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Abstract

We define and characterize renegotiation-proof equilibria of coordination games with pre-play communication in which players have private preferences over the feasible coordinated outcomes. These are such that players never miscoordinate, players coordinate on their jointly preferred outcome whenever there is one, and players communicate only the ordinal part of their preferences. This set of renegotiation proof equilibrium strategies does not depend on the distribution of private preferences, and is thus robust to changes in players' beliefs. Moreover, these equilibria are interim Pareto efficient and evolutionarily stable.

Keywords: coordination games, renegotiation-proof, equilibrium entrants, secret handshake, incomplete information, evolutionary robustness.

JEL codes: C72, C73, D82

1 Introduction

Coordination is an important aspect of successful economic and social interaction. Yet, in many economic and social interactions coordination is the primary but not the only goal, and often the parties involved have private preferences over which joint action they would like to coordinate on. Moreover, and especially because of this private information, these situations often seem to necessitate communication. These are the situations that we are here interested in.

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As an example of such economic interactions consider a proposed research joint venture between two firms (see, e.g., Katz, 1986; Vonortas, 2012). The two firms have undertaken some research on their own before and have individually found solutions as to what methods and tools, such as software and accounting system, to use. When collaborating it would probably make sense for the two firms to agree on one of these two solutions. It is plausible to assume that the two firms have private preferences over which of the two solutions they would like to implement in the joint venture. They may have learned to love or hate whatever system they are using. These firms then communicate before each deciding with which system they will work in their joint venture.

As an example of such social interaction consider the problem of two pedestrians in everyday traffic, say at a busy train station, finding themselves face-to-face and trying to get past each other. In many cases, each person has private preferences as to which side they would like to pass the other person, as only they know whether they want to head more to the left or the right just after this encounter. Pedestrians often use non-verbal communication to signal their preferred direction (e.g., a slight movement to the left or right, a tilt of the head, a glimpse in a certain direction).[1] Casual observation suggests that for pedestrians often there is no uniform social norm such as "always pass on the right" as there is for cars.[2]

The questions we are after are as follows. Do we expect to see, likely inefficient, miscoordination in such settings? Even if there is coordination will it solve the problem efficiently? Will communication help achieve (more or less efficient) coordination, when this could not be achieved without communication? How subtle (complex) would we expect emerging social norms to be in such an environment with incomplete information?

Our baseline model for these situations is a symmetric two-action two-player coordination games with private preferences over which action to coordinate on and with public pre-play cheap-talk communication. We denote the two actions by "left" (L) and "right" (R). We normalize the payoff of miscoordination to zero to both players. The players' private preferences are summarized by a number $u \in [0, 1]$ (sampled from an arbitrary atomless distribution, independent of the opponent's value) that describes a player's payoff when players coordinate on action R. Coordination on the action L yields a payoff of $1 - u$. The solution concept we employ is that of renegotiation-proof Bayes Nash equilibrium.[3] To be specific we, implicitly, allow players to renegotiate their initial plan of play after communicating and, hence, after learning what such communication reveals. When renegotiating the players adopt a Pareto-improving plan of play if one exists, possibly involving additional communication. A renegotiation-proof equilibrium is then one in which parties, on the equilibrium path of play, never have an incentive to renegotiate.

[1] The example is motivated by Goffman (1971, Chapter 1, p. 6): "Take, for example, techniques that pedestrians employ in order to avoid bumping into one another. [...] There are an appreciable number of such devices; they are constantly in use and they cast a pattern on street behavior. Street traffic would be a shambles without them."

[2] Car traffic rules motivated the evolutionary analysis of complete information coordination games in Young (1998).

[3] We adapt previous notions of renegotiation-proofness to this setting. See Section 9 for a detailed discussion.

We believe that this model and solution concept fits both kinds of problems, those in the economic sphere as well as those in the social sphere. It does so, however, for different reasons in the two spheres. In the economic sphere we think of this solution concept as the outcome of a conscious effort on behalf of both interacting parties to coordinate on the best available outcome. Renegotiation-proofness is then a requirement that when the interacting parties identify a Pareto-improvement they will try to convince each other to adopt it. In the social sphere, we think of Bayes Nash equilibrium as a necessary condition for a social norm.[4] Renegotiation proofness is then our way to model another necessary condition for evolutionary stability: stability with respect to a secret handshake mutation (by a small group of individuals in the population of pedestrians) as suggested in e.g., Robson (1990) for complete information games. If individuals coordinate on a Pareto-inferior equilibrium then the following "mutant" group could successfully enter: these mutants use a hitherto unused message (the secret handshake) and then play the incumbent equilibrium if their opponent does not reciprocate this handshake, and play a Pareto-superior equilibrium (if one exists) if their opponent also uses the same handshake. Mutants, thus, obtain superior payoffs over incumbents.

Our main result is then as follows. A strategy in our setup is a renegotiation-proof equilibrium strategy if and only if it satisfies three properties: it is coordinated, mutual preference consistent, and has essentially binary communication. A strategy is coordinated if the two players always end up choosing the same action: players never ultimately miscoordinate. A strategy is mutual preference consistent if whenever two players have the same (ordinal) preference for coordinating on one rather than another action, they manage to do so. This entails that in such a strategy, players reveal enough about their type for them to be able to do this: they need to reveal the ordinal content of their preferences. A strategy has essentially binary communication if this is essentially all they communicate. They do not communicate the strength of their preferences. They only may use additional communication (or simply more messages) in order to determine how to play when it emerges that the two players have opposing ordinal preferences: one would like to coordinate on one action, the other on another. There is then a variety of possible behaviors, all, however, are coordinated and independent of the players' strength of preferences.[5]

Interestingly, and we believe crucially for the empirical plausibility of these equilibria, the set of renegotiation-proof equilibria is completely independent of the distribution of private preferences. This is so despite the fact that in the absence of communication different distributions of private preferences can give rise to very different predictions. For some distributions only a uniform coordinated norm of always choosing the same action is an evolutionary stable equilibrium, whereas for others only miscoordinated equilibria are evolutionary stable, see Sandholm (2007).

[4]This is also suggested in (Goffman, 1971, p.xx): "[T]he rules of an order are necessarily such as to preclude the kind of activity that would have disrupted the mutual dealings, making it impractical to continue with them."

[5]These possible behaviors range from employing a "fallback" norm of, e.g., pedestrians passing each other on the right, to using random messages to implement a joint lottery, akin to a public toss of dice or coin, to decide which action to play as in Aumann and Maschler (1968).

Given the solution concept that we employ, communication, thus, leads to coordination and this coordination is relatively efficient. We show in Section 6 that renegotiation-proof equilibria of the game with communication are Pareto-efficient at the interim stage and Pareto-superior (i.e., better for all types of players) over all equilibria of the game without communication. Renegotiation-proof equilibria do not implement the first-best outcome (which maximizes the ex-ante expected payoff), in fact no equilibrium does. More interestingly perhaps, renegotiation proof equilibria do not necessarily maximize the ex-ante payoff among all equilibria (though, they maximize the ex-ante payoff with respect to all coordinated equilibria). Interpreting renegotiation-proof equilibria as a social norm, we note that they represent a more subtle norm than a simple "always drive on the right" norm. The norm dictates the use of some communication and then adapting behavior in a fairly efficient way to the outcome of this communication.

Section 8 provides a series of robustness checks and extensions of our baseline model. We show that all our results hold for any length of pre-play communication, and for more than two players (assuming that any miscoordination yields a payoff of zero to all players). We also extend our results to general (possibly asymmetric) two-action coordination games. The result that the three properties imply renegotiation-proofness holds in general. The converse result of renegotiation-proofness implying the three properties holds under the additional assumption, satisfied in our baseline model, that for all types the coordinated action yielding the higher payoff is also this type's risk-dominant action (i.e., it is the best-reply against an opponent choosing her action uniformly).

We also present somewhat weaker results for coordination games with more than two actions. Specifically, we show that a specific, in fact "fairest" strategy among all strategies with the three properties, remains a renegotiation-proof equilibrium strategy in this more general setup, and that any renegotiation-proof equilibrium strategy must be coordinated whenever both players send the same message, and it must be mutual-preference consistent. Finally, we study a variant of our baseline model in which a few types have dominant actions. We show that in this setup, there is a unique renegotiation-proof equilibrium strategy among the strategies satisfying our three properties, which does depend on the distribution of types with dominant actions.

The paper proceeds as follows. Section 2 provides the two-action two-player coordination game with private preferences and public pre-play communication. Section 3 defines Bayes Nash equilibrium and the three key properties that renegotiation-proof equilibria have. Section 4 defines the concept of renegotiation-proofness appropriately adapted to our incomplete information strategic setting. Section 5 states the main theorem and a sketch of its proof. Section 6 discusses the efficiency properties of renegotiation-proof equilibria. Section 7 discusses additional, classical, notions of evolutionary stability beyond the secret handshake stability implied by renegotiation-proofness. Section 8 provides a series of robustness checks and extensions. Section 9 concludes with a discussion of related literature and our main takeaways. Proofs are given in the appendix (A) for results in Sections 5 and 6, and the online appendices (B-E) on our homepages, for the remaining results.

2 Baseline Model

We consider a setup in which two agents with private idiosyncratic preferences play a two-action coordination game that is preceded by pre-play cheap-talk. Extensions are presented in Section 8.

Players and Types There are two players in ex-ante symmetric positions. Players can choose one of two actions, L and R. Each player has a privately known "value" or "type". The two players' values are independently drawn from a common atomless distribution with cumulative distribution function F on the unit interval $U = [0,1]$ and with density f. To make it interesting we make the assumption, throughout the paper, that $F(1/2) \in (0,1)$. This means that not all player types agree on their preferred outcome in the coordination game below.

Payoff Matrix For any realized pair of types, u and v, the players play a coordination game given by the following payoff matrix, where the first entry is the payoff of the player of type u (choosing row) and the second entry is the payoff of the player of type v (choosing column).

Table 1: Payoff Matrix of the Coordination Game

		Type v	
		L	R
Type u	L	1-u, 1-v	0, 0
	R	0, 0	u, v

We call this game the *coordination game without communication* and denote it by Γ.

Pre-Play Communication After learning their type, but before playing this coordination game, the two players each simultaneously send a publicly observable message from a finite set of messages M (satisfying $2 \leq |M| < \infty$), with $\Delta(M)$ the set of all probability distributions over messages in M. We assume that messages are costless, i.e., cheap talk. We call the game, so amended, the *coordination game with communication* and denote it by $\langle \Gamma, M \rangle$.

Strategies A player's (ex-ante) strategy in the coordination game with communication is then a pair $\sigma = (\mu, \xi)$, where $\mu : U \to \Delta(M)$ is a (Lebesgue measurable) *message function* that describes which (possibly random) message is sent for each possible realization of the agent's type, and $\xi : M \times M \to U$ is an *action function* that describes the maximal type (cutoff type) that chooses L as a function of the observed message profile; that is, when an agent who follows strategy (μ, ξ) observes a message profile (m, m') (message m sent by the agent, and message m' sent by the opponent), then the agent plays L if her type u is at most $\xi(m, m')$ (i.e., if $u \leq \xi(m, m')$), and she plays R if $u > \xi(m, m')$. Let Σ be the set of all strategies in the game $\langle \Gamma, M \rangle$.

Remark 1. In principle, we should allow more general action functions $\xi : U \times M \times M \to \triangle\{L, R\}$, which specify the probability an agent chooses L as a function of the observed message profile and the agent's type. It is simple to see, however, and proven in Lemma 1 in Appendix A.1, that any “generalized” strategy is dominated by a strategy that uses a cutoff action function in the second stage. The intuition, is that following the observation of any pair of messages, lower types always gain more (less) than higher types from choosing L (R). We, thus, simplify our notation by considering only cutoff action functions of the form $\xi : M \times M \to U$.[6]

Let $\mu_u(m)$ denote the probability, given message function μ, that a player sends message m if she is of type u. Let $\mu(m) = \mathbb{E}_u[\mu_u(m)]$ be the mean probability that a player of a random type sends message m. Let $\mathrm{supp}(\mu) = \{m \in M \mid \mu(m) > 0\}$ denote the support of μ. With a slight abuse of notation we write $\xi(m, m') = L$ when all types play L (i.e., when $\xi(m, m') = 1$), and we write $\xi(m, m') = R$ when all types play R (i.e., when $\xi(m, m') = 0$).

3 Equilibrium Strategies

In this section we define the standard notion of (Bayesian Nash) equilibrium strategies, present properties that renegotiation-proof equilibria turn out to have, and present examples of equilibria in the coordination game with communication with and without these properties.

Definition Given a strategy profile (σ, σ') and a type profile $u, v \in U$, let $\pi_{u,v}(\sigma, \sigma')$ denote the interim (pre-communication) expected payoff of a player of type u who follows strategy σ and faces an opponent of type v who follows strategy σ'. Formally, for $\sigma = (\mu, \xi)$ and $\sigma' = (\mu', \xi')$,

$$\begin{aligned}\pi_{u,v}(\sigma, \sigma') &= \sum_{m\in M}\sum_{m'\in M} \mu_u(m)\,\mu_v(m')\Big((1-u)\mathbf{1}_{\{u\le\xi(m,m')\}}\mathbf{1}_{\{v\le\xi'(m',m)\}} + \\ &+ \quad u\mathbf{1}_{\{u>\xi(m,m')\}}\mathbf{1}_{\{v>\xi'(m',m)\}}\Big),\end{aligned}$$

where $\mathbf{1}_{\{x\}}$ is the indicator function equal to 1 if statement x is true and zero otherwise. Let

$$\pi_u(\sigma, \sigma') = \mathbb{E}_v[\pi_{u,v}(\sigma, \sigma')] \equiv \int_{v=0}^{1} \pi_{u,v}(\sigma, \sigma')\, f(v)\, dv$$

denote the expected interim payoff of a player of type u who follows strategy σ and faces an opponent with a random type who follows strategy σ'. Finally, let,

$$\pi(\sigma, \sigma') = \mathbb{E}_u[\pi_u(\sigma, \sigma')] \equiv \int_{u=0}^{1} \pi_u(\sigma, \sigma')\, f(u)\, du$$

denote the ex-ante expected payoff of an agent who uses strategy σ against strategy σ'.

A strategy σ is a *(symmetric Bayesian Nash) equilibrium strategy* if $\pi_u(\sigma, \sigma) \ge \pi_u(\sigma', \sigma)$ for each $u \in U$ and each strategy $\sigma' \in \Sigma$. Let $\mathcal{E} \subseteq \Sigma$ denote the set of all equilibrium strategies of $\langle \Gamma, M \rangle$.

[6] The arbitrary choice that the threshold type plays L does not play any role in our analysis, given the assumption that the distribution of types F is without atoms.

Three Key Properties We call a strategy $\sigma = (\mu, \xi) \in \Sigma$ *mutual-preference consistent* if whenever $u, v < 1/2$ then $\xi(m, m') = \xi(m', m) = L$ for all $m \in \text{supp}(\mu_u)$ and all $m' \in \text{supp}(\mu_v)$ and whenever $u, v > 1/2$ then $\xi(m, m') = \xi(m', m) = R$ for all $m \in \text{supp}(\mu_u)$ and all $m' \in \text{supp}(\mu_v)$. I.e., players with the same ordinal preference coordinate on their mutually preferred outcome.

We call it *coordinated* if $\xi(m, m') = \xi(m', m) \in \{L, R\}$ for any pair of messages $m, m' \in \text{supp}(\mu)$. A coordinated strategy never leads to miscoordination after any (used) message pair.

For any strategy $\sigma = (\mu, \xi) \in \Sigma$ and for any message $m \in M$, define the expected probability of a player's opponent playing L conditional on this player sending message $m \in M$, as

$$\beta^\sigma(m) = \int_{u=0}^{1} \sum_{m' \in \text{supp}(\mu_u)} \mu_u(m') \mathbf{1}_{\{u \leq \xi(m', m)\}} f(u) du.$$

We say that strategy σ has *(essentially) binary communication* if there are two numbers $0 \leq \underline{\beta}^\sigma \leq \overline{\beta}^\sigma \leq 1$ such that for all messages $m \in M$ we have $\beta^\sigma(m) \in [\underline{\beta}^\sigma, \overline{\beta}^\sigma]$, for all messages $m \in M$ such that there is a type $u < 1/2$ with $\mu_u(m) > 0$ we have $\beta^\sigma(m) = \overline{\beta}^\sigma$, and for all messages $m \in M$ such that there is a type $u > 1/2$ with $\mu_u(m) > 0$ we have $\beta^\sigma(m) = \underline{\beta}^\sigma$. Note that, as defined here, a strategy with no communication also has (essentially) binary communication.

In Appendix B we show that no single one of these three properties is implied by the other two. Clearly, a strategy that has binary communication and is coordinated must be an equilibrium. In Appendix B we also show that no other combination of two of these three properties implies that a strategy is an equilibrium. Finally, we show that mutual-preference consistency implies that players reveal their ordinal type (i.e., whether their type is below or above $1/2$).

Consider a strategy that is coordinated and mutual-preference consistent and has binary communication. Then there is an $\alpha^\sigma \in [0, 1]$ such that we can write

$$\underline{\beta}^\sigma = (1 - F(1/2))\, \alpha^\sigma \quad \text{and} \quad \overline{\beta}^\sigma = F(1/2) + (1 - F(1/2))\, \alpha^\sigma,$$

where α^σ is the probability of coordination on L conditional on one player having type $u < 1/2$ and the other type $v > 1/2$. We refer to α^σ as the *left-tendency* of a strategy σ that is coordinated and mutual-preference consistent and has binary communication.

Remark 2. The set of strategies with the above three properties (coordinated, mutual-preference consistent, and binary communication) is, essentially, one-dimensional because the left-tendency $\alpha^\sigma \in [0, 1]$ of such a strategy σ describes all payoff-relevant aspects. Two strategies with the same left-tendency can only differ in the way in which the players implement the joint lottery when they have different preferred outcomes, but these implementation differences are non-essential, as the probability of the joint lottery inducing the players to play L remains the same.

Examples of Equilibria Satisfying All Properties The following strategies, denoted by σ_L, σ_R, and σ_C, are prime examples (that play a special role in later sections) of strategies that are all mutual-preference consistent, coordinated, and have binary communication.

The strategies σ_L and σ_R are given by the pairs (μ^*, ξ_L) and (μ^*, ξ_R), respectively. The message function μ^* has the property that there are messages $m_L, m_R \in M$ such that message m_L indicates a preference for L and m_R a preference for R, that is

$$\mu^*(u) = \begin{cases} m_L & u \leq \frac{1}{2} \\ m_R & u > \frac{1}{2}. \end{cases}$$

The action functions ξ_L and ξ_R are defined as follows:

$$\xi_L(m, m') = \begin{cases} R & m = m' = m_R \\ L & \text{otherwise,} \end{cases} \qquad \xi_R(m, m') = \begin{cases} L & m = m' = m_L \\ R & \text{otherwise.} \end{cases}$$

This means that the "fallback norm" of σ_L (which is applied when the agents have different preferred outcomes) is to coordinate on L, while the fallback norm of σ_R is to coordinate on R. In other words the left-tendency of σ_L is one and the left-tendency of σ_R is zero.

Strategy $\sigma_C = (\mu_C, \xi_C)$ has the "fallback norm" to use a joint lottery to randomly choose the coordinated outcome. The agents implement this by each agent simultaneously sending a random bit such that the coordinated outcome depends on whether the random bits are equal or not.

Assume that $|M| \geq 4$. We denote four distinct messages as $m_{L,0}, m_{L,1}, m_{R,0}, m_{R,1} \in M$, where we interpret the first subscript (R or L) as the agent's preferred direction, and the second subscript (0 or 1) as a random binary number chosen with probability 1/2 each by the agent. Formally, the message function μ_C is defined as follows:

$$\mu_C(u) = \begin{cases} \frac{1}{2}m_{L,0} \oplus \frac{1}{2}m_{L,1} & u \leq \frac{1}{2} \\ \frac{1}{2}m_{R,0} \oplus \frac{1}{2}m_{R,1} & u > \frac{1}{2}, \end{cases}$$

where $\alpha m \oplus (1-\alpha)m'$ is a lottery with a probability of α on message m and $1-\alpha$ on message m'.

In the second stage, if both agents share the same preferred outcome they play it. Otherwise, they coordinate on L if their random numbers differ, and coordinate on R otherwise. Formally:

$$\xi_C(m, m') = \begin{cases} R & (m, m') \in \{(m_{R,0}, m_{R,0}), (m_{R,0}, m_{R,1}), (m_{R,0}, m_{L,0}), (m_{R,1}, m_{L,1}) \\ & \quad (m_{R,1}, m_{R,1}), (m_{R,1}, m_{R,0}), (m_{L,0}, m_{R,0}), (m_{L,1}, m_{R,1})\} \\ L & \text{otherwise.} \end{cases}$$

Note that among all strategies that satisfy the three properties, strategies σ_L and σ_R are the *simplest* in terms of the number of "bits" needed to implement the message function. Strategy σ_C is in a certain sense *fairest*: conditional on a coordination conflict, i.e., conditional on one agent having type between 0 and $1/2$ and the other between $1/2$ and 1 both agents expect the same payoff. By contrast strategy σ_L favors types below $1/2$, and strategy σ_R favors types above $1/2$.

Examples of Equilibria Not Satisfying Some of the Properties The coordination game with communication $\langle \Gamma, M \rangle$ admits many more equilibria that satisfy only some or even none of the three properties defined above. It has, for instance, two simple babbling equilibria, in which agents ignore the communication and apply a uniform norm of always playing L (or R). These equilibria are coordinated and trivially have binary communication, but are not mutually preference consistent.

Depending on the distribution of types, the game can also have more inefficient babbling equilibria in which agents sometimes miscoordinate. Specifically, if there exists a type $x \in (0, 1)$ satisfying $x = F(x)$, then there is a babbling equilibrium in which agents ignore messages and choose L if and only if her type is below x. Such a babbling equilibrium (trivially) has binary communication but does not satisfies the other two properties defined above. Note that, as by assumption $F(0) = 0$ and $F(1) = 1$, all babbling equilibria can be identified with an $x \in [0, 1]$ that satisfies $F(x) = x$.

Remark 3 (Robustness of equilibria without communication). When there is no pre-play communication (i.e., $|M| = 1$), then these babbling equilibria of course constitute all equilibria. Arguably, a plausible equilibrium refinement in setups without communication is robustness to small perturbations in the behavior of the population (e.g., requiring Lyapunov stability of the best-reply dynamics, or continuous stability à la Eshel, 1983). Adapting the analysis of Sandholm (2007) to the current setup implies that an equilibrium is robust in this sense if and only if the density of the distribution of types at the relevant threshold x (with $x = F(x)$) is less than one. In particular, if the distribution of types satisfies $f(0), f(1) > 1$, then there exists $x \in (0, 1)$ satisfying $x = F(x)$ and $f(x) < 1$. The corresponding equilibrium, which entails inefficient miscoordination is then robust to small perturbations. Thus, coordination games without communication are likely to induce substantial miscoordination if the density of extreme types is high (i.e., if $f(0), f(1) > 1$).

The game also admits equilibria in which agents reveal some information about the cardinality of their preferences (i.e., some information beyond only stating if $u \leq 1/2$ or $u > 1/2$). One simple example of such an equilibrium for a specific distribution function F is given as Example 1 in Section 6. It is not completely straightforward to construct such examples for all possible distributions F if we consider only a single round of communication as we do in the main body of this paper. In Section 8 we show, however, that our main results continue to hold also if we allow for multiple rounds of communication. For the case of multiple rounds of communication it is relatively straightforward to construct examples of equilibria that do not have binary communication and that, therefore, reveal some cardinal content of the players' preferences. For simplicity assume that the distribution of types, F, is symmetric around $1/2$. That is $f(x) = f(1 - x)$ for all $x \in [0, 1]$, or equivalently,

$F(x) = 1 - F(1-x)$ for all $x \in [0,1]$. In particular, we have that $F(1/2) = 1/2$.

The equilibrium is such that there is an x satisfying $0 < x < 1/2$ such that in the first round of communication players indicate whether their preferences are "extreme" (i.e., $u \leq x$ or $u > 1-x$) or "moderately left" ($x < u \leq 1/2$) or "moderately right". In the second round individuals only reveal real additional information if in the first round one sent the extreme message and the other a moderate message, in which case the extreme type now reveals which side she prefers ($u \leq 1/2$ or $u > 1/2$). In this case joint play is dictated by the extreme type's preferences. If both send the extreme message in the first round then there is no more communication and both extreme types follow their inclination (play L if $u \leq 1/2$ and R otherwise). This leads to miscoordination with a conditional probability of a half. Any two moderate types eventually coordinate, either on their mutually preferred outcome or, using essentially σ_C employing communication to induce a joint fair lottery over both playing L and both playing R. In Appendix C we write this strategy down formally and show that for any symmetric distribution F there is an $x \in (0, 1/2)$ such that this strategy is indeed a Nash equilibrium of the coordination game with two rounds of communication.

4 Definition of Renegotiation-Proofness

For any given strategy in Σ, employed by both players, in the game $\langle \Gamma, M \rangle$, communication and knowledge of this strategy leads to updated, and, possibly, different and asymmetric information about the two agents' types. Suppose the updated distributions of types are given by some distribution functions G and H. The two agents then face a (possibly asymmetric) game of coordination without communication, which we shall denote by $\Gamma(G, H)$. Note that the original game (without communication) Γ is then given by $\Gamma(F, F)$.

Let f_m be the type density conditional on the agent following a given strategy in the game $\langle \Gamma, M \rangle$ and sending a message m that is sent with positive probability given this strategy.[7] That is,

$$f_m(u) = \frac{f(u)\mu_u(m)}{\mu(m)},$$

and let F_m be the cumulative distribution function associated with density f_m.

We allow players to renegotiate (only) after communication. Players in their renegotiation can use any new finite message set, $\tilde{M}$.[8] Given a strategy of the game $\langle \Gamma, M \rangle$, employed by both players, we denote the induced "renegotiation" game after a positive probability message pair $m, m' \in M$ by $\langle \Gamma(F_m, F_{m'}), \tilde{M} \rangle$. For a pair of strategies σ, σ' of such a renegotiation game $\langle \Gamma(G, H), \tilde{M} \rangle$ define

[7] The density f_m depends on the given strategy in the game $\langle \Gamma, M \rangle$. For aesthetic reasons we refrain from giving this strategy a name and omit to indicate this obvious dependence in our notation.

[8] All of our main results remain the same if one limits renegotiating players to use the original set of messages M, as long as the set M includes at-least four messages (i.e., $|M| \geq 4$).

the *post-communication* expected payoffs for a type u agent by

$$\pi_u^H(\sigma,\sigma') = \mathbb{E}_{v\sim H}\left[\pi_{u,v}(\sigma,\sigma')\right] \equiv \int_{v=0}^{1} \pi_{u,v}(\sigma,\sigma')\, h(v)\, dv.$$

Define $\mathcal{E}(G,H)$ as the set of all (possibly asymmetric) equilibrium profiles of the coordination game with communication $\langle\Gamma(G,H),\tilde{M}\rangle$ for some finite message set $\tilde{M}$. Furthermore let $\mathcal{S}(G)$ denote the set of all symmetric equilibrium strategies of the coordination game with communication $\langle\Gamma(G,G),\tilde{M}\rangle$ for some finite message set $\tilde{M}$. With a slight abuse of notation for any strategy σ of the game $\langle\Gamma,M\rangle$ we denote its babbling prescription after message pair $m,m'\in M$, i.e., in the game $\langle\Gamma(F_m,F_{m'}),\tilde{M}\rangle$ by σ as well; that is, we denote by σ the strategy in $\langle\Gamma(F_m,F_{m'}),\tilde{M}\rangle$ in which the player chooses her message uniformly (regardless of her type), and then plays according to what the original strategy σ has induced her to play in $\langle\Gamma,M\rangle$ after observing the message profile (m,m').

Definition 1. We say that an equilibrium strategy $\sigma\in\mathcal{E}$ is *post-communication equilibrium Pareto-dominated* if either there is a message $m\in\mathrm{supp}(\mu)$ and an equilibrium $\tilde{\sigma}\in\mathcal{S}(F_m)$ such that $\pi_u^{F_m}(\sigma,\sigma)\leq\pi_u^{F_m}(\tilde{\sigma},\tilde{\sigma})$ for all $u\in\mathrm{supp}(F_m)$ with strict inequality for some $u\in\mathrm{supp}(F_m)$, or there is a pair of messages $m\neq m'\in\mathrm{supp}(\mu)$ and an equilibrium profile $\tilde{\sigma}\in\mathcal{E}(F_m,F_{m'})$ such that $\pi_u^{F_{m'}}(\sigma,\sigma)\leq\pi_u^{F_{m'}}(\tilde{\sigma})$ and $\pi_v^{F_m}(\sigma,\sigma)\leq\pi_v^{F_m}(\tilde{\sigma})$ for all $u\in\mathrm{supp}(F_m)$ and all $v\in\mathrm{supp}(F_{m'})$ with strict inequality for some $u\in\mathrm{supp}(F_m)$ or some $v\in\mathrm{supp}(F_{m'})$.

Definition 2. An equilibrium strategy $\sigma=(\mu,\xi)\in\mathcal{E}$ is *renegotiation-proof* if it is not post-communication equilibrium Pareto-dominated.[9]

The motivation for renegotiation-proofness is that if the agents can communicate prior to playing the game, then it seems plausible that they can further communicate after observing the realized messages. If there is an observed pair of messages after which the original equilibrium induces the agents to play a strategy profile with a low payoff, then, arguably, the agents can use an additional round of communication to renegotiate the existing "bad" equilibrium of the current induced game, and to coordinate their play on a Pareto-improving equilibrium (which weakly improves the payoff of all possible types of both players). Our refinement of renegotiation-proofness requires that no such Pareto-improving equilibria exists in any induced game with additional communication.

We have chosen to define a mild notion of renegotiation-proofness because it already suffices for the sharp characterization given in Theorem 1. Our refinement is mild in the following ways: we allow players to renegotiate only after observing their realized messages (but not before), and when players play a symmetric induced game, we allow them only to implement an alternative symmetric equilibrium (rather, then also allowing them to play asymmetric equilibria, in which an agent's behavior may explicitly depend on its role in the game).

[9]Section 9 provides a thorough discussion of how our notion of renegotiation-proofness compares to and is motivated by earlier notions in the literature, including our alternative interpretation of renegotiation-proofness as a requirement of (evolutionary) secret hand-shake stability in the sense of Robson, 1990.

5 Main Result

With all this in place we can state our main result.

Theorem 1. *A strategy of the game with communication* $\langle \Gamma, M \rangle$ *is a renegotiation-proof equilibrium strategy if and only if it is mutual-preference consistent, coordinated, and has binary communication.*

Sketch of proof; see Appendix A.2 for the formal proof. The "if" part, i.e., that any strategy satisfying the three properties must be renegotiation-proof, is fairly straightforward. We here provide a sketch of the proof of the "only if" part. The proof in the Appendix is split into three lemmas, each showing that one of the three properties must hold.

Lemma 2 proves that a renegotiation-proof equilibrium strategy must be coordinated: If play after any message pair is not coordinated then it is Pareto-inferior (given the information about players' types implicit in this message pair) to either σ_L, σ_R, or σ_C. To see this suppose first that both players use thresholds below $1/2$. Then this strategy is Pareto-dominated by σ_R as types above $1/2$ gain because σ_R induces their first-best outcome, and types below $1/2$ gain because σ_R yields a higher coordination probability and a higher probability of the opponent playing this type's preferred action L. Analogously, an equilibrium in which both players use thresholds above $1/2$ is Pareto-dominated by σ_L. Suppose, finally, that player one uses threshold $x < 1/2$, while player two uses threshold $x' > 1/2$. Observe that $x < 1/2$ (resp., $x' > 1/2$) can be an equilibrium threshold only if player two (resp., player one) plays L with an average probability of less (resp., more) than $1/2$. This, implies that players in these equilibria coordinate with a probability of at most $1/2$, and one can show that such a low coordination probability implies that these equilibria are Pareto-dominated by σ_C.

Next, we show in Lemma 3 that a renegotiation-proof equilibrium strategy must have binary communication. The reason for this is that if a strategy is coordinated, then different messages, not knowing the realized opponent message, can only lead to different ex-ante probabilities of coordination on L (and R). Thus, any type who favors L, i.e., any type $u < 1/2$ will choose a message to maximize this probability, while any type $u > 1/2$ will choose a message to minimize this probability. Thus, effectively only two kinds of messages are used in a coordinated equilibrium strategy.

Using both lemmas we finally show in Lemma 4 that a renegotiation-proof equilibrium strategy must be mutual-preference consistent. Given that it is coordinated we know that any message pair will lead to either coordination on L or on R. If it is not mutual-preference consistent then, without loss of generality, there are two types $u, u' < 1/2$ that, with positive probability, send a message pair (m, m') that leads them to coordinate on R. But then all types who send this message pair would be weakly better off (and some strictly better off) if instead of coordinating on R they use strategy σ_R, which would allow them to coordinate on L if and only if both types are below $1/2$.[10] □

[10] In our proof we actually prove a slightly stronger result. Any equilibrium that is not renegotiation-proof is in fact

6 On Efficiency

In this section we investigate the efficiency properties of renegotiation-proof equilibria. We first argue that the first-best outcome cannot be achieved by any equilibrium of any coordination game with communication. We then provide an example of an equilibrium with high ex-ante payoffs that is, however, not renegotiation-proof. We then show that all renegotiation-proof equilibria, while not necessarily ex-ante payoff optimal among all equilibria, are at least interim (pre-communication) Pareto efficient. Finally we show that at least one of the two "extreme" renegotiation-proof equilibria, σ_L and σ_R, provides the highest ex-ante payoff among all coordinated equilibria, and that any equilibrium without communication is Pareto-dominated by either one of these extreme renegotiation-proof equilibria or by the action-symmetric renegotiation-proof equilibrium σ_C.

First Best The first-best ex-ante payoff can only be induced by a strategy that is coordinated and such that the coordinated outcome depends heavily on the cardinal preferences of the two agents. Specifically, the first best strategy is one that induces coordination on L whenever $u + v \leq 1$ and coordination on R otherwise. The first-best ex-ante payoff can, thus, only be induced by a strategy in which each agent reveals her type, and the two agents then choose the favorite outcome, L or R, of the more extreme type (i.e., the type that is farther away from $1/2$). Note that this strategy is not an equilibrium: Each player has an incentive to present a more extreme type than her real type (e.g., all types $u > 1/2$ would claim to have type 1).[11]

High payoff of Non-Coordinated Equilibria Equilibria with miscoordination (which are not renegotiation-proof due to Theorem 1) may induce agents to credibly reveal some cardinal information about their type. This can happen if there is a message that induces higher probabilities of coordinating on the agent's favored outcome but also higher probabilities of miscoordination compared with some other available message. Such a message can then be chosen by extreme types with u far from $1/2$, while moderate types with u closer to $1/2$ choose the other message. Such equilibria with miscoordination may induce a higher ex-ante payoff, if the benefit of signaling the extremeness of the type outweighs the loss due to miscoordination. Consider the following example.

Example 1. For simplicity we let the distribution of types F be discrete with four atoms $1/10 + \epsilon$, $1/2 - \epsilon$, $1/2 + \epsilon$, $9/10 - \epsilon$, with a probability of $1/4$ on each atom and $\epsilon > 0$ sufficiently small.[12] The game admits three babbling equilibria: always coordinating on L or always coordinating on R, both with an ex-ante payoff of $1/2$, and playing L if and only if the type is less than $1/2$ with an ex-ante payoff

such that after some message pair equilibrium play is Pareto-dominated by a renegotiation-proof equilibrium strategy, while no renegotiation-proof equilibrium strategy is Pareto-dominated by any equilibrium strategy after any message pair. See the related discussion of internal and external consistency requirements for a definition of renegotiation-proof strategies in repeated games in e.g. Ray (1994).

[11]Note that the ex-ante efficient strategy could only be exactly implemented in a coordination game with communication with a continuum message set. For finite message sets it could only be approximated. For the same reason describe above no such approximation could be an equilibrium.

[12]One can easily adapt the example to an atomless distribution of types, in which each atom is replaced with a continuum of nearby types.

of $^7/_{20} < ^1/_2$, for all ϵ sufficiently small. Theorem 1 (together with the symmetry of the distribution F) implies that with communication, any renegotiation-proof equilibrium strategy (in particular σ_L and σ_R) induces the same expected ex-ante payoff of $^3/_5 > ^1/_2$ for any ϵ sufficiently small.

This game also has a (non-renegotiation-proof) equilibrium strategy with miscoordination that yields a higher ex-ante payoff than the renegotiation-proof payoff of $^3/_5$ provided the message set M has sufficiently many elements. To simplify the presentation we here allow the players to use public correlation devices to determine their joint play after sending messages, which can be approximately implemented by a sufficiently large message set (à la Aumann and Maschler, 1968; see the similar construction in Case (II) of the proof of Proposition 8 in Appendix E.7.3). Let $m_L, m_l, m_r, m_R \in M$ and consider strategy $\sigma = (\mu, \xi)$ as follows. Let $\mu(^1/_{10} + \epsilon) = m_L$, $\mu(^1/_2 - \epsilon) = m_l$, $\mu(^1/_2 + \epsilon) = m_r$, and $\mu(^9/_{10} - \epsilon) = m_R$, and let $\xi(m_a, m_b) = L$ if $a, b \in \{L, l\}$, $\xi(m_a, m_b) = R$ if $a, b \in \{r, R\}$, $\xi(m_L, m_r) = L$, $\xi(m_l, m_R) = R$, $\xi(m_l, m_r)$ be a joint lottery to coordinate on L or R with probability $^1/_2$ each, and, finally, let $\xi(m_L, m_R)$ be a joint lottery to coordinate on L or R with probability $^3/_{10}$ each, and to play the inefficient mixed equilibrium (in which each type plays her favored outcome with probability $^9/_{10} - \epsilon$) with probability $^4/_{10}$. It is straightforward to verify that, for e.g., $\epsilon = ^1/_{100}$, this strategy is indeed an equilibrium strategy with an ex-ante payoff of around 0.627 which is higher than the ex-ante payoff of $^3/_5$ of all the renegotiation-proof equilibria. This equilibrium strategy is not coordinated (also not mutually preference consistent or has binary communication) and, hence, by Theorem 1 the strategy is not renegotiation-proof.

Interim (pre-communication) Pareto Optimality An (ex-ante) symmetric *(type-dependent) outcome function* is a function $\phi : [0,1]^2 \to \Delta\left(\{L, R\}^2\right)$ assigning to each pair of types a possibly correlated action profile with the condition that $\phi_{u,v}(a, b) = \phi_{v,u}(b, a)$ for any $a, b \in \{L, R\}$, where $\phi_{u,v} \equiv \phi(u, v)$.[13] We interpret $\phi_{u,v}$ as the correlated action profile played by the two players when a player of type u meets a player of type v. Let Φ be the set of all such functions.

Any strategy of any coordination game with communication (with any finite message set) induces an outcome function in Φ, but not all outcome functions in Φ can be generated by a strategy of a given coordination game with communication. One can interpret the set of outcome functions Φ as the outcome functions that can be implemented by a social planner who perfectly observes the types of both players, and as a function of that can force them to play arbitrarily.

For each type $u \in [0, 1]$, let $\pi_u(\phi)$ denote the expected payoff of a player of type u under outcome function ϕ, i.e.,

$$\pi_u(\phi) = \mathbb{E}_v\left[(1-u)\,\phi_{u,v}(L, L) + u\phi_{u,v}(R, R)\right].$$

A strategy is interim (pre-communication) Pareto-dominated if there is a feasible outcome, which

[13]We restrict attention to symmetric outcome functions here for two reasons. First, it makes the paper conceptually consistent, given that the subject of the paper, coordination games with communication, is a class of (ex-ante) symmetric games. Second, this prevents us from having to here introduce player subscripts which we do not need anywhere else in the paper. Proposition 1 below, however, also holds even if we allow asymmetric outcome functions.

is weakly better for all types, and strictly better for some types.

Definition 3. A strategy $\sigma \in \Sigma$ is *interim (pre-communication) Pareto-dominated* by a type-dependent outcome function $\phi \in \Phi$ if $\pi_u(\sigma, \sigma) \leq \pi_u(\phi)$ for each type $u \in [0,1]$ with the inequality being strict for a positive measure set of types. A strategy $\sigma \in \Sigma$ is *interim (pre-communication) Pareto optimal* if it is not interim (pre-communication) Pareto-dominated by any $\phi \in \Phi$.

Proposition 1. *Every renegotiation-proof strategy of a coordination game with communication is interim (pre-communication) Pareto optimal.*

Sketch of proof; see Appendix A.3 for the formal proof. Recall that due to Theorem 1 and Remark 2, any renegotiation-proof equilibrium strategy σ is characterized by its left-tendency α^σ. In order for an outcome function ϕ to improve the payoff of any type $u < 1/2$ (resp., $u > 1/2$) relative to the payoff induced by σ, it must be that ϕ induces any type $u < 1/2$ (resp., $u > 1/2$) to coordinate on L with probability larger (resp., smaller) than α^σ. This, in turn, implies that the probability of two players coordinating on L, conditional on the players having different preferred outcomes, must be larger (resp., smaller) than α^σ. However, these two requirements contradict each other. □

Above we have given an example of an equilibrium strategy that provides a higher ex-ante payoff than any renegotiation-proof equilibrium. This strategy involved a certain degree of miscoordination. In the following proposition we show that any equilibrium without miscoordination, i.e., any coordinated equilibrium must have an ex-ante expected payoff that is less than or equal to the maximal ex-ante payoff of the two "extreme" renegotiation-proof strategies σ_L and σ_R.

Proposition 2. *Let $\sigma \in \mathcal{E}$ be a coordinated equilibrium strategy. Then*

$$\pi(\sigma, \sigma) \leq \max\{\pi(\sigma_L, \sigma_L), \pi(\sigma_R, \sigma_R)\}.$$

Sketch of proof; see Appendix A.3 for the formal proof. Let α^σ be the probability of two players who each follow σ to coordinate on L, conditional on the players having different preferred outcomes. It is simple to see that σ is dominated by the renegotiation-proof equilibrium strategy with the same left-tendency α^σ, and that the payoff of the latter strategy is a convex combination of the payoffs of σ_L and σ_R, which implies that $\pi(\sigma, \sigma) \leq \max\{\pi(\sigma_L, \sigma_L), \pi(\sigma_R, \sigma_R)\}$. □

Remark 4 (All-stage renegotiation-proofness à la Benoit and Krishna (1993)). One could refine the notion of renegotiation-proofness to allow agents to renegotiate to a Pareto-improving equilibrium additionally also in earlier stages: in the interim stage before observing the realized messages induced by the original equilibrium, and in the ex-ante stage before each agent observes her own type. This more restrictive definition of renegotiation-proofness à la Benoit and Krishna (1993) would: call the strategies satisfying our definition of renegotiation-proofness *ex-post renegotiation-proof* strategies; say that an ex-post renegotiation-proof strategy is *interim renegotiation-proof* if it is not Pareto-dominated (in the original game after each agent observes her own type, yet before observing the realized message profile) by any ex-post renegotiation-proof strategy; and say that an interim renegotiation-proof strategy is *all-stage renegotiation-proof* if there is no other interim renegotiate-proof strategy that induces a higher ex-ante expected payoff to both players (before each player knows her own type). Clearly, any ex-post renegotiation-proof strategy that satisfies interim Pareto optimality is an interim renegotiation-proof strategy. Proposition 2 implies that either σ_L or σ_R maximizes the ex-ante payoff among all interim renegotiation-proof strategies, and, thus, either σ_L or σ_R is an all-stage renegotiation-proof strategy. Moreover, if $\pi(\sigma_R, \sigma_R) \neq \pi(\sigma_L, \sigma_L)$, then one can show that either σ_L or σ_R is the unique coordinated strategy that maximizes the ex-ante payoff, which implies that it is the unique all-stage renegotiation-proof strategy.

Next, we show that σ_L or σ_R provides a strictly higher ex-ante expected payoff than any equilibrium of the game without communication (and thus any babbling equilibrium of the game with communication). Recall from Remark 3 and the text preceding it that in the coordination game without communication any equilibrium is characterized by a cutoff value $x \in [0,1]$ such that $x = F(x)$ with the interpretation that types $u \leq x$ play L and types $u > x$ play R.

Let $\pi_u(x, x')$ denote the payoff of an agent with type u who follows a strategy with cutoff x and faces a partner of unknown type who follows a strategy with cutoff x', which is given by

$$\pi_u(x, x') = \mathbf{1}_{\{u \leq x\}} F(x')(1-u) + \mathbf{1}_{\{u > x\}} F(x')\, u,$$

and let $\pi(x, x') = \mathbb{E}_u[\pi_u(x, x')]$ be the ex-ante expected payoff of an agent who follows strategy x and faces a partner who follows x'. Our final result shows that any (possibly asymmetric) equilibrium in the game without communication is Pareto-dominated by either σ_L, σ_R or σ_C.

Corollary 1. *Let (x, x') be a (possibly asymmetric) equilibrium in the coordination game without communication. Then either $\pi_u(x, x') \leq \pi_u(\sigma_L, \sigma_L)$ for all types $u \in U$, or $\pi_u(x, x') \leq \pi_u(\sigma_R, \sigma_R)$ for all types $u \in U$, or $\pi_u(x, x') \leq \pi_u(\sigma_C, \sigma_C)$ for all types $u \in U$.*

Corollary 1 is immediately implied by Lemma 2 in Appendix A.2, and the sketch of proof of the lemma is presented as part of the sketch of proof of Theorem 1.

7 Evolutionary Stability

A common interpretation of a Nash equilibrium is a convention that is reached as a result of a process of social learning when similar games are repeatedly played within a large population. This interpretation seems very apt, for instance, if we think of our motivating example of how pedestrians avoid bumping into each other. Specifically, consider a population in which a pair of agents from a large population are occasionally randomly matched and play the coordination game with communication $\langle\Gamma, M\rangle$. The agents can observe past behavior of other agents playing similar games in the past. It seems plausible that the aggregate behavior of the population would gradually converge into a self-enforcing convention, which is a symmetric Nash equilibrium of $\langle\Gamma, M\rangle$.[14]

We consider renegotiation-proofness as a necessary condition for evolutionary stability by capturing the idea of stability with respect to secret handshake mutations as in Robson (1990). See our discussion in Section 9 for more details. In this section we report results from Appendix D in which we investigate the evolutionary stability properties of both σ_L and σ_R (the results can be extended to all renegotiation-proof equilibrium strategies).

In Appendix D.1 we show that strategies σ_L and σ_R are neutrally stable strategies (NSS) in the sense of Maynard Smith and Price (1973), and evolutionary stable strategies (ESS) if $|M| = 2$. This implies that σ_L and σ_R are robust to the presence of a small proportion of experimenting agents who behave differently than the rest of the population.

We are not quite satisfied with this result for three reasons. First, neutral stability is not the strongest form of evolutionary stability, although in games with cheap talk this is typically the strongest form of stability one can expect owing to the freedom that unused messages provide mutants, see e.g., Banerjee and Weibull (2000).[15] Second, our game, owing to the incomplete information modeled here as a continuum variable, has a continuum of strategies, especially in the action phase after messages are observed. But with a continuum of strategies the notion of even an ESS is not sufficient to imply local convergence to the equilibrium from nearby states. The reason for this, see e.g., Oechssler and Riedel (2002), is that ESS for continuum models only considers the possibly large strategy deviation of a small proportion of individuals but not the small strategy deviation of possibly a large proportion of individuals. Third and finally our cheap talk game here is a two stage game and is thus an extensive form game. It is well-known that extensive-form games do not admit ESSs of the entire game (unless they are strict equilibria, see Selten, 1980), and it seems reasonable to explore the stability of equilibrium behavior in each stage separately.

To address these issues we investigate two additional evolutionary stability properties. We investigate the evolutionary stability in each subgame that is on the equilibrium path.[16] Evolution will not necessarily place huge restrictions on play in unreached subgames (see e.g., Nachbar, 1990; Gale, Binmore, and Samuelson, 1995), but should do so for subgames reached with positive probability.

[14] See, e.g., the "mass action" interpretation of a Nash equilibrium already present given by Nash (1950). See also Weibull, 1995; Sandholm, 2010 for a textbook introduction.

[15] We here do not consider set-valued concepts of evolutionary stability such as *evolutionary stable sets* (Swinkels, 1992 and the related analysis in e.g., Balkenborg and Schlag, 2001, 2007), nor the perturbation-based concept of *limit-ESS* (Selten, 1983; Heller, 2014), which lies in between ESS and NSS.

[16] Note that in our renegotiation-proofness concept we do not impose any restrictions on unreached subgames beyond that the strategy in the whole game must be an equilibrium strategy. In particular we do not require play in unreached subgames to be a Nash equilibrium nor do we require renegotiation-proofness in unreached subgames. So we here also do not demand evolutionary stability in unreached subgames.

To address this we therefore study the evolutionary stability properties of strategies σ_L and σ_R at the message level, taking as given the action behavior in the second stage, and at the action level, choosing action cutoffs after messages that are observed with positive probability.

In Appendix D.2 we show that, holding action behavior fixed at either ξ_L or ξ_R, the message function μ^* used by σ_L and σ_R is weakly dominant, a stronger property than being an NSS.[17]

In Appendix D.3 we investigate the evolutionary stability properties of the action choice induced by σ_L and σ_R after observed message pairs. As choosing an action as a function of a player's type is equivalent to choosing a cutoff from a continuum (the unit interval) we employ a stability concept designed for such cases. The issue is further complicated by the fact that, owing to the asymmetry after unequal messages, we need to employ a multidimensional stability concept. The literature provides one in the form of a neighborhood invader strategy developed for the double-population case by Cressman (2010) building on earlier work by Eshel and Motro (1981) and Apaloo (1997), among others. We there show that the action choice induced by σ_L and σ_R indeed constitutes a neighborhood invader strategy after each pair of possible messages.

8 Extensions

In this section we present informally various extensions and robustness checks of our main results. We postpone the detailed formal analysis to Appendix E.

Multiple Rounds of Communication In Appendix E.1 we show that all of our results hold in a setup in which the pre-play communication phase includes multiple rounds. Players observe messages after each round and can, thus, condition their message choice and then their final action choice on the history of observed message pairs. Renegotiation then possibly takes place once at the end of this communication phase but before the final action choices are made.

Multi-Dimensional Set of Types In Appendix E.2 we study general symmetric two-action two-player coordination games, where miscoordination may have different payoffs to the L and R players. The "if" direction of Theorem 1 still holds in this general setup: any strategy that satisfies the three key properties is renegotiation-proof. Theorem 2 shows that the "only if" direction holds as well, under the restriction of "unambiguous coordination preferences", which requires that for all feasible types the preferred coordinated action is also this type's risk-dominant action (i.e., their best reply against an opponent who plays each action uniformly; see, Harsanyi and Selten, 1988).

[17]For dynamic evolutionary processes it is not always true that weakly dominated strategies are eliminated. See e.g., Weibull (1995), Hart (2002), Kuzmics (2004), Kuzmics (2011), Laraki and Mertikopoulos (2013), Bernergård and Mohlin (2019) for a discussion of this issue. Note also that Kohlberg and Mertens (1986) made it a desideratum for any concept of *strategic* stability that it does not include weakly dominated strategies and strategies that survive popular refinements such as trembling hand perfection of Selten (1975) or proper equilibrium of Myerson (1978) cannot be weakly dominated.

An example in Appendix 2 suggests that without this restriction (i.e., with stag-hunt-like types for which the payoff-dominant action does not coincide with the risk-dominant action), equilibria with miscoordination may be renegotiation-proof.

More Than Two Players Suppose there are $n \geq 2$ players. The payoff of each player of type u is equal to u if all players play R, it is equal to $1-u$ if all players play L, and it is equal to zero if not all the players play the same action. In Appendix E.3 we show that our main result (Theorem 1) as well as appropriate versions of the efficiency results hold in this setup.

Asymmetric Coordination Games Appendix E.4 shows that all our results hold in asymmetric (two-action two-player) coordination games, in which the distributions of types of the two players' positions may differ.

Coordination Games with More Than 2 Actions In Appendix E.5 we analyze coordination games with more than two actions. In this setup we are only able to prove weaker variants of our main result. First, we show that σ_C remains a renegotiation-proof equilibrium strategy in this more general setup (by contrast, strategies σ_L and σ_R might not be equilibria in this setup). While we do not have a full characterization of the set of renegotiation-proof equilibrium strategies, we are able to show that every renegotiation-proof equilibrium strategy must satisfy weaker counterparts of some of the key properties: be coordinated in cases in which both players send the same message, and be mutual-preference consistent.

Extreme Types with Dominant Actions Appendix E.6 extends our analysis to a setup in which some types find one of the actions a dominant action for them. We show that with these extreme types there exists, essentially, a unique renegotiation-proof equilibrium strategy that satisfies the three key properties, where the left-tendency of this strategy is equal to the share of extreme types for which L is their dominant action among all extreme types. In this setup moderate "leftists", i.e., types with $u \in (0, 1/2)$, gain if there are more extreme "leftists" than extreme "rightists" in the sense that the above essentially unique strategy induces a higher probability of coordination on left when two agents with different preferred outcomes meet.

9 Discussion

While our notion of renegotiation-proofness may not be exactly the same as any notion in the literature as far as we can see, it is, however, inspired by previous notions and appropriately adapted to the problem at hand. At the heart of renegotiation-proofness is the idea that people will find it hard to ignore obvious Pareto-improving alternatives to any suggestions as to how they should behave or what choices they should (collectively) make. When people are forward looking, their anticipation of revisions of plans that are not Pareto-efficient can constrain possible equilibrium behavior. Renegotiation-proofness concepts have been developed in the context of infinitely and finitely repeated games with complete information in e.g., Farrell and Maskin (1989), Van Damme (1989), Bernheim and Ray (1989), Evans and Maskin (1989), and in e.g., Benoit and Krishna (1993) and Wen (1996), respectively. There is a sizable literature on the renegotiation-proofness of contracts in the presence of asymmetric information possibly starting with Hart and Tirole (1988) and Dewatripont (1989) and recent contributions in Maestri (2017) and Strulovici (2017).

The closest concept to ours may be that of posterior efficiency of Forges (1994) (building on Holmström and Myerson (1983)). Forges (1994) argues that the final outcome of a mechanism (or here strategy profile) will not necessarily fully reveal all initially privately held information. Posterior efficiency then only demands that the outcome be efficient given the information that the people can infer from the outcome of the mechanism alone. Similarly we here demand that the strategy profile prescribes an action profile after messages are sent that is efficient given the information revealed by the messages sent. Thus, our players have more information than just the prescribed action profile as they in fact observe also the messages sent which in our case typically provide additional information. See also Kawakami (2016, page 897) on this point. Our definition of renegotiation-proofness necessarily also differs from Forges (1994) posterior efficiency in that in our strategic setting we impose the additional (sequential rationality) requirement that agents play an equilibrium action profile given the information they have. The domain of problems in Forges (1994) is the domain of Bayesian collective choice problems, in which agents only choose which message to send but do not make any other strategic choices. The contribution of our paper is not the possibly slightly novel solution concept, but the characterization of this, we believe in our context most appropriate solution concept, for our problem of communication in coordination game with private values.

Another "path" to our paper is the literature on costless pre-play communication, especially when paired with an evolutionary analysis. There are various strands of literature on this topic, but perhaps the most germane is the literature that started with studying costless pre-play communication before players engage in a complete information coordination game by Robson (1990) (see also the earlier related notion of "green beard effect" in Hamilton, 1964; Dawkins, 1976). This has spurred a sizable literature including Sobel (1993), Blume, Kim, and Sobel (1993), Wärneryd (1993), Kim and Sobel (1995), Bhaskar (1998), and Hurkens and Schlag (2003). Simplifying, the general insights are as follows. Suppose that a complete information coordination game has two Pareto-rankable equilibria. Then the Pareto-inferior equilibrium is not evolutionary stable as it can be invaded by mutants who use a previously unused message as a secret handshake: if their opponent does not use the same handshake they simply play the inferior Pareto equilibrium (as do all incumbents), but if their opponent also uses the secret handshake they both play the Pareto-superior equilibrium. Suppose the game has an equilibrium that is not Pareto-dominated by another equilibrium but is Pareto-dominated by some non-equilibrium strategy profile. Then the same argument would suggest that the so Pareto-dominated equilibrium is unstable, yet the mutant strategy profile - by virtue of not being an equilibrium - is itself also unstable. To avoid this one can appeal to the notion of "robustness to equilibrium entrants" introduced by Swinkels (1992) that only those mutants are considered that when they play against each other mutants play an equilibrium. Then, for instance, everyone defecting is the unique strategy in the prisoner's dilemma that is stable with respect to equilibrium entrants (mutants). Our notion of renegotiation-proofness has a similar flavor as we request that a renegotiation-proof equilibrium is both an equilibrium and efficient among equilibria. Given the incomplete information in our model we believe that the most appropriate place for applying this idea is at the stage after messages are sent, essentially at the posterior stage in the language of Forges (1994) as explained above.[18]

Another related literature deals with stable equilibria in coordination games with private values, but without pre-play communication. Sandholm (2007) (extending earlier results of Fudenberg and Kreps, 1993; Ellison and Fudenberg, 2000) shows that mixed Nash equilibria of the game with complete information can be purified in the sense of Harsanyi (1973) in an evolutionary stable way (see also Remark 3). Finally, two related papers analyse stag-hunt games with private values. Baliga and Sjöström (2004) show that introducing pre-play communication induces a new equilibrium in which the Pareto-dominant action profile is played with a high probability. Jelnov, Tauman, and Zhao (2018) show that in some cases a small probability to have another interaction can have a substantial influence on the set of equilibrium outcomes in stag-hunt games with private values.

This paper improves our understanding of if, and if so, how and what kind of coordination can be achieved in situations in which there are players who can freely communicate and who face what is primarily a coordination problem but have private preferences over which action they would rather coordinate on. This paper takes us we believe one step closer towards what Schelling (1960, p. 54, Chapter I.3) termed the "most interesting and most important situations" that entail a coordination problem while taking into the account the possibility of some conflict of interest (with private information about the extent of conflict). Crawford and Haller (1990, p.592), in their seminal work on complete information coordination problems without communication, state that

> "[t]he possibility of pre-play communication raises an important question about our analysis. Because our players have identical preferences, sensible use of costless communication would presumably solve the coordination problems we study. But characterizing the effects of costless communication requires overcoming formidable multiple-equilibrium problems, as in Crawford and Sobel (1982). [...] In real relationships, differences in players' preferences impede truthful communication."

Our paper is an attempt to provide exactly this by applying the plausible refinement of renegotiation-proofness to obtain sharp predictions. The main takeaways of our analysis are as follows (each of which induces empirically testable predictions). First, very little communication (namely, an

[18] Recently, Newton (2017) provides an evolutionary foundation for players developing the ability to renegotiate into a Pareto-better outcome ("collaboration" in the terminology of Newton).

opportunity to communicate two bits) significantly alters the predicted play (relative to the setup with no communication, see Remark 3), by implying the testable predictions that players will always coordinate, and that players will always play a jointly-preferred outcome if such an outcome exists. The second insight is that, regardless of the length and breadth of the communication, players can only communicate their ordinal preferences (i.e, what is their preferred outcome), but they cannot use cheap-talk to credibly communicate the intensity of their preferences (i.e., how much one coordinated outcome is better than another).

This latter insight may be relevant for anti-trust policy. When oligopolistic firms try to collude it is often the case that successful collusion depends on the firms' private preferences. For example, consider firms trying to implement a market sharing agreement by which a firm sells in certain regions whereas the rivals sell in other regions, where each firm has private information about which regions they would prefer to serve. Our findings strengthens the importance of not allowing even a very brief form of explicit communication between oligopolistic competitors, as this can substantially facilitate collusion.[19]

A Proofs

A.1 Undominated Action Strategies

In this appendix we show that our restriction to threshold action functions is without loss of generality, in the sense that each generalized action function is dominated by a threshold strategy.

Let $\Gamma(F,G)$ be a coordination game without communication (possibly played after observing a pair of messages in the original game $\langle\Gamma, M\rangle$). A generalized strategy in this game is a measurable function $\eta : U \to \Delta(\{L,R\})$ that describes a mixed action as a function of the player's type. A generalized strategy in $\Gamma(F,G)$ corresponds to a generalized action function $\xi : U \times M \times M \to \triangle\{L,R\}$ (as defined in Remark 1), given a specific pair of observed messages (m,m'), i.e., $\eta(u) \equiv \xi(u,m,m')$.

A pair of generalized strategies $\eta, \tilde{\eta}$ are almost-surely realization equivalent (abbr., *equivalent*), which we denote by $\eta \approx \tilde{\eta}$, if they induce the same behavior with probability one, i.e., if

$$\mathbb{E}_{u\sim F}\left[\eta(u) \neq \tilde{\eta}(u)\right] \equiv \int_{u\in U} f(u)\, \mathbf{1}_{\{\eta(u)\neq\tilde{\eta}(u)\}} du = 0.$$

It is immediate that two equivalent generalized strategies always induce the same (ex-ante) payoff, i.e., that $\pi(\eta,\eta') = \pi(\tilde{\eta},\eta')$ for each generalized strategy η'.

[19]A specific example is the 1997 series of FCC ascending auctions allocating licenses for slices of the electromagnetic spectrum, in which some firms used the trailing digits of their bids to reveal information on their preferred geographic areas and frequencies, and the firms used this information to collude (see Cramton and Schwartz, 2000).

A generalized strategy is a *cutoff strategy* if there exists a type $x \in [0,1]$ such that $\eta(u) = L$ for each $u < x$ and $\eta(u) = R$ for each $u > x$. A generalized strategy η is *strictly dominated* by generalized strategy $\tilde{\eta}$ if $\pi(\eta, \eta') < \pi(\tilde{\eta}, \eta')$ for any generalized strategy η' of the opponent.

The following result shows that any generalized strategy is either equivalent to a cutoff strategy, or it is strictly dominated by a cutoff strategy.

Lemma 1. *Let η be a generalized strategy. Then there exists a cutoff strategy $\tilde{\eta}$, such that either η is equivalent to $\tilde{\eta}$, or η is strictly dominated by $\tilde{\eta}$.*

Proof. If $\mathbb{E}_{u \sim F}[\eta_u(L)] = 1$ (resp., $\mathbb{E}_{u \sim F}[\eta_u(L)] = 0$), then η is equivalent to the cutoff strategy of always playing L (resp., R). Thus, suppose that $\mathbb{E}_{u \sim F}[\eta_u(L)] \in (0,1)$. Let $x \in (0,1)$ be such that $F(x) = \mathbb{E}_{u \sim F}[\eta_u(L)] = \int_u \eta_u(L) f(u) du$. Let $\tilde{\eta}$ then be the cutoff strategy with cutoff x, i.e.,

$$\tilde{\eta}_u(L) = \begin{cases} 1 & u \leq x \\ 0 & u > x. \end{cases}$$

Assume that η and $\tilde{\eta}$ are not equivalent, i.e., $\eta \not\approx \tilde{\eta}$. Let η' be an arbitrary generalized strategy of the opponent. By construction strategies η and $\tilde{\eta}$ induce the same average probability of choosing L. Strategies $\tilde{\eta}$ and η differ in that $\tilde{\eta}$ induces lower types to choose L with a higher probability, and higher types to choose L with a lower probability, i.e., $\eta_u(L) \leq \tilde{\eta}_u(L)$ for any type $u \leq x$ and $\eta_u(L) \geq \tilde{\eta}_u(L)$ for any type $u > x$. The fact that $\eta \not\approx \tilde{\eta}$ and $\mathbb{E}_{u \sim F}[\eta_u(L)] \in (0,1)$ imply that the inequalities are strict for a positive measure of types, i.e.,

$$0 < \int_{u<x} f(u) \mathbf{1}_{\{\eta(u)<\tilde{\eta}(u)\}} du \text{ and } 0 < \int_{u>x} f(u) \mathbf{1}_{\{\eta(u)>\tilde{\eta}(u)\}} du.$$

The fact that lower types always gain more (resp., less) from choosing L (*resp., R*) relative to higher types, where the inequality is strict unless the opponent always plays R (resp., L), implies that $\pi(\eta, \eta') < \pi(\tilde{\eta}, \eta')$. □

A.2 Proof of Theorem 1

We first prove the "if" part of the theorem. Suppose $\sigma = (\mu, \xi) \in \Sigma$ is mutual-preference consistent, coordinated, and has binary communication.

As σ is *mutual-preference consistent* it must satisfy $\text{supp}(F_m) \subseteq [0, 1/2]$ or $\text{supp}(F_m) \subseteq [1/2, 1]$ for any message $m \in \text{supp}(\mu)$, a property we term *ordinal preference revealing* in Appendix B. Consider any message pair $m, m' \in \text{supp}(\mu)$. There are three cases to consider. Suppose first that both $\text{supp}(F_m), \text{supp}(F_{m'}) \subseteq [0, 1/2]$. Then as σ is *mutual-preference consistent* we have that

$\xi(m, m') = \xi(m', m) = L$. Thus ξ describes best response behaviour after this message pair. Moreover this behavior is the best possible outcome for any type in $[0, 1/2]$ and thus for any type in $\text{supp}(F_m)$ and $\text{supp}(F_{m'})$. The second case of both $\text{supp}(F_m), \text{supp}(F_{m'}) \subseteq [1/2, 1]$ is analogous. Suppose finally, that, w.l.o.g., $\text{supp}(F_m) \subseteq [0, 1/2]$ and $\text{supp}(F_{m'}) \subseteq [1/2, 1]$. As σ is *coordinated* we have that $\xi(m, m') = \xi(m', m) = L$ or $\xi(m, m') = \xi(m', m) = R$. Action function ξ, therefore, again describes best response behavior. Moreover, one player always obtains her most preferred outcome. Any way to improve the outcome for the other player would require the former player to deviate from her most preferred outcome. Thus, there is no message set $\tilde{M}$ such that an equilibrium strategy σ' in the game $\langle \Gamma(F_m, F_{m'}), \tilde{M} \rangle$ Pareto dominates σ after this message pair.

All this shows that action function ξ is a best response to μ and to itself given μ and that, moreover, it cannot be post-communication equilibrium Pareto-improved upon. It remains to be shown that the message function μ is optimal given the opponent chooses $\sigma = (\mu, \xi)$.

Consider type $u \in [0, 1/2]$ and consider this type's choice of message. As σ has *binary communication* and is *coordinated* different messages $m \in M$ can only trigger different probabilities of coordinating on L with a highest likelihood of such coordination for any message $m \in \text{supp}(\mu_u)$. Type u is, therefore, indifferent between any message $m \in \text{supp}(\mu_u)$ and weakly prefers any $m \in \text{supp}(\mu_u)$ over any message $m' \notin \text{supp}(\mu_u)$. An analogous statement holds for types $u \in [1/2, 1]$. This concludes the proof of the "if" part of the theorem.

We prove the "only if" part in three lemmas, each proving one of the three properties.

Lemma 2. *Every renegotiation-proof equilibrium strategy $\sigma = (\mu, \xi)$ is coordinated.*

Proof. We need to show that for any message pair $m, m' \in \text{supp}(\mu)$ either

$$\xi(m, m') \geq \sup\{u \mid \mu_u(m) > 0\} \text{ or } \xi(m, m') \leq \inf\{u \mid \mu_u(m) > 0\}.$$

Let $m, m' \in \text{supp}(\mu)$ and assume to the contrary that

$$\inf\{u \mid \mu_u(m) > 0\} < \xi(m, m') < \sup\{u \mid \mu_u(m) > 0\}.$$

As σ is an equilibrium, we must have that also

$$\inf\{u \mid \mu_u(m') > 0\} < \xi(m', m) < \sup\{u \mid \mu_u(m') > 0\}.$$

Otherwise the m' message sender plays L with probability one or R with probability one, in which case the m message sender's best response would be to play L (or R) regardless of her type. Let $x = \xi(m, m')$ and $x' = \xi(m', m)$. We now show that the equilibrium (x, x') of the game without coordination $\Gamma(F_m, F_{m'})$ is equilibrium Pareto-dominated in this game by either σ_L, σ_R, or σ_C.

There are three cases to be considered. Case 1: Suppose that $x, x' \leq 1/2$. We now show that in this case the equilibrium (x, x') is Pareto-dominated by σ_R. Consider the player who sent message m.

Case 1a: Consider a type $u \leq x$. Then we have

$$(1-u)F_{m'}(\tfrac{1}{2}) + u\left(1 - F_{m'}(\tfrac{1}{2})\right) \geq (1-u)F_{m'}(x'),$$

with the left-hand-side such a u-type's payoff under strategy profile σ_R and the right-hand-side the payoff under strategy profile (x, x'). The inequality follows from the fact that $u\,(1 - F_{m'}(1/2)) \geq 0$ and $F_{m'}(1/2) \geq F_{m'}(x')$ by the fact that $F_{m'}$ is non-decreasing (as it is a cumulative distribution function). This inequality is strict for all u except for $u = 0$ in the case $x' = 1/2$.

Case 1b: Now consider a type u with $x < u \leq 1/2$. Then we have

$$(1-u)F_{m'}(\tfrac{1}{2}) + u\left(1 - F_{m'}(\tfrac{1}{2})\right) > u\,(1 - F_{m'}(x'))\,,$$

with the left-hand-side such a u-type's payoff under strategy profile σ_R and the right-hand-side the payoff under strategy profile (x, x'). The inequality follows from the observations that by $u \leq 1/2$ we have that $1 - u \geq u$ and therefore $(1-u)F_{m'}(1/2) + u\,(1 - F_{m'}(1/2)) \geq u$.

Case 1c: Finally, consider a type $u > 1/2$. Then we have

$$u > u\,(1 - F_{m'}(x'))\,.$$

with the left-hand-side such a u-type's payoff under strategy profile σ_R and the right-hand-side the payoff under strategy profile (x, x').

The analysis for the player who sent message m' is analogous.

Case 2: Suppose that $x, x' \geq 1/2$. The analysis is analogous to Case 1 if we replace σ_R with σ_L.

Case 3: Suppose, w.l.o.g. for the remaining cases, that $x \leq 1/2 \leq x'$. The equilibrium (x, x') in this case is Pareto-dominated by σ_C. To see this, consider the player who sent message m.

Case 3a: Consider a type $u \leq x$. Then we have

$$(1-u)\left[F_{m'}(\tfrac{1}{2}) + \tfrac{1}{2}\left(1 - F_{m'}(\tfrac{1}{2})\right)\right] + u\tfrac{1}{2}\left(1 - F_{m'}(\tfrac{1}{2})\right) > (1-u)F_{m'}(x'),$$

with the left-hand-side such a u-type's payoff under strategy profile σ_C and the right-hand-side the payoff under strategy profile (x, x'). The inequality follows from the observations that $F_{m'}(x') \leq 1/2$ by the fact that $F_{m'}(x') = x$ when (x, x') is an equilibrium.

Case 3b: Now consider a type u with $x < u \leq 1/2$. Then we have

$$(1-u)\left[F_{m'}(\tfrac{1}{2}) + \tfrac{1}{2}\left(1 - F_{m'}(\tfrac{1}{2})\right)\right] + u\tfrac{1}{2}\left(1 - F_{m'}(\tfrac{1}{2})\right) > u\left(1 - F_{m'}(x')\right),$$

with the left-hand-side such a u-type's payoff under strategy profile σ_C and the right-hand-side the payoff under strategy profile (x, x'). The inequality follows from the observations that given $u \leq 1/2$ we have $1 - u \geq u$ and, thus,

$$(1-u)\left[F_{m'}(\tfrac{1}{2}) + \tfrac{1}{2}\left(1 - F_{m'}(\tfrac{1}{2})\right)\right] + u\tfrac{1}{2}\left(1 - F_{m'}(\tfrac{1}{2})\right) \geq u.$$

Case 3c: Finally, consider a type $u > 1/2$. Then we have

$$u\left[\left(1 - F_{m'}(\tfrac{1}{2})\right) + \tfrac{1}{2}F_{m'}(\tfrac{1}{2})\right] + (1-u)\tfrac{1}{2}F_{m'}(\tfrac{1}{2}) > u\left(1 - F_{m'}(x')\right),$$

with the left-hand-side such a u-type's payoff under strategy profile σ_C and the right-hand-side the payoff under strategy profile (x, x'). The inequality follows from the observations that $F_{m'}(1/2) > 0$ and $F_{m'}(1/2) \leq F_{m'}(x')$ by the fact that $F_{m'}$ is non-decreasing.

The analysis for the player who sent message m' is analogous. □

Lemma 3. *Every renegotiation-proof equilibrium strategy σ has binary communication.*

Proof. Let σ be a renegotiation-proof strategy. Recall that

$$\beta^\sigma(m) = \int_{u=0}^{1} \sum_{m' \in M} \mu_u(m') \mathbf{1}_{\{u \leq \xi(m,m')\}} f(u) du.$$

As σ is coordinated by Lemma 2, the payoff to a u-type from sending message m can be written as

$$(1-u)\beta^\sigma(m) + u\left(1 - \beta^\sigma(m)\right).$$

For a type $u < 1/2$ the problem of choosing a message to maximize her payoffs is thus equivalent to choosing a message that maximizes $\beta^\sigma(m)$. We thus must have that there is a $\overline{\beta}^\sigma \in [0, 1]$ such that for all $u < 1/2$ and all $m \in \text{supp}(\mu_u)$ $\beta^\sigma(m) = \overline{\beta}^\sigma$. Analogously, we must have a $\underline{\beta}^\sigma \in [0, 1]$ such that for all $u > 1/2$ and all $m \in \text{supp}(\mu_u)$ $\alpha^\sigma(m) = \overline{\beta}^\sigma$. Clearly also $\underline{\beta}^\sigma \leq \overline{\beta}^\sigma$. □

Lemma 4. *Every renegotiation-proof equilibrium strategy σ is mutual-preference consistent.*

Proof. By Lemma 2 a renegotiation-proof equilibrium strategy $\sigma = (\mu, \xi)$ is coordinated. Suppose it is not mutual-preference consistent. This means that there is either a message pair (m, m') such that there are types $u, v < 1/2$ with $m \in \text{supp}(\mu_u)$ and $m' \in \text{supp}(\mu_v)$ such that play after (m, m') is coordinated on R, or a message pair (m, m') such that there are types $u, v > 1/2$ with $m \in \text{supp}(\mu_u)$

and $m' \in \text{supp}(\mu_v)$ such that play after (m, m') is coordinated on L. Consider, w.l.o.g., the first case. Then strategy σ_R provides a Pareto improvement in the game $\left\langle \Gamma(F_m, F_{m'}), \hat{M} \right\rangle$ a message set $\hat{M}$ with at least two elements: it does not affect the payoff of any type greater than $1/2$ after message pair (m, m') and strictly improves the payoff to all types $u, v < 1/2$. $\square$

A.3 Proofs of Section 6 (On Efficiency)

Proof of Proposition 1. By Theorem 1 and Remark 2 the equilibrium payoff of a renegotiation-proof strategy σ is determined by its left-tendency $\alpha \equiv \alpha^\sigma \in [0, 1]$ and, for each type $u \in [0, 1/2]$ given by

$$\pi_u(\sigma, \sigma) = (1-u)\left[F(\tfrac{1}{2}) + \alpha\left(1 - F(\tfrac{1}{2})\right)\right] + u(1-\alpha)\left[1 - F(\tfrac{1}{2})\right],$$

and for each type $u \in (1/2, 1]$ given by

$$\pi_u(\sigma, \sigma) = (1-u)\alpha F(\tfrac{1}{2}) + u\left[\left(1 - F(\tfrac{1}{2})\right) + F(\tfrac{1}{2})(1-\alpha)\right].$$

The payoff to a u-type for a given outcome function ϕ is given by

$$\pi_u(\phi) = (1-u)\,\mathbb{E}_v\phi_{u,v}(L, L) + u\mathbb{E}_v\phi_{u,v}(R, R).$$

Now suppose that ϕ interim (pre-communication) Pareto dominates σ. Then $\pi_u(\phi) \geq \pi_u(\sigma, \sigma)$ for all $u \in [0, 1]$ with a strict inequality for a positive measure of u. As $\pi_u(\sigma, \sigma)$ is a convex combination of two payoffs, this implies that

$$\mathbb{E}_v\phi_{u,v}(L, L) \geq F(\tfrac{1}{2}) + \alpha\left(1 - F(\tfrac{1}{2})\right) \text{ for any } u \leq 1/2 \text{ and}$$

$$\mathbb{E}_v\phi_{u,v}(R, R) \geq \left(1 - F(\tfrac{1}{2})\right) + F(\tfrac{1}{2})(1-\alpha) \text{ for any } u > 1/2,$$

with at least one of the inequalities holding strictly for a positive measure of types. We can write

$$\mathbb{E}_v\phi_{u,v}(L, L) = F(\tfrac{1}{2})\mathbb{E}_{\{v \leq 1/2\}}\phi_{u,v}(L, L) + \left(1 - F(\tfrac{1}{2})\right)\mathbb{E}_{\{v > 1/2\}}\phi_{u,v}(L, L),$$

where, for instance, $\mathbb{E}_{\{v>1/2\}}$ denotes the expectation conditional on $v > 1/2$. To have

$$F(\tfrac{1}{2})\mathbb{E}_{\{v \leq 1/2\}}\phi_{u,v}(L, L) + \left(1 - F(\tfrac{1}{2})\right)\mathbb{E}_{\{v > 1/2\}}\phi_{u,v}(L, L) \geq F(\tfrac{1}{2}) + \alpha\left(1 - F(\tfrac{1}{2})\right)$$

for any $u \leq 1/2$, by the fact that $\mathbb{E}_{\{v \leq 1/2\}}\phi_{u,v}(L, L) \leq 1$, we must then have that $\mathbb{E}_{\{v>1/2\}}\phi_{u,v}(L, L) \geq \alpha$ for any $u \leq 1/2$ and, by an analogous argument that $\mathbb{E}_{\{v<1/2\}}\phi_{u,v}(R, R) \geq 1 - \alpha$, for any $u > 1/2$, with at least one of the inequalities holding strictly for a positive measure of types.

This implies that

$$\mathbb{E}_{\{u<1/2\}}\mathbb{E}_{\{v>1/2\}}\phi_{u,v}(L,L) \geq \alpha \text{ and } \mathbb{E}_{\{u>1/2\}}\mathbb{E}_{\{v<1/2\}}\phi_{u,v}(R,R) \geq 1-\alpha,$$

with at least one of the two inequalities holding strictly. By the symmetry of ϕ we have $\phi_{u,v}(R,R) = \phi_{v,u}(R,R)$ and thus,

$$\mathbb{E}_{\{u<1/2\}}\mathbb{E}_{\{v>1/2\}}\phi_{u,v}(L,L) + \mathbb{E}_{\{u<1/2\}}\mathbb{E}_{\{v>1/2\}}\phi_{u,v}(R,R) > 1,$$

a contradiction to ϕ being a probability distribution, and thus, to our supposition. □

The proof of Proposition 2 uses the following lemma (which is of some independent interest).

Lemma 5. *Let $\sigma \in \mathcal{E}$ be a coordinated equilibrium strategy. Then there is a renegotiation-proof strategy σ' such that either σ and σ' are interim (pre-communication) payoff equivalent or σ' interim (pre-communication) Pareto dominates σ.*

Proof. Let $\sigma = (\mu,\xi) \in \mathcal{E}$ be coordinated. For each message $m \in M$, let $p_m \in [0,1]$ be the probability that the players coordinate on L, conditional on the agent sending message m:

$$p_m = \sum_{m' \in M} \mu(m')\,\mathbf{1}_{\{\xi(m,m')=L\}}.$$

As σ is coordinated $1-p_m$ is then the probability that the players coordinate on R, conditional on the agent sending message m.

Let $\bar{p} = \max_{m\in M} p_m$ be the maximal probability, and let $\underline{p} = \min_{m\in M} p_m$ be the minimal probability. By definition $\underline{p} \leq \bar{p}$. As σ is an equilibrium strategy, $\underline{p} < \bar{p}$ would imply that all types $u < 1/2$ send a message inducing probability $\bar{p}$ and all type $u > 1/2$ send a message inducing probability $\underline{p}$. We, therefore, must have that the expected payoff of a type $u \leq 1/2$ is given by

$$\pi_u(\sigma,\sigma) = \bar{p}(1-u) + (1-\bar{p})\,u,$$

and the expected payoff of any type $u > 1/2$ is equal to

$$\pi_u(\sigma,\sigma) = \underline{p}(1-u) + \left(1-\underline{p}\right) u.$$

This is also true if $\underline{p} = \bar{p}$. Note that for every type $u < 1/2$ this type's expected payoff strictly increases in $\bar{p}$ and for every type $u > 1/2$ this type's expected payoff strictly decreases in $\underline{p}$.

We consider three cases. Suppose first that $\underline{p} \leq \bar{p} \leq F(1/2)$. Then let $\sigma' = \sigma_R$. This strategy is also coordinated and its induced payoffs can be written in the same form as those for strategy σ with

$\underline{p}' = 0$ and $\bar{p}' = F(1/2)$. Thus, we get that $\pi_u(\sigma', \sigma') \geq \pi_u(\sigma, \sigma)$ for every $u \in [0, 1]$. This implies that σ is either interim (pre-communication) payoff equivalent to or Pareto-dominated by $\sigma' = \sigma_R$.

The second case where $F(1/2) \leq \underline{p} \leq \bar{p}$ is analogous to the first one, with $\sigma' = \sigma_L$.

In the final case $\underline{p} < F(1/2) < \bar{p}$. Let $\alpha \in [0, 1]$ be such that $F(1/2) + (1 - F(1/2))\alpha = \bar{p}$ and let σ' be a renegotiation-proof strategy with left tendency α. Then $\underline{p} \geq \alpha F(1/2)$ and by construction σ' is either interim (pre-communication) payoff equivalent to or Pareto dominates σ. □

Proof of Proposition 2. By Lemma 5 we have that every coordinated equilibrium strategy σ is interim (pre-communication) Pareto-dominated by some renegotiation-proof strategy with some left tendency $\alpha \in [0, 1]$ denoted by σ_α. We thus have that $\pi(\sigma, \sigma) \leq \pi(\sigma_\alpha, \sigma_\alpha)$.

The ex-ante expected payoff of to a u type under strategy σ_α is given by

$$\pi_u(\sigma_\alpha, \sigma_\alpha) = (1 - u)\left[F(\tfrac{1}{2}) + \alpha\left(1 - F(\tfrac{1}{2})\right)\right] + u(1 - \alpha)\left(1 - F(\tfrac{1}{2})\right) \text{ for } u \leq 1/2 \text{ and}$$

$$\pi_u(\sigma_\alpha, \sigma_\alpha) = (1 - u)\,\alpha F(\tfrac{1}{2}) + u\left[1 - F(\tfrac{1}{2}) + (1 - \alpha)F(\tfrac{1}{2})\right] \text{ for } u > 1/2.$$

It is straightforward to verify that for every u,

$$\pi_u(\sigma_\alpha, \sigma_\alpha) = \alpha\pi_u(\sigma_1, \sigma_1) + (1 - \alpha)\pi_u(\sigma_0, \sigma_0).$$

As $\sigma_1 = \sigma_L$ and $\sigma_0 = \sigma_R$ and as for all $u \in [0, 1]$ $\pi_u(\sigma_\alpha, \sigma_\alpha)$ is the same convex combination of $\pi_u(\sigma_L, \sigma_L)$ and $\pi_u(\sigma_R, \sigma_R)$ we get that

$$\pi(\sigma_\alpha, \sigma_\alpha) = \alpha\pi(\sigma_1, \sigma_1) + (1 - \alpha)\pi(\sigma_0, \sigma_0),$$

which implies that $\pi(\sigma, \sigma) \leq \pi(\sigma_\alpha, \sigma_\alpha) \leq \max\{\pi(\sigma_L, \sigma_L), \pi(\sigma_R, \sigma_R)\}$. □

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