} \leq 0\,. \tag{B.13}$$

Inserting equilibrium taxes into outputs, it turns out that the most restrictive NN-constraint requires $d < \frac{3}{2\gamma}$. There is no need for a BCA-constraint since $t_1^{BF*} > t_2^{BF*}$ and $t_1^{BF*}(1 - \varphi) = 0 \geq t_2^{BF*}$ always hold.

B.6 CB-regime

Using Table (1) in the text and eq. (B.1), equilibrium outputs are: $x_{11}^{CB} = x_{21}^{CB} = \frac{A - t_1}{3}$, $x_{12}^{CB} = x_{22}^{CB} = \frac{A - t_2}{3}$. The first-order conditions are given by:

$$\frac{\partial W_1^{CB}}{\partial t_1} = \frac{1}{3}\left(-2t_1 + 2\gamma d\left(\frac{4A - 2t_1 - 2t_2}{3}\right)\right) = 0\,, \text{ and}$$
$$\frac{\partial W_2^{CB}}{\partial t_2} = \frac{1}{3}\left(-2t_2 + 2\,(1-\gamma)\,d\left(\frac{4A - 2t_1 - 2t_2}{3}\right)\right) = 0.$$

For the second derivatives we find: $\frac{\partial^2 W_1}{\partial {t_1}^2} = -\frac{2}{3} - \frac{4\gamma d}{9} < 0$, $\frac{\partial^2 W_1}{\partial t_1 \partial t_2} = -\frac{4\gamma d}{9} < 0$, $\frac{\partial^2 W_2}{\partial {t_2}^2} = -\frac{2}{3} - \frac{4(1-\gamma)d}{9} < 0$ and $\frac{\partial^2 W_2}{\partial t_2 \partial t_1} = -\frac{4(1-\gamma)d}{9} < 0$. Hence, $\frac{\partial^2 W_1}{\partial {t_1}^2}\frac{\partial^2 W_2}{\partial {t_2}^2} - \frac{\partial^2 W_1}{\partial t_1 \partial t_2}\frac{\partial^2 W_2}{\partial t_2 \partial t_1} = \frac{4}{9} + \frac{8d}{27} > 0$.

Solving the first-order conditions yields the equilibrium CB-taxes:

$$t_1^{CB*} = \frac{4A\gamma d}{2d+3} = D_1'(e)\,, \tag{B.14}$$

$$t_2^{CB*} = \frac{4A(1-\gamma)d}{2d+3} = D_2'(e)\,. \tag{B.15}$$

Inserting equilibrium taxes into outputs, the NN-constraint requires $d < \frac{3}{4\gamma-2}$ if $\gamma > 0.5$ and if $\gamma = 0.5$ no constraint is required.

B.7 Ranges of Parameter Values

We summarise the conditions that satisfy the NN-constraints and the BCA-constraints in the following table. We assume those constraints to hold. If we conduct a comparison across regimes, we assume the most restrictive condition, referred to as the feasible range below, to hold.

Table B.1: Feasible Range of the Parameters Values

Regime/Constraint	NN-constraint	BCA-constraint
Social Optimum	/	/
PB	$d < \frac{1}{2(2\gamma-1)}$	/
BI	$d < \frac{1}{3\gamma-1}$	$d > \frac{1}{6(3\gamma-1)}$
BIE	$d < \frac{6}{19\gamma-8}$	$d \geq \frac{9}{2(61\gamma-23)}$
BF	$d < \frac{3}{2\gamma}$	/
CB	$d < \frac{3}{4\gamma-2}$	/
Feasible Range	$d < \overline{d}\,(\gamma) = \frac{1}{3\gamma-1}$	$d \geq \underline{d}\,(\gamma) = \frac{9}{2(61\gamma-23)}$

B.8 Proof of Proposition 5

Using the equilibrium tax levels in eq. (B.4) to eq. (B.15) and the constraints in Table B.1, we conduct the following comparisons.

1. We start by ranking equilibrium taxes. i) For country 1, we have $t_1^{BIE*} > t_1^{BI*}$ for all $\gamma \leq \frac{29}{37} \approx 0.783$ and if $d < \frac{9}{37\gamma-29}$ for all $\gamma > 0.783$ where also this condition holds as long as the NN-constraint holds. In addition, $t_1^{BF*} > t_1^{BI*}$ for all $d > 0$. Finally, $t_1^{BF*} \geq t_1^{BIE*}$ if $d \geq \frac{3}{2(9\gamma-2)}$, which implies that $\varphi^* \leq 1$. However, if $d < \frac{3}{2(9\gamma-2)}$, $\varphi^* > 1$ and we have $t_1^{BIE*} > t_1^{BF*}$. Regarding country 2, we have $t_2^{BI*} > t_2^{BIE*}$ if $d > \frac{9}{37\gamma-29} < 0$ for all $\gamma < \frac{29}{37} \approx 0.78$, if $\gamma = 0.78$, and if $d < \frac{9}{37\gamma-29}$ for all $\gamma > 0.78$, which must hold due to the NN-constraint. In addition, $t_2^{BI*} > t_2^{BF*}$ is always true for all $d > 0$. Therefore, $t_2^{BI*} > t_2^{BIE*}, t_2^{BF*}$. We also have $t_2^{BF*} \geq t_2^{BIE*}$ if $d \leq \frac{3}{2(9\gamma-2)}$, which implies that $\varphi^* \geq 1$. If $d > \frac{3}{2(9\gamma-2)}$, i.e., $\varphi^* < 1$, we have $t_2^{BIE*} > t_2^{BF*}$. Note that $t_2^{BIE*} = t_2^{BF*} = 0$ if $\gamma = 0.5$, irrespective of φ^*.
   ii) Comparing the equilibrium tax level under the PB-regime with the BCA-regimes leads to the following: $t_1^{BI*} > t_1^{PB*}$ for all $\gamma \leq 0.607$ and if $d < \frac{\sqrt{8016\gamma^2-6624\gamma+1209}+63-84\gamma}{2(8\gamma^2+33\gamma-23)}$ for all $\gamma > 0.607$ where the NN-constraint is sufficient for this condition to hold. Given the ranking of country 1's tax levels under

the BCA-regimes, country 1 sets a higher tax level under the BCA-regimes compared to the PB-regime. For country 2, we have: $t_2^{BI*} > t_2^{PB*}$ if $d < \breve{d} = -\frac{\sqrt{3600\gamma-1200\gamma^2+849}-12\gamma-3}{2(8\gamma^2-21\gamma-5)}$. The NN-constraint is sufficient for this condition to hold for all $\breve{\gamma} > 0.57$, i.e., for all $(1-\gamma) < (1-\breve{\gamma})$, that is, if the damage evaluation in country 2 is not too large. While if $d \geq \breve{d}$ for all $\gamma \leq \breve{\gamma}$ (or $(1-\gamma) \geq (1-\breve{\gamma})$), we have $t_2^{BI*} \leq t_2^{PB*}$. Similarly, $t_2^{BIE*} > t_2^{PB*}$ if $d < \dddot{d} = -\frac{\sqrt{3212\gamma-2711\gamma^2+1828}-13\gamma-26}{2(20\gamma^2-27\gamma-8)}$. This condition must hold for all $\gamma \geq \dddot{\gamma} = 0.66$, i.e. $(1-\gamma) \leq (1-\dddot{\gamma})$ as long the NN-constraint, i.e., $d < \overline{d}(\gamma)$, holds, while we could have $t_2^{BIE*} \leq t_2^{PB*}$ if $d \geq \dddot{d}$ for all $(1-\gamma) > (1-\dddot{\gamma})$. Finally, $t_2^{BF*} > t_2^{PB*}$ if $d < \ddot{d} = \frac{2\gamma-7+\sqrt{292\gamma^2-244\gamma+193}}{4(4\gamma^2-3\gamma+2)}$ where the NN-constraint, $d < \overline{d}(\gamma)$, is sufficient for this condition to hold for all $\gamma > \ddot{\gamma} = 0.673$, i.e., for all $(1-\gamma) < (1-\ddot{\gamma})$. However, if $\gamma \leq \ddot{\gamma}$, or equivalently $(1-\gamma) \geq (1-\ddot{\gamma})$, we could have $t_2^{BF*} \leq t_2^{PB*}$ if $d \geq \ddot{d}$.

iii) Comparing the equilibrium tax levels under unilateral BCA-regimes with the CB-regime leads to the following:

For country 1, one possibility is to have $t_1^{BCA*} < t_1^{CB*}$. We find that $t_1^{CB*} > t_1^{BF*}$ if $d < \frac{3(2\gamma-1)}{2(\gamma^2-\gamma+1)}$ where the NN-constraint is sufficient to guarantee that this condition holds for all $\gamma \geq \mathring{\gamma} = 0.73$. Furthermore, $t_1^{CB*} > t_1^{BIE*}$ for all $\gamma \geq 0.865$ and if $d < -\frac{3(31\gamma-17)}{2(10\gamma^2+11\gamma-17)}$ for all $\gamma < 0.865$. However, also the NN-constraint is sufficient to guarantee this condition for all $0.66 \leq \gamma < 0.865$. In addition, $t_1^{BCA*} < t_1^{CB*}$ also holds if $\gamma \in [0.61, 0.73)$ if $d < \frac{3(2\gamma-1)}{2(\gamma^2-\gamma+1)}$. However, if $\gamma < 0.61$, $t_1^{BCA*} > t_1^{CB*}$ could be the case. With respect to country 2, $t_2^{BI*} > t_2^{CB*}$ if $d < -\frac{3\left(\sqrt{929\gamma^2+594\gamma-79}-31\gamma-3\right)}{4(4\gamma^2-51\gamma+11)}$, which violates the BCA-constraint for all values of γ. Hence, we always have $t_2^{BI*} < t_2^{CB*}$. Given the ranking of taxes in i), we have $t_2^{BCA*} < t_2^{CB*}$.

iv) $t_1^{CB*} > t_1^{PB*}$ if $d < \frac{3}{4\gamma-2}$ for all $\gamma > 0.5$. This is exactly the NN-constraint under the CB-regime (see table B.1) and, hence, it must hold. For $\gamma = 0.5$, $t_1^{PB*} < t_1^{CB*}$ is easily checked. In addition, $t_2^{CB*} > t_2^{PB*}$ if $d > -\frac{3}{4\gamma-2}$ for all $\gamma > 0.5$. which must hold as d is a positive parameter. Again, for $\gamma = 0.5$, $t_2^{CB*} > t_2^{PB*}$ is easily checked.

2. We rank global emissions across regimes. We have $e^{PB*} - e^{BI*} = \frac{A(4\gamma d+7\gamma d+22)}{(d+2)(\gamma d+8d+18)} > 0$. Similarly, $e^{PB*} - e^{BIE*} = \frac{A(20\gamma d+5\gamma d+42)}{(d+2)(5\gamma d+14d+36)} > 0$. In addition, we have $e^{PB*} > e^{BF*}$ if $d < \frac{3}{2\gamma-1}$, which must hold due to the NN-constraint. We also find that $e^{BIE*} > e^{BI*}$ if $d < \frac{9}{37\gamma-29}$, which holds as long the NN-constraint holds. Furthermore, $e^{BF*} > e^{BI*}$ for all $\gamma < 0.7$ and if $d > \frac{9}{2(31\gamma-2)}$ for all $\gamma \geq 0.7$, while if $d < \frac{9}{2(31\gamma-2)}$ for all $\gamma \geq 0.7$, we have $e^{BF*} < e^{BI*}$. Finally, the comparison between the BIE- and the BF-regime depends on the optimal rebate rate. More specifically, if $d \geq \frac{3}{2(9\gamma-2)}$, implying $\varphi^* \leq 1$, we have $e^{BF*} \geq e^{BIE*}$, while if $d < \frac{3}{2(9\gamma-2)}$, i.e., $\varphi^* > 1$, $e^{BF*} < e^{BIE*}$. Comparison between the PB- and the CB regime follows directly from result iv) above. That is, $e^{PB*} > e^{CB*}$. It can be easily checked that $e^{BI*} - e^{CB*} > 0$ and $e^{BF*} - e^{CB*} > 0$. Hence, $e^{BCA*} > e^{CB*}$, given $e^{BIE*} > e^{BI*}$ always hold.

B.9 Proof of Proposition 6

i) $t^{S*} > t_1^{PB*}$ if $\gamma = 0.5$ and if $d < -\frac{8\gamma-13}{8(2\gamma-1)}$ for all $\gamma > 0.5$. The NN-constraint (either the most restrictive or the one under the PB-regime only) is sufficient for this condition to hold. Similarly, $t^{S*} > t_2^{PB*}$ if $\gamma = 0.5$ and if $d > -\frac{5+8\gamma}{16\gamma-8}$ for all $\gamma > 0.5$, which must hold as the right hand-side term is negative and d is a positive parameter.

ii) In order to obtain a sufficient condition for both countries to set a higher tax under the BCA- and the CB-regime than in the social optimum, we need to compare the lowest non-cooperative tax level with the socially optimal tax. Since taxes of country 1 are always larger than that of country 2 under all regimes, we will choose the lowest tax level which country 2 could choose to compare it with the socially optimal tax. It turns out that the most strict condition follows from: $t_2^{BIE*} > t^{S*}$ if $d < \tilde{d}(\gamma) = \frac{\sqrt{17161\gamma^2+58292\gamma-7676}-131\gamma-62}{8(73\gamma-20)}$. However, this condition violates the BCA-constraint for all $\gamma < \tilde{\gamma} \approx 0.77$, while for $\gamma \geq \tilde{\gamma}$, we need $d < \tilde{d}(\gamma)$. Comparing global emissions under the non-cooperative outcomes and the social optimum leads to the following: $e^{S*} > e^{CB*}$ if $d < 0.5$ for all $\gamma > 0.52$, while if countries are nearly symmetric, i.e., $\gamma \leq 0.52$, $e^{S*} < e^{CB*}$. A sufficient condition for the ranking in Proposition 6 (ii) follows from comparing global emissions under the social optimum with global emissions under the BIE/BF-regime. We find $e^{BCA*} < e^{S*}$ for all $\gamma > 0.61$ if $d < \check{d}$, where $\check{d} = \frac{21}{2(37-5\gamma)}$ for all $\gamma \leq 0.75$ and $\check{d} = \frac{3}{2(\gamma+4)}$for all $\gamma > 0.75$. In both cases, $\check{d} < 0.5$ for all γ, which leads to the ranking in Proposition 6(ii).

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