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Meng-Wei Chen†, Yu Chen‡, Zhen-Hua Wu§ and Ningru Zhao¶

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Abstract

We study how government intervention affects innovation and entrepreneurship, using a model in which two agents (e.g., one entrepreneur and one venture capitalist) engage in teamwork to launch a new business in which a moral hazard problem may persist for both parties. One feature of our model is that the government’s financial support (grant) may have (positive) externalities on the teamwork of both parties, but is also constrained by budget costs. We compare two major forms of government intervention: indirect intervention and hybrid intervention. Contrasted to the case without government intervention, indirect government intervention always raises the efforts of both parties and promotes social surplus (welfare) while hybrid government intervention may not always raise the efforts of both parties or promote social surplus. Hybrid government intervention may, however, deliver even higher social surplus than indirect government intervention when the government’s share in the enterprise is dominant and its marginal contribution to the project is sufficiently high.

Key Words: Government intervention, moral hazard, innovation, entrepreneurship

JEL Classifications: D80, H20, O30, O38

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1 Introduction

In most entrepreneurship on innovative business strategies or projects, the use of teamwork is ubiquitous. Meanwhile, the government typically has a strong tendency to intervene in innovation and entrepreneurship to raise social welfare. On the one hand, innovation in the private sector may not be sufficiently desirable due to potential market failure. Takalo et al. (2013)\cite{21} argue that “the private sector is likely to invest suboptimally in R&D because of appropriability problems and potential market failures in the provision of private funding to R&D.” Freeman and Soete (1997)\cite{5} argue that the level of private firms’ R&D might be lower than the socially optimal level, which is due to the risk and high uncertainty in the R&D process. On the other hand, government intervention may correct the distortion of market failure to some degree. Nelson (1959)\cite{15} and Arrow (1962)\cite{1} argue that government funding is essential for fundamental research due to the risk of market failure. Martin and Scott (2000)\cite{12} posit that “The knowledge inappropriability and uncertainty in obtaining returns for long-term commitment often lead to firms’ under-investment in R&D, which calls for impetus from the public sector.” Link and Siegel (2007)\cite{11} point out that technological developments often involve costs that go beyond the financial and technical capabilities of most private firms, and require government assistance.

Nevertheless, there has been long-standing debate about the specific means used for government intervention in promoting innovation, from the perspectives of both theory and practice. For instance, debate over the ideal role of government in the economy seems to be polarized between neoliberalism, which favors market-led development, and statism, which favors government intervention (Yeung et al. 2000)\cite{27}. Masters and Delbecq (2008)\cite{13} examine the design of grants, contracts, public-private partnerships, and other payment mechanisms used by governments and philanthropic donors to complement private investment. The authors focus on the role of ex post prizes in the innovation process, and identify a combination of circumstances under which alternative mechanisms and a new kind of prize payment could accelerate and guide the innovation process. The major innovation
policy tools used in practice may include intellectual property, subsidies, tax incentives, prizes and contests, and public production and procurement, etc. Takalo (2012)[20] reviews the economic justifications for a wide variety of public innovation policies, and compares existing policy tools. Clearly, different means of government intervention may lead to different innovation performance and outcomes. As Wang (2018)[23] points out, the ideal role of government in the economy is also partly due to the difficulties of assessing the impact of government intervention on innovation performance, given the presence of various confounding factors. Eventually, “any public innovation policy tool should only be judged on whether it yields a net increase in social welfare” (Takalo 2012)[20]. Meanwhile, it becomes more and more important to evaluate different means of government intervention. As Sakakibara (2001)[19] points out, there is increased interest in OECD countries in the evaluation of government programs for innovation and technology; this is driven in part by budgetary stringency and in part by a greater concern for accountability and transparency in government programs (OECD, 1997)[17].

This paper studies how government intervention will affect innovation and entrepreneurship in a case in which two agents (e.g., one entrepreneur and one investor) engage in teamwork to launch an innovative business enterprise or R&D project. The moral hazard problem is present for both parties, since their efforts in the enterprise are normally hidden, but jointly affect the probability of the outcome of the risky project. Greater effort will raise the probability of success of the business project. This setting is similar to that of Yang (2010)[25], and Yang et al. (2018)[26].

In addition, we consider two major forms of government intervention: indirect intervention and hybrid intervention. With indirect intervention, the government offers a subsidized scheme, consisting of an up-front payment (grant) and an ex post prize for a successful project, to stimulate innovation, but it will not participate in the project directly (without acquiring any share of the enterprise). Kalil (2006)[10] states that under certain circumstances, inducement prizes may act as a useful complement to grants and contracts as
a way to encourage technological innovation. The government can establish a goal without
determining who is in the best position to reach the goal or what the most promising
technical approach is. In practice, the subsidized scheme can take either the form of a
direct subsidy or tax credit. These are also two widely used instruments for supporting
R&D in empirical studies (Aerts et al., 2004; Almus and Czarnitzki, 2003; David et al.,
2000; Hall and van Reenen, 2000; Martin and Scott, 2000). In the hybrid intervention,
the government not only offers the incentive scheme, but also participates in the project
directly by acquiring some share of the enterprise. For instance, according to Alperovych et
al. (2015)[2], “Many governments have attempted to achieve the mentioned benefits of VC
financing by initiating their own programs, often through independent government-sponsored
VC (GVC) investment funds.” NRF(2015)[16] shows that the Singapore government started
a series of five-year national plans for science and technology, and set up a Technopreneurship
Innovation Fund to promote high-tech entrepreneurship by co-investing with venture
capitalists in new businesses.

A feature of our model is that the government’s up-front grant has (positive) externalities
on the teamwork of both parties, but is also constrained by budget costs. The development
of some technology may involve high cost, and it also may be hard for new firms to
finance their projects. In these circumstances, innovative projects might need government
assistance in terms of providing a certain amount of start-up funds or charging lower rent
for work space. In other words, the existence of government grants could not only provide a
mechanism to lower the entrance barrier for innovative projects, but also boost innovation
activities if the market fails and entrepreneurs find it difficult to start their projects. As
Holmström and Tirole (1997)[8] point out, outside investors are wary of investing in the
projects of entrepreneurs who cannot put down a sufficient amount of their own capital.
If entrepreneurs do not retain a sufficient stake in project outcomes, financiers cannot
be sure about the entrepreneurs’ motivation. This creates a funding gap by which even
unambiguously profitable projects are not launched if the entrepreneurs do not have enough
liquid assets. González and Pazó (2008) argue that such public interventions are primarily intended to reduce the effective cost of R&D, induce firms to invest in research, and improve the efficiency of innovation activities. Moreover, the setting of cost synergies is similar to Edmans et al. (2011) and Yang et al. (2018). Meanwhile, a government clearly has to consider its potential costs of providing any up-front grant. This is related to budget stringency and a greater concern for accountability and transparency in government programs (OECD, 1997).

Our main finding is that the government’s indirect intervention is always preferred to not intervening, but it may be dominated by hybrid intervention. Indirect intervention always raises the efforts of both parties and promotes social surplus (welfare) relative to the case without government intervention. In the indirect intervention, the up-front grant also plays the role of incentive provision, just as an ex post prize does, basically due to its externality effect. By contrast, the government’s hybrid intervention may not always raise the efforts of both parties or promote social surplus relative to the case without government intervention. The trade-off is between the marginal contribution to the project and the government’s share in the enterprise from its direct participation. Hybrid intervention may deliver an even higher social surplus than the government’s indirect intervention when the government’s share in the enterprise is dominant and its marginal contribution to the project is sufficiently high. The spillover effect of innovation usually strengthens our results in favor of government intervention.

This paper contributes to the theory of government intervention in innovation and entrepreneurship in terms of contract design. A number of studies concern the contracting relationship between a government funder and an innovative firm or researcher (see Wright (1983); Fu et al. (2012); Che et al. (2017); and Rietzke and Chen (2018); among many others). However, unlike our work, these studies do not address the teamwork in entrepreneurship between agents or the share structure in the enterprise. Hirsch (2006)

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1This fact is somewhat opposite to the dominant importance of prizes over grants in most existing literature, e.g., Masters and Delbecq (2008), and Murray et al. (2012).
examines the effects of public policy programs that aim at internalizing spillover due
to successful innovation in a sequential double-sided moral hazard double-sided adverse
selection framework, in which the government can only subsidize one entrepreneur and the
entrepreneur makes a take-it-or-leave-it offer to a venture capitalist. They exert efforts
sequentially. However, in our model, the government intervenes in an enterprise already set
up by two key agents. They exert efforts simultaneously. The government may have more
options with direct participation in the enterprise. Moreover, Hirsch (2006) [9] claims that
ex ante grants and some types of investment grants depend strongly on the characteristics
of the project: In certain cases they not only offer no further incentives, but even destroy
contract mechanisms and so worsen the outcome. By contrast, our analysis still supports
the incentive provision of grants, even without consideration of adverse selection.

2 Basics

Let us consider an environment with two agents, $A_1$ and $A_2$ (e.g., an innovative entrepreneur
and an investor or venture capitalist), who are collaborating on a risky entrepreneurial project
with a binary outcome: success or failure. $e_1 \in [0, 1]$ is the hidden effort chosen by $A_1$. $A_2$ can
also provide effort $e_2 \in [0, 1]$ to reinforce $A_1$’s effort, such as providing expertise, consulting,
business clients, or extra investment. Then, the probability that the project succeeds is
$\rho(e_1, e_2)$. For tractability,

$$\rho(e_1, e_2) = \frac{m_1 e_1 + m_2 e_2}{2}.$$ 

For $i = 1, 2$, $m_i \in [0, 1]$ represents the marginal contribution of $A_i$’s effort to the success
of the project. If the project succeeds, it can generate a constant revenue $W > 0$. Otherwise,
it does not yield any revenue. After choosing effort level $e_i$, $A_i$ must pay cost $C_i(e_i)$.

The two agents have made an agreement about the shares within an established
enterprise. Specifically, they would divide the potential revenue according to the share
proportions: $\beta$ and $1 - \beta$, where $\beta \in (0, 1)$ is the share $A_1$ owns, and naturally $1 - \beta \in (0, 1)$
is the share $A_2$ owns. Therefore, $A_1$’s payoff function is

$$U_1 = \begin{cases} 
\beta W - C_1(e_1), & \text{with } \rho(e_1, e_2) \\
-c_1(e_1), & \text{with } 1 - \rho(e_1, e_2)
\end{cases},$$

and $A_2$’s payoff function is

$$U_2 = \begin{cases} 
(1 - \beta) W - C_2(e_2), & \text{with } \rho(e_1, e_2) \\
-c_2(e_2), & \text{with } 1 - \rho(e_1, e_2)
\end{cases}.$$  

### 3 Benchmark Model

We first consider a pure market benchmark without government intervention. In this benchmark, the game between two agents unfolds in two stages. In stage 1, $A_1$ and $A_2$ simultaneously choose effort level $e_1$ and $e_2$, respectively. In stage 2, the outcome is realized according to $\rho(e_1, e_2)$ and the outcome is divided according to the division $(\beta, 1 - \beta)$.

Thus, we examine the two agents’ optimal decisions. They simultaneously choose effort levels, given the sharing rule $(\beta, 1 - \beta)$. We further assume that the costs function for $A_i$ take the quadratic form:

$$C_i(e_i) = ce_i^2,$$

where cost parameter $c > 0$. Here we consider a symmetric cost parameter over two agents for expository simplicity and focus on the different influences of their contributions to the success of the project. It also implies $A_i$’s productive efficiency can be represented by $m_i$. Given symmetric cost parameter $c$, the greater $m_i$ is, the higher $A_i$’s productive efficiency will be.

For $A_1$, its optimal effort level is determined by

$$e_1^* \in \max_{e_1} \beta \left( \frac{m_1 e_1 + m_2 e_2}{2} \right) W - ce_1^2.$$ (1)
For $A_2$, its optimal decision is determined by

$$e_2^* \in \max_{e_2} (1 - \beta) \left( \frac{m_1e_1 + m_2e_2}{2} \right) W - ce_2^2.$$  \hspace{1cm} (2)

Simultaneously solving the two maximization problems in Equations 1 and 2, we obtain the optimal efforts as follows:

$$e_1^* = \frac{\beta m_1 W}{4c},$$

$$e_2^* = \frac{(1 - \beta) m_2 W}{4c}.$$

Furthermore, we present the comparative statics results below.

**Proposition 1.** In the benchmark without government intervention,

1. for both $A_1$ and $A_2$, the larger share of the outcome always induces higher levels of efforts, i.e., the effort levels of $A_1$ and $A_2$ are increasing in their shares.
2. given the sharing rule, the effort levels of $A_1$ and $A_2$ are increasing in the revenue of the project, $W$.
3. given the sharing rule, the effort levels of $A_1$ and $A_2$ are increasing in their marginal contributions to success.

**Proof.** See the Appendix.

In this benchmark case, the optimal effort of an agent increases with his share of the outcome, the revenue of the project, and his marginal contribution to success. The higher levels of these parameters will all lead to higher expected revenue, and therefore induce the agents’ higher effort levels.

Moreover, we provide the welfare analysis in this benchmark. It would be interesting to know how agents’ welfare is associated with the changes in their marginal contributions and their shares in the benchmark model. Comparative statics results show that for each

\footnote{Note that as long as $W$ is positive, participation constraints always hold; that is, $e_i^* > 0$.}
agent, their welfare is increasing in their own and their partner’s marginal contributions. This result is straightforward, because an increase in an agent’s marginal contribution would induce an increase in the probability of success for their project. Therefore, agents would benefit from an increase of their own and their partner’s marginal contributions.

In addition, our benchmark model implies that the change in an agent’s welfare is determined by a relation between relative marginal contribution and the difference between the agent’s share measured in their own share. Specifically, each agent’s own utility increases with an increase in their own share when the square term of the ratio of their own marginal contribution to the teammate’s marginal contribution is larger than the difference in shares relative to his own share, e.g. for \( A_1 \), \((\frac{m_1}{m_2})^2 > \left( \frac{2\beta-1}{\beta} \right) \).

**Proposition 2.** In the benchmark without government intervention, \( A_1 \)’s utility is increasing in its marginal contribution to success \( m_1 \) \((\partial U_1/\partial m_1 > 0)\) and \( A_2 \)’s marginal contribution to success \( m_2 \) \((\partial U_1/\partial m_2 > 0)\), and increasing in its share \((\partial U_1/\partial \beta > 0)\) if and only if \((\frac{m_1}{m_2})^2 > \left( \frac{2\beta-1}{\beta} \right)\); \( A_2 \)’s utility is increasing in its marginal contribution to success \( m_2 \) and \( A_1 \)’s marginal contribution to success \( m_1 \), and is increasing in its share \((\partial U_2/\partial (1 - \beta) > 0)\) if and only if \((\frac{m_2}{m_1})^2 > \left( \frac{1-2\beta}{1-\beta} \right)\).

**Proof.** See the Appendix.

Proposition 2 shows that the agent’s welfare increases when the project’s success possibility is increased, which is induced by an increase in the agent’s marginal contribution. Basically, a larger share rewards one agent, and therefore increase his effort and also his welfare. Furthermore, total surplus is increasing in \( A_1 \)’s share when his relative contribution is sufficiently large relative to his relative share. Specifically, given that \( A_1 \) has a higher marginal contribution than \( A_2 \), although a higher incentive for \( A_1 \) from a higher share accompanies \( A_2 \)’s lower effort and lower welfare, the reduction in \( A_2 \)’s welfare will be dominated by an increase in \( A_1 \)’s welfare. Increasing \( A_1 \)’s share will mitigate the mismatching of relative contribution and relative revenue (in terms of share). This is summarized in Proposition 3 below.
Proposition 3. Total surplus is increasing in $A_1$’s share ($\frac{\partial U}{\partial \beta} > 0$) if and only if $(\frac{m_1}{m_2})^2 > (\frac{\beta}{1-\beta})$, and in $A_2$’s share if and only if $(\frac{m_2}{m_1})^2 > (\frac{1-\beta}{\beta})$. Total surplus increases with $A_1$’s marginal contribution to success $m_1$ and $A_2$’s marginal contribution to success $m_2$.

Proof. See the Appendix.

4 Indirect Government Intervention

In the benchmark without government intervention, the optimal decisions of $A_1$ and $A_2$ are only determined by their own allocation of interests and costly effort inputs. In reality, however, the government also intervenes in entrepreneurship in different ways. One of the most prevalent ways is indirect government intervention—that is, the government only designs a supporting policy to maximize social welfare without acquiring any share of the enterprise. For simplicity, we temporally ignore spillover effects of the innovation or entrepreneurship. The presence of a spillover effect will clearly favor government intervention and enhance our consequent results.

The government’s indirect intervention policy is a pair of state-contingent transfers (subsidies), $\{g, p\}$, where $p \in \mathbb{R}$ represents the prize for the project if it succeeds and $g \in \mathbb{R}$ denotes the up-front payment (grant) for the project even if it fails. This policy can also be treated as the equivalent of a tax credit. Moreover, we assume a limited liability constraint: $g, p \geq 0$.

The up-front payment, $g$, will be paid in advance and is intended to reduce the costs of $A_1$ and $A_2$ in reality. Therefore, we have the following cost function for $A_i$,

$$C_i(e_i) = c(e_i - \gamma g)^2,$$

where $\gamma \geq 0$ is a parameter reflecting an externality effect of up-front payment over the agents’ cost; that is, up-front payment can reinforce the agent’s effort and reduce their costs. For expository tractability, we assume a symmetric externality effect with identical $\gamma$ for
both agents. Compared to the cost functions for $A_1$ and $A_2$ in the case without government
intervention, such cost functions keep the properties of twice differentiable in effort levels,
and are strictly convex. Differently, the up-front payment from the government enters the
cost functions of $A_1$ and $A_2$ and reduces the costs of both $A_1$ and $A_2$.

Up-front payment will also incur cost to the government, due to opportunity costs of the
budget, etc. We assume this takes a quadratic form as $\delta g^2$, where $\delta > 0$ represents the cost
parameter. Thus, the government’s problem $[P1]$ is to design an indirect policy to maximize
social welfare, as follows:

$$
\max_{g, p, \bar{e}_1, \bar{e}_2} \left( \frac{m_1 \bar{e}_1 + m_2 \bar{e}_2}{2} \right) W - c(\bar{e}_1 - \gamma g)^2 - c(\bar{e}_2 - \gamma g)^2 - \delta g^2,
$$
given the IC constraints:

$$
\bar{e}_1 \in \max_{e_1} \left( \frac{m_1 e_1 + m_2 e_2}{2} \right) \beta (W + p) + \beta g - c(e_1 - \gamma g)^2,
$$

$$
\bar{e}_2 \in \max_{e_2} \left( \frac{m_1 e_1 + m_2 e_2}{2} \right) [(1 - \beta)(W + p)] + (1 - \beta)g - c(e_2 - \gamma g)^2,
$$

and limited liability constraints: $g \geq 0$, and $p \geq 0$.

By solving the government’s optimal problem under IC constraints, optimal effort levels
under the government’s indirect intervention $(p^*, g^*)$ are

$$
\bar{e}_1^* = \frac{\beta m_1 (W + p^*)}{4c} + \gamma g^*;
$$

$$
\bar{e}_2^* = \frac{(1 - \beta) m_2 (W + p^*)}{4c} + \gamma g^*.
$$

Note that not only the prize $p$ but also the grant $g$ may boost both agents’ effort levels.
In other words, the grant has the effect of incentive provision, as does the prize. This is
mainly because the grant has cost-reduction effect. A higher grant will offset the cost of
effort exertion and moral hazard. Therefore, agents are more willing to exert higher efforts
to raise the chance of success and their expected utilities. Moreover, the optimal policy for the government’s indirect intervention is given by

\[ p^* = (\frac{\theta_1}{\theta_2} - 1)W, \]

\[ g^* = \frac{\gamma(m_1 + m_2)W}{4\delta}, \]

where \( \theta_1 = \beta m_1^2 + (1 - \beta)m_2^2 \) and \( \theta_2 = \beta^2 m_1^2 + (1 - \beta)^2 m_2^2. \)

After we get optimal levels for effort and the government intervention policy, we can compare the level of efforts and the welfare in indirect government intervention with the efforts in the pure market benchmark.

**Proposition 4.** The equilibrium effort levels of \( A_1 \) and \( A_2 \) induced by the government’s optimal policy are higher than the effort levels without government intervention.

**Proof.** See the Appendix.

**Proposition 5.** The equilibrium level of total surplus induced by the government’s optimal policy are higher than total surplus without government’s intervention.

**Proof.** See the Appendix.

Both grant (through the externality effect) and prize induce higher efforts and expected revenue. Therefore, each agent will work hard to earn more. Furthermore, higher efforts lead to higher social welfare. This can also be regarded as a rationale to support the prevalence of indirect government intervention.

# 5 Hybrid Government Intervention

In addition to indirect government intervention, the government may also directly participate in the enterprise. This is frequently observed in many European and East Asian countries.

\(^3\)Note that \( \theta_1 > \theta_2 \) given \( \beta \in (0, 1) \).
We call this hybrid intervention. For facility of comparing with the pure indirect intervention, we can hypothetically consider a situation in which the government replaces one incumbent agent, say $A_2$, by acquiring his share of the enterprise, or there exists some potential private participant in the enterprise in the market. The government will serve as a partner of $A_1$.

Now $m_1$ and $m_g$ are the marginal contributions for $A_1$ and the government, respectively. The government controls its effort $\tilde{a} \in [0, 1]$ in the project and gives $A_1$ a subsidized contract $\{\tilde{g}, \tilde{p}\}$ for his effort. The government will keep the same share as $A_2$.

Thus, the government problem [P2] is to maximize total surplus by selecting an optimal triple of an incentive contract, and a recommendation for $A_1$’s effort and its own effort to satisfy the incentive compatibility constraint over $A_1$:

$$\max_{\tilde{g}, \tilde{p}, \tilde{e}_1, \tilde{a}} \left( \frac{m_1 \tilde{e}_1 + m_g \tilde{a}}{2} \right) W - c(\tilde{e}_1 - \gamma \tilde{g})^2 - c(\tilde{a} - \gamma \tilde{g})^2 - \delta \tilde{g}^2$$

s.t. $\tilde{e}_1 \in \max_{e_1} \left( \frac{m_1 e_1 + m_g \tilde{a}}{2} \right) \beta (W + \tilde{p}) + \beta \tilde{g} - c(e_1 - \gamma \tilde{g})^2$.

By solving [P2], we obtain the optimal level for the government’s intervention policy.

The optimal levels for a prize under the hybrid intervention is

$$\tilde{p}^* = \frac{1 - \beta}{\beta} W,$$

and the optimal level for a government grant is

$$\tilde{g}^* = \frac{\gamma (m_1 + m_g) W}{4\delta}.$$

$A_1$’s optimal effort level is

$$\tilde{e}_1^* = \frac{\delta m_1 W + c\gamma^2(m_1 + m_g) W}{4c\delta},$$

and the government’s optimal effort level is
\[ \tilde{a}^* = \frac{\delta m_2 W + c\gamma^2 (m_1 + m_2)W}{4c\delta}. \]

5.1 Comparison with the Benchmark Model

In this section, we compare agents’ optimal effort under the hybrid intervention case with our benchmark case, as well as the difference in total welfare between hybrid model and benchmark model. Our model shows that under the hybrid case, \( A_1 \)’s effort is raised due to the government’s direct participation and its incentive provision. Nevertheless, the government’s optimal effort is determined by the magnitude of \( m_g \) relative to \( m_2 \). If the productive efficiency of a directly participating government is not sufficiently small relative to the outside market participant, a directly participating government would provide higher effort than the benchmark without government. We summarize this in the proposition below.

**Proposition 6.** \( A_1 \)’s effort under hybrid intervention is higher than its effort without government intervention. If \( m_g > (1 - \beta)m_2 \), then the government’s effort under hybrid intervention is larger than the replaced agent’s effort in the market case.

**Proof.** See the Appendix.

Next, we compare welfare in the hybrid case with welfare in the benchmark case.

**Proposition 7.** If \( m_g > m_2 \) or \( m_1 > m_2 \), the welfare under hybrid government invention is better than the welfare without government invention.

**Proof.** See the Appendix.

Hybrid government intervention can raise social welfare when the outside market participant’s productive efficiency is lower than that of the remaining participant or that of the government.
5.2 Comparison with Indirect Intervention

Next, we compare the hybrid intervention case with the indirect intervention case to see its potential dominance.

**Proposition 8.** If \( \beta < \frac{1}{2} \) and \( m_g > m_2 \), then \( \tilde{e}_1 > \bar{e}_1 \); that is, \( A_1 \)'s effort under hybrid intervention is larger than its effort under indirect intervention. If \( \beta > \frac{1}{2} \) and \( m_g < m_2 \), then \( \tilde{e}_1 < \bar{e}_1 \). Moreover, if \( \frac{m_g}{m_2} > \frac{(1-\beta)}{\beta} \) and \( m_g > m_2 \) we have \( \tilde{a} > \bar{e}_2 \). If \( \frac{m_g}{m_2} < \frac{(1-\beta)}{\beta} \) and \( m_g < m_2 \), we have \( \tilde{a} < \bar{e}_2 \).

**Proof.** See the Appendix.

We can see that when the government is dominant in the enterprise, and the marginal contribution of the government is larger than that of the outside market participant, \( A_1 \) will take higher effort in the hybrid intervention case. When the government is not dominant or efficient relative to the outside market participant, \( A_1 \) will make lower effort in the hybrid intervention case. When the government’s productive efficiency is higher (lower) than that of the outside market participant, and its productive efficiency relative to that of the outside market participant is higher (lower) than its relative share, the government will take higher (lower) effort in the hybrid intervention case.

Next, we compare total welfare between the hybrid intervention case and the indirect intervention case. We provide the sufficient conditions for the welfare superiority of hybrid intervention and that of indirect intervention below.

**Proposition 9.** If \( \beta < \frac{1}{2} \) and \( (\frac{m_g}{m_2})^2 > \left(\frac{1-\beta}{\beta}\right) \), then welfare under hybrid government intervention will be larger than welfare under indirect government intervention. If \( \beta > \frac{1}{2} \) and \( (\frac{m_g}{m_2})^2 < \left(\frac{1-\beta}{\beta}\right) \), then welfare under hybrid government intervention will be smaller than welfare under indirect government intervention.

**Proof.** See the Appendix.

If the government’s share is dominant in the enterprise, and the government’s relative productive efficiency to \( A_2 \)'s productive efficiency is larger than his relative share in
the enterprise, then hybrid government intervention will be more desirable than indirect intervention. In this case, the government’s incentive to enter the enterprise is well induced, according to the governance structure that assigns a high share with its high production efficiency in the enterprise. Thus the government can play a sufficient role in the enterprise and then bring higher business efficiency and total surplus.

On the other hand, if $A_1$ is dominant in the enterprise, and the government’s relative productive efficiency to $A_2$’s productive efficiency is larger than his relative share in the enterprise, then hybrid government intervention will be less desirable than indirect intervention. Thus, the government only plays a minor role in the enterprise. Therefore, its direct participation cannot bring extra benefit to social welfare.

6 Discussion

In the analysis above, we temporarily ignore the spillover effect from the project. Normally, innovative spillover effect is positive. More specifically, it is a positive additional term in social welfare and increasing in the efforts of the project. Therefore, it is predictable that the presence of a spillover effect will favor intervention. There could also be different spillover effects in indirect and hybrid interventions. For instance, the government may value the probability of success or a low-risk project more in a hybrid intervention.

A long-term relationship in government intervention is also worth studying. However, long-term rationality may substantially complicate the comparison between different interventions. This will further twist the government’s behavior. In different stages of the project or enterprise, the environment, in terms of a set of parameters, may vary, and therefore different comparative results may emerge. It is very likely that the government can consider direct participation in the start-up stage of the enterprise, but exit the enterprise at some point in the long run.

Let us summarize policy implications of our analysis. First, indirect intervention is
always desirable in supporting innovation and entrepreneurship relative to leaving it to the pure market. In particular, although the grant is not performance-dependent, it is still an important, useful component of the indirect government intervention policy. It can still provide incentive, as the prize does, since the grant can normally induce the externality effect over the agents in terms of cost reduction. Nevertheless, there is no universal solution that entails sticking with one fixed pattern when intervening in innovation and entrepreneurship. The governance structure of the enterprise matters. In a governance structure of the enterprise, if the government plays a minor role with a low productive efficiency, then hybrid intervention with the government’s direct participation will be less desirable. However, if the governance structure of the enterprise is in favor of the government with a high productive efficiency, hybrid intervention with the government’s direct participation will be more desirable. There are quite a few successful examples in practice. For instance, semiconductor giant TSMC was cofounded by the Taiwanese government with dominant shares. In recent years, the Chinese government has also strongly supported many high-tech enterprises through direct intervention. Many successful firms have emerged, e.g. Lenovo, iFlytek, etc. In these cases, governments normally entered the enterprises with dominant roles and provided more crucial support for start-up businesses, including financial assistance, government procurement, a signaling effect, etc.
References


A Appendix

Proof of Proposition 1

Proof. The optimal effort for $A_1$ is to solve the following optimization problem:

$$\max_{e_1} \beta \left( \frac{m_1 e_1 + m_2 e_2}{2} \right) W - c e_1^2.$$ 

Its first-order condition yields

$$\beta \left( \frac{m_1}{2} \right) W - 2 c e_1 = 0.$$ 

The optimal effort for $A_2$ is to solve the following optimization problem:

$$\max_{e_2} (1 - \beta) \left( \frac{m_1 e_1 + m_2 e_2}{2} \right) W - c e_2^2.$$ 

Its first-order condition yields

$$(1 - \beta) \left( \frac{m_2}{2} \right) W - 2 c e_2 = 0.$$ 

By simultaneously solving the two equations, we obtain the optimal effort as follows:

$$e_1^* = \frac{\beta m_1 W}{4c},$$

$$e_2^* = \frac{(1 - \beta) m_2 W}{4c}.$$ 

(1) Taking $e_1^*$’s first derivative with respect to $\beta$ and $e_2^*$’s first derivative with respect to $1 - \beta$, we have the following results:

$$\frac{\partial e_1^*}{\partial \beta} = \frac{m_1 W}{4c},$$
\[ \frac{\partial e^*_2}{\partial (1 - \beta)} = \frac{m_2 W}{4c}. \]

Since \( W > 0 \) and \( m_1, m_2 \in (0, 1] \), we have \( \frac{\partial e^*_2}{\partial \beta} > 0 \) and \( \frac{\partial e^*_2}{\partial (1 - \beta)} > 0 \).

(2) Taking \( e^*_1 \)'s and \( e^*_2 \)'s first derivative with respect to \( W \), we have the following results:

\[ \frac{\partial e^*_1}{\partial W} = \frac{\beta m_1}{4c}, \]
\[ \frac{\partial e^*_2}{\partial W} = \frac{(1 - \beta)m_1}{4c}. \]

Since \( m_1, m_2 \in (0, 1] \), \( \beta \in (0, 1) \), and \( c > 0 \), we have \( \frac{\partial e^*_1}{\partial W} > 0 \) and \( \frac{\partial e^*_2}{\partial W} > 0 \).

(3) Taking \( e^*_1 \)'s first derivative with respect to \( m_1 \) and \( e^*_2 \)'s first derivative with respect to \( m_2 \), we have the following results:

\[ \frac{\partial e^*_1}{\partial m_1} = \frac{\beta W}{4c} > 0, \]
\[ \frac{\partial e^*_2}{\partial m_2} = \frac{(1 - \beta)W}{4c} > 0. \]

Since \( W > 0 \), \( \beta \in (0, 1) \), and \( c > 0 \), we have \( \frac{\partial e^*_1}{\partial m_1} > 0 \) and \( \frac{\partial e^*_2}{\partial m_2} > 0 \).

Proof of Proposition 2

Proof. Substituting the result of \( e^*_1 \), \( e^*_2 \) into Equations 1 and 2, we have \( A_1 \)'s utility listed as follows:

\[ U_1 = \frac{\beta^2 m_1^2 W^2 + 2\beta(1 - \beta)m_2^2 W^2}{16c}, \]

and \( A_2 \)'s utility is as follows:

\[ U_2 = \frac{2\beta(1 - \beta)m_1^2 W^2 + (1 - \beta)^2m_2^2 W^2}{16c}. \]
The total surplus is as follows:

\[ U = U_1 + U_2 = \frac{(2\beta - \beta^2)m_1^2W^2 + (1 - \beta^2)m_2^2W^2}{16c}. \]

Taking the first derivative of \( A_1 \)'s utility with respect to \( \beta \), we have the following results:

\[ \frac{\partial U_1}{\partial \beta} = \frac{\beta m_1^2W^2 + (1 - 2\beta)m_2^2W^2}{8c}. \]

We have \( \frac{\partial U_1}{\partial \beta} > 0 \) when \( m_1, m_2 \in (0, 1) \), and if \( \frac{m_1}{m_2} > \left( \frac{2\beta - 1}{\beta} \right)^{\frac{1}{2}} \). Otherwise, the utility of \( A_1 \) is decreasing in \( A_1 \)'s share, \( \frac{\partial U_1}{\partial \beta} < 0 \), given that \( \frac{m_1}{m_2} < \left( \frac{2\beta - 1}{\beta} \right)^{\frac{1}{2}} \) and \( \frac{1}{2} < \beta < 1 \).

The partial derivative of \( A_1 \)'s utility with respect to \( m_1 \) is as follows:

\[ \frac{\partial U_1}{\partial m_1} = \frac{\beta^2m_1W^2}{8c}. \]

Therefore, \( A_1 \)'s utility is increasing in its marginal contribution of its effort to the success of the project, \( \frac{\partial U_1}{\partial m_1} > 0 \), since we have \( m_1, m_2 \in (0, 1) \), \( W > 0 \), and \( \beta \in (0, 1) \).

The partial derivative of \( A_1 \)'s utility with respect to \( m_2 \) is as follows:

\[ \frac{\partial U_1}{\partial m_2} = \frac{\beta(1 - \beta)m_2W^2}{4c}. \]

\( A_1 \)'s utility is increasing in agent \( A_2 \)'s marginal contribution of agent \( A_2 \)'s effort to the success of the project, \( \frac{\partial U_1}{\partial m_2} > 0 \), given \( m_1, m_2 \in (0, 1) \), \( \beta \in (0, 1) \), and \( W > 0 \).

Again, taking the first derivative of \( A_2 \)'s utility with respect to \( \beta \), we have the following results:

\[ \frac{\partial U_2}{\partial (1 - \beta)} = -\frac{\partial U_2}{\partial \beta} = \frac{(1 - \beta)m_2^2W^2 - (1 - 2\beta)m_1^2W^2}{8c}. \]

We then have \( \frac{\partial U_2}{\partial (1 - \beta)} > 0 \) when \( \frac{m_2}{m_1} > \left( \frac{1 - 2\beta}{1 - \beta} \right)^{\frac{1}{2}} \), since \( m_1, m_2 \in (0, 1) \), \( W > 0 \). Otherwise, \( \frac{\partial U_2}{\partial (1 - \beta)} < 0 \) if \( \frac{m_2}{m_1} < \left( \frac{1 - 2\beta}{1 - \beta} \right)^{\frac{1}{2}} \) and \( 0 < \beta < \frac{1}{2} \) hold.
The partial derivative of $A_2$'s utility with respect to $m_2$ is as follows:

$$\frac{\partial U_2}{\partial m_2} = \frac{(1 - \beta)^2 m_2^2 W^2}{8c}.$$ 

Therefore, we have $\frac{\partial U_2}{\partial m_2} > 0$, given $m_1, m_2 \in (0, 1)$, $\beta \in (0, 1)$, and $W > 0$.

The partial derivative of $A_2$'s utility with respect to $m_1$ is as follows:

$$\frac{\partial U_2}{\partial m_1} = \frac{\beta(1 - \beta)m_1 W^2}{4c} > 0.$$ 

Therefore, we have $\frac{\partial U_2}{\partial m_1} > 0$, given $m_1, m_2 \in (0, 1)$, $\beta \in (0, 1)$, and $W > 0$. \qed

Proof of Proposition 3

Proof. The partial derivative of total surplus with respect to $A_1$’s share is

$$\frac{\partial U}{\partial \beta} = \frac{(1 - \beta)m_1^2 W^2 - \beta m_2^2 W^2}{8c},$$

and the partial derivative of total surplus with respect to $A_2$’s share is

$$\frac{\partial U}{\partial (1 - \beta)} = -\frac{\partial U}{\partial \beta} = \frac{\beta m_2^2 W^2 - (1 - \beta)m_1^2 W^2}{8c}.$$

First, we can see that $\frac{\partial U}{\partial \beta} > 0$ when $\frac{m_1}{m_2} > \left(\frac{\beta}{1 - \beta}\right)^{\frac{1}{2}}$. Otherwise, the total surplus is decreasing in $A_1$’s share, $\frac{\partial U}{\partial \beta} < 0$, when $\frac{m_1}{m_2} < \left(\frac{\beta}{1 - \beta}\right)^{\frac{1}{2}}$. Second, as for the impact of the changes in $A_2$’s share on the total surplus, we have that $\frac{\partial U}{\partial (1 - \beta)} > 0$ when $\frac{m_2}{m_1} > \left(\frac{1 - \beta}{\beta}\right)^{\frac{1}{2}}$; otherwise, $\frac{\partial U}{\partial (1 - \beta)} < 0$ if $\frac{m_1}{m_2} < \left(\frac{1 - \beta}{\beta}\right)^{\frac{1}{2}}$.

Moreover, given that $m_1, m_2 \in (0, 1)$, $\beta \in (0, 1)$, and $W > 0$, we found not only that the partial derivative of total surplus with respect to $A_1$’s marginal contribution to success is positive, since $\frac{\partial U}{\partial m_1} = \frac{(2\beta - \beta^2)m_1 W^2}{8c} > 0$, but the total surplus increases as $A_2$’s marginal contribution to success increases as well, because $\frac{\partial U}{\partial m_2} = \frac{(1 - \beta^2)m_2 W^2}{8c} > 0$. \qed
Solution to P1 Problem

First, we solve the optimal problem for \( A_1 \) and \( A_2 \). Optimal efforts for both \( A_1 \) and \( A_2 \) are

\[
\bar{e}_1^* = \frac{\beta m_1(W + p)}{4c} + \gamma g,
\]

and

\[
\bar{e}_2^* = \frac{(1 - \beta)m_2(W + p)}{4c} + \gamma g.
\]

The expected payoff when the project is successful in the social welfare problem is

\[
\frac{m_1\bar{e}_1^* + m_2\bar{e}_2^*}{2}W = \left[ \frac{\beta m_1^2 + (1 - \beta)m_2^2}{8c} (W + p) + 4c\gamma (m_1 + m_2) g \right] W.
\]

Let \( \theta_1 = \beta m_1^2 + (1 - \beta)m_2^2 \), we have the expected project reward is

\[
\frac{m_1\bar{e}_1^* + m_2\bar{e}_2^*}{2}W = \frac{\theta_1(W + p) + 4c\gamma (m_1 + m_2) g}{8c} W.
\]

Given that the cost function for agent \( A_1 \) is

\[
c(\bar{e}_1^* - \gamma g)^2 = c \left( \frac{\beta m_1(W + p) + 4c\gamma g}{4c} - \gamma g \right) = \frac{[\beta m_1(W + p)]^2}{16c},
\]

and agent \( A_2 \)'s cost function is

\[
c(\bar{e}_2^* - \gamma g)^2 = c \left( \frac{(1 - \beta)m_2(W + p) + 4c\gamma g}{4c} - \gamma g \right) = \frac{[(1 - \beta)m_2(W + p)]^2}{16c}.
\]

The total cost function in the social welfare problem would be

\[
c(\bar{e}_1^* - \gamma g)^2 + c(\bar{e}_2^* - \gamma g)^2 = \frac{[\beta^2 m_1^2 + (1 - \beta)^2m_2^2](W + p)^2}{16c} = \frac{\theta_2(W + p)^2}{16c},
\]

where \( \theta_2 = \beta^2 m_1^2 + (1 - \beta)^2m_2^2 \). Note that \( \theta_1 > \theta_2 \), because \( \beta > \beta^2 \) and \( (1 - \beta) > (1 - \beta)^2 \).
given $\beta \in (0, 1)$. After simplifying, the optimal question can be modified as follows:

$$\max_{g,p} U_{S} = \frac{\theta_1 (W + p) + 4c\gamma (m_1 + m_2) g W - \theta_2 (W + p)^2}{8c} - \frac{\delta g^2}{16c}$$

s.t. $p \geq 0, g \geq 0$.

As a result, its KKT conditions yield

$$p \geq 0; \quad \frac{\partial U_{S}}{\partial p} \leq 0; \quad p \frac{\partial U_{S}}{\partial p} = 0,$$

$$g \geq 0; \quad \frac{\partial U_{S}}{\partial g} \leq 0; \quad g \frac{\partial U_{S}}{\partial g} = 0.$$

The first order condition for $p$ is

$$\frac{\partial U_{S}}{\partial p} = \frac{\theta_1 W}{8c} - \frac{\theta_2 (W + p)}{8c}.$$

If $p = 0$, then $\frac{\partial U_{S}}{\partial p} > 0$, which conflicts with $\frac{\partial U_{S}}{\partial p} \leq 0$. Therefore, we know that the government would have a positive prize for a project, $p > 0$. Under the circumstance that $p > 0$, we could solve for the optimal $p$ by setting that $\frac{\partial U_{S}}{\partial p} = 0$. Hence, the following result for optimal $p$ is

$$p^* = \frac{\theta_1 W}{\theta_2} - W.$$

In a similar method, the first-order condition for $g$ is

$$\frac{\partial U_{S}}{\partial g} = \frac{\gamma (m_1 + m_2) W}{2} - 2\delta g.$$

Following the same logic, we get the following result for optimal $g$:

$$g^* = \frac{\gamma (m_1 + m_2) W}{4\delta}.$$
After obtaining $g^*$ and $p^*$, we could derive the optimal effort for $A_1$ and $A_2$ under indirect intervention as follows:

$$
\bar{e}^*_1 = \frac{(c\gamma^2 + \delta)\beta^2m_1^3W + [(1-\beta)c\gamma^2 + \delta\beta](1-\beta)m_1m_2^2W + (\beta^2m_1^2 + (1-\beta)^2m_2^2)c\gamma^2m_2W}{4c\delta(\beta^2m_1^2 + (1-\beta)^2m_2^2)},
$$

$$
\bar{e}^*_2 = \frac{(c\gamma^2 + \delta)(1-\beta)^2m_3^2W + [c\beta\gamma^2 + \delta(1-\beta)]\beta m_2^2m_2W + (\beta^2m_1 + (1-\beta)^2m_2)c\gamma^2m_1W}{4c\delta(\beta^2m_1^2 + (1-\beta)^2m_2^2)}.
$$

**Proof of Proposition 4**

**Proof.** We have the following optimal efforts for $A_1$ and $A_2$ derived from the case without government intervention:

$$
e^*_1 = \frac{\beta m_1W}{4c},
$$

$$
e^*_2 = \frac{(1-\beta)m_2W}{4c},
$$

and optimal efforts for $A_1$ and $A_2$ derived from the case with indirect government’s intervention:

$$
\bar{e}^*_1 = \frac{\beta m_1(W+p^*) + 4\gamma g^*}{4},
$$

$$
\bar{e}^*_2 = \frac{(1-\beta)m_2(W+p^*) + 4\gamma g^*}{4}.
$$

Therefore, the difference in the equilibrium efforts level of $A_1$ under the benchmark model and government intervention is

$$
\bar{e}^*_1 - e^*_1 = \frac{\beta m_1p^*}{4c} + \gamma g^*.
$$

For agent $A_2$, the difference in the equilibrium efforts level under the benchmark model and government’s intervention is

$$
\bar{e}^*_2 - e^*_2 = \frac{(1-\beta)m_2p^*}{4c} + \gamma g^*.
Because \( p^* > 0 \) and \( g^* > 0 \), we have that both differences are greater than zero.

**Proof of Proposition 5**

**Proof.** For the case without government intervention, we have following welfare equation:

\[
\left( \frac{m_1 \epsilon_1^* + m_2 \epsilon_2^*}{2} \right) W - c \epsilon_1^* \epsilon_2^*.
\]

For the case with indirect government intervention, we have following welfare equation:

\[
\left( \frac{m_1 \bar{\epsilon}_1^* + m_2 \bar{\epsilon}_2^*}{2} \right) W - c (\bar{\epsilon}_1^* - \gamma g^*)^2 - c (\bar{\epsilon}_2^* - \gamma g^*)^2 - \delta g^2.
\]

The welfare difference (\( \Delta \)) is as follows:

\[
\Delta = \frac{m_1 W}{2} (\bar{\epsilon}_1^* - \epsilon_1^*) + \frac{m_2 W}{2} (\bar{\epsilon}_2^* - \epsilon_2^*) + c \epsilon_1^* \epsilon_2^* - c (\bar{\epsilon}_1^* - \gamma g^*)^2 + c \epsilon_2^* \epsilon_2^* - c (\bar{\epsilon}_2^* - \gamma g^*)^2 - \delta g^2.
\]

Through calculation, the reduced form of the welfare difference (\( \Delta \)) is as follows:

\[
\frac{(2\theta_1 W - 2\theta_2 W + \theta_2 p^*) p^*}{16c} + \left( \frac{\gamma (m_1 + m_2) W}{2} - \delta g^* \right) g^*.
\]

Given \( \theta_1 > \theta_2 \), \( p^* = (\frac{\theta_1}{\theta_2} - 1) W \), \( g^* = \frac{\gamma (m_1 + m_2) W}{48} \), the above equation is greater than zero without any conditions. Therefore, with the government’s indirect intervention, social welfare will definitely increase. We would like to point out that given \( \beta g^* < c (\bar{\epsilon}_1^* - \gamma g^*)^2 \) and \( (1 - \beta) g^* < c (\bar{\epsilon}_2^* - \gamma g^*)^2 \) hold, firms would not default on purpose to obtain ex ante grants.

**Solution to P2 Problem**

First, we solve the IC constraint for \( A_1 \) and find the optimal effort for \( A_1 \). The F.O.C for \( A_1 \)’s problem yields

\[
\bar{\epsilon}_1 = \frac{\beta m_1 (W + \bar{p})}{4c} + \gamma \bar{g}.
\]
Then we substitute $\tilde{e}_1$ into the government’s problem and have

$$\max \left( \frac{m_1 \left( \frac{\beta m_1 (W + \tilde{p})}{4c} + \gamma \tilde{g} \right)}{2} + m_2 \tilde{a} \right) W - c \left( \frac{\beta m_1 (W + \tilde{p})}{4c} + \gamma \tilde{g} - \gamma \tilde{g} \right)^2 - c(\tilde{a} - \gamma \tilde{g})^2 - \delta \tilde{g}^2$$

s.t. $\tilde{g} \geq 0, \tilde{p} \geq 0; \tilde{a} \geq 0$.

Rewrite this to be:

$$\max \left( \frac{\beta m_1^2 (W + \tilde{p})}{8c} + \frac{\gamma m_1 \tilde{g}}{2} + \frac{m_2 \tilde{a}}{2} \right) W - \frac{\beta^2 m_1^2 (W + \tilde{p})^2}{16c} - c(\tilde{a} - \gamma \tilde{g})^2 - \delta \tilde{g}^2$$

s.t. $\tilde{g} \geq 0, \tilde{p} \geq 0; \tilde{a} \geq 0$.

The KKT conditions yield:

$$\frac{\partial U_S}{\partial \tilde{a}} = \frac{m_2}{2} W - 2c(\tilde{a} - \gamma \tilde{g}) \leq 0; \tilde{a} \geq 0,$n

$$\frac{\partial U_S}{\partial \tilde{g}} = \frac{\gamma m_1}{2} W - 2c(\tilde{a} - \gamma \tilde{g})(-\gamma) - 2\delta \tilde{g} \leq 0; \tilde{g} \geq 0,$n

$$\frac{\partial U_S}{\partial \tilde{p}} = \frac{\beta m_1^2 W}{8c} - \frac{\beta^2 m_1^2 (W + \tilde{p})}{8c} \leq 0; \tilde{p} \geq 0.$$

For $\tilde{a}$, from its KKT conditions, we know that if $\tilde{a} = 0$, we have $\frac{\partial U_S}{\partial \tilde{a}} = \frac{m_2}{2} W + 2c\gamma \tilde{g} > 0$ because $\tilde{g} \geq 0$. Therefore, we would have $\tilde{a} > 0$ such that $\frac{\partial U_S}{\partial \tilde{a}} = \frac{m_2}{2} W - 2c(\tilde{a} - \gamma \tilde{g}) = 0$.

Hence, in this question, $\tilde{a} = \frac{m_2}{4c} W + \gamma \tilde{g}$.

Similarly, for $\tilde{g}$, if $\tilde{g} = 0$, we have $\frac{\partial U_S}{\partial \tilde{g}} = \frac{\gamma m_1}{2} W + 2c\gamma \tilde{a} > 0m$, since $\tilde{a} \geq 0$. Therefore, we need $\tilde{g} > 0$, then $\frac{\partial U_S}{\partial \tilde{g}} = \frac{\gamma m_1}{2} W + 2c\gamma(\tilde{a} - \gamma \tilde{g}) - 2\delta \tilde{g} = 0$. Hence, $\tilde{g} = \frac{\gamma m_1 W + 4c\gamma \tilde{a}}{4c\gamma^2 + 4\delta}$.

As for $\tilde{p}$, when $\tilde{p} = 0$, we have $\frac{\partial U_S}{\partial \tilde{p}} = \frac{\beta m_1^2 W}{8c} - \frac{\beta^2 m_1^2 W}{8c} > 0$, because of $\beta > \beta^2$. Therefore, $\tilde{p} > 0$, then $\frac{\partial U_S}{\partial \tilde{p}} = \frac{\beta m_1^2 W}{8c} - \frac{\beta^2 m_1^2 (W + \tilde{p})}{8c} = 0$. Hence, $\tilde{p}^* = \frac{1 - \beta}{\beta} W$.

In sum, we have the following solutions to the government’s problem. The optimal levels for government intervention are

$$\tilde{p}^* = \frac{1 - \beta}{\beta} W,$$
and
\[ \bar{g}^* = \frac{\gamma (m_1 + m_g)W}{4\delta}. \]

The optimal effort levels for agent \( A_1 \) and government are
\[ \tilde{e}^*_1 = \frac{\delta m_1 W + c \gamma^2 (m_1 + m_g) W}{4 c \delta}, \]
and
\[ \tilde{a}^* = \frac{\delta m_g W + c \gamma^2 (m_1 + m_g) W}{4 c \delta}. \]

**Proof of Proposition 6**

*Proof.* Recall that agents’ optimal effort levels in our benchmark model are
\[ e^*_1 = \frac{\beta m_1 W}{4 c}, \]
\[ e^*_2 = \frac{(1 - \beta) m_2 W}{4 c}. \]

Therefore, the difference between \( A_1 \)’s effort under different situations is
\[ \tilde{e}^*_1 - e^*_1 = \frac{\delta m_1 W + c \gamma^2 (m_1 + m_g) W - \delta \beta m_1 W}{4 c \delta} = \frac{\delta (1 - \beta) m_1 W + c \gamma^2 (m_1 + m_g) W}{4 c \delta}. \]

Given the dividend share \( \beta \in (0, 1) \) and the existence of hybrid intervention, we know that \( \tilde{e}^*_1 - e^*_1 > 0 \), which means that \( A_1 \)’s effort under the hybrid case is higher than its effort without any government intervention.

The difference between the optimal effort of the government in the hybrid case and the
effort of agent $A_2$ in the indirect intervention case is as follows:

$$\tilde{a}^* - e_2^* = \frac{\delta m_g W + c\gamma^2 (m_1 + m_g) W - \delta (1 - \beta) m_2 W}{4c\delta} = \frac{\delta (m_g - m_2) W + c\gamma^2 (m_1 + m_g) W + \delta \beta m_2 W}{4c\delta}.$$ 

Therefore, a sufficient condition for the optimal effort of the government in the hybrid case greater than the effort of agent $A_2$ in the indirect intervention case is $m_g > (1 - \beta)m_2$. \qed

Proof of Proposition 7

Proof. Total surplus in the benchmark model without government intervention is

$$\left(\frac{m_1 e_1^* + m_2 e_2^*}{2}\right) W - c(e_1^*)^2 - c(e_2^*)^2.$$ 

Total welfare under the hybrid government intervention is

$$\left(\frac{m_1 \tilde{e}_1^* + m_g \tilde{a}_2^*}{2}\right) W - c(\tilde{e}_1^*-\gamma \tilde{g}^*)^2 - c(\tilde{a}_2^*-\gamma \tilde{g}^*)^2 - \delta \tilde{g}^*.$$ 

The difference $\Delta$ between welfare under the hybrid government intervention and the benchmark case is

$$\Delta = \frac{\delta(1 - \beta)^2 m_1^2 W^2 + \delta(m_g^2 - m_2^2) W^2 + c\gamma^2 (m_1 + m_g)^2 W^2 + \delta \beta^2 m_2^2 W^2}{16c\delta}.$$ 

Therefore, when $m_g^2 \geq m_2^2$, the total welfare under hybrid government intervention is better than the total welfare of without government invention. \qed

Proof of Proposition 8

Proof. Recall that in the hybrid intervention case,

$$\tilde{e}_1^* = \frac{\delta m_1 W + c\gamma^2 (m_1 + m_g) W}{4c\delta},$$
\[ \tilde{a}^* = \frac{\delta m_g W + c \gamma^2 (m_1 + m_g) W}{4c \delta}, \]

and in the indirect intervention case,

\[ \tilde{e}_1^* = \frac{\beta m_1 (W + p^*)}{4c} + \gamma g^*, \]

\[ \tilde{e}_2^* = \frac{(1 - \beta) m_2 (W + p^*)}{4c} + \gamma g^*. \]

First, we can compare the firm’s efforts in different cases below given \( p^* = \left( \frac{\theta_1}{\theta_2} - 1 \right) W \), \( g^* = \frac{\gamma (m_1 + m_g) W}{4 \delta} \), \( \theta_1 = \beta m_1^2 + (1 - \beta) m_2^2 \), and \( \theta_2 = \beta^2 m_1^2 + (1 - \beta)^2 m_2^2 \).

\[ \tilde{e}_1^* - \tilde{e}_1^* = \frac{m_1 W (1 - \beta) m_2^2 [\beta m_g - (1 - \beta) - \beta]}{4c \beta^2 m_1^2 + (1 - \beta)^2 m_2^2} + \frac{\gamma^2 W (m_g - m_2)}{4 \delta}. \]

Therefore, we have \( \tilde{e}_1^* - \tilde{e}_1^* > 0 \) if \([(1 - \beta) - \beta] > 0 \) and \( m_g > m_2 \), and \( \tilde{e}_1^* - \tilde{e}_1^* < 0 \), if \([(1 - \beta) - \beta] < 0 \) and \( m_g < m_2 \).

For the effect difference between government and agent \( A_2 \), we have the following results:

\[ \tilde{a}^* - \tilde{e}_2^* = \frac{W}{4c} \left\{ \frac{\beta m_2^2 [\beta m_g - (1 - \beta) m_2] + (1 - \beta)^2 m_2^2 [m_g - m_2]}{\beta^2 m_1^2 + (1 - \beta)^2 m_2^2} \right\} + \frac{\gamma^2 W (m_g - m_2)}{4 \delta}. \]

Therefore, we have \( \tilde{a}^* - \tilde{e}_2^* > 0 \) when \( m_g > m_2 \) and \( \beta m_g - (1 - \beta) m_2 > 0 \), i.e., \( \frac{m_g}{m_2} > \frac{(1 - \beta)}{\beta} \) or \( \frac{m_g}{(1 - \beta)} > m_2^2 \). \( \tilde{a}^* - \tilde{e}_2^* < 0 \) when \( m_g < m_2 \) and \( \beta m_g - (1 - \beta) m_2 < 0 \), i.e., \( \frac{m_g}{m_2} < \frac{(1 - \beta)}{\beta} \) or \( \frac{m_g}{(1 - \beta)} < m_2^2 \).

**Proof of Proposition 9**

*Proof.* Social welfare under the government’s indirect intervention is

\[ \bar{U} = \left( \frac{m_1 \tilde{e}_1^* + m_2 \tilde{e}_2^*}{2} \right) W - c (\tilde{e}_1^* - \gamma g^*)^2 - c (\tilde{e}_2^* - \gamma g^*)^2 - \delta g^2. \]
When a government implements a hybrid intervention, social welfare is

$$
\tilde{U} = \left( \frac{m_1\tilde{e}_1^* + m_g\tilde{a}^*}{2} \right) W - c(\tilde{e}_1^* - \gamma\tilde{g}^*)^2 - c(\tilde{a}^* - \gamma\tilde{g}^*)^2 - \delta\tilde{g}^*.
$$

Therefore, the difference in social welfare ($\Delta$) is

$$
\Delta = \tilde{U} - U = \frac{m_1 W}{2} (\tilde{e}_1^* - e_1^*) + \frac{W}{2} (m_g\tilde{a}^* - m_2\tilde{e}_2^*) - c[(\tilde{e}_1^* - \gamma\tilde{g}^*)^2 - (\tilde{e}_1^* - \gamma g^*)^2] - c[(\tilde{a}^* - \gamma\tilde{g}^*)^2 - (\tilde{e}_2^* - \gamma g^*)^2] - \delta(\tilde{g}^* - g^*).
$$

To calculate $\Delta = \tilde{U} - U$, we start from the last term,

$$
\delta(\tilde{g}^* - g^*)^2 = \frac{\delta\gamma W}{4\delta}(m_g - m_2)(\frac{\gamma(m_1 + m_2)W}{4\delta} + \frac{\gamma(m_1 + m_g)W}{4\delta}).
$$

Therefore, we have $\delta(\tilde{g}^* - g^*)^2 > 0$ when $m_g - m_2 > 0$.

We then look at the difference in the cost function between the government in the hybrid case and $A_2$ in the indirect case. We know that $\tilde{a}^* - \gamma\tilde{g}^* = \frac{m_g W}{4c}$ and $\tilde{e}_2^* - \gamma g^* = \frac{(1-\beta)m_2(W + p^*)}{4c}$. Therefore, we have the following result:

$$
c[(\tilde{a}^* - \gamma\tilde{g}^*)^2 - (\tilde{e}_2^* - \gamma g^*)^2] = \frac{W}{4}[m_g + (1 - \beta)m_2\theta_1\theta_2] W [m_g - (1 - \beta)m_2\theta_1\theta_2].
$$

Therefore $c[(\tilde{a}^* - \gamma\tilde{g}^*)^2 - (\tilde{e}_2^* - \gamma g^*)^2] > 0$ when $m_g > m_2$ and $\beta m_g - (1 - \beta)m_2 > 0$, i.e., $\frac{m_g}{m_2} > \frac{(1-\beta)}{\beta}$. Next, we take care of the cost difference for firms in different cases:

$$
c[(\tilde{e}_1^* - \gamma\tilde{g}^*)^2 - (\tilde{e}_1^* - \gamma g^*)^2] = \frac{m_1 W}{4} [1 + \beta\theta_1\theta_2 ] [1 - \beta\theta_1\theta_2].
$$

Hence, $c[(\tilde{e}_1^* - \gamma\tilde{g}^*)^2 - (\tilde{e}_1^* - \gamma g^*)^2] > 0$ when $[(1 - \beta) - \beta| or $\beta < 1/2$ given $[1 - \beta\theta_1\theta_2] = \frac{(1-\beta)m_2^2[(1-\beta)-\beta]}{\beta^2m_1^2+(1-\beta)^2m_2^2}

Moreover, we have the following result:

$$
\frac{W}{2} (m_g\tilde{a}^* - m_2\tilde{e}_2^*) = \frac{W}{2}[m_g m_2 W}{4c} - \frac{m_2(1 - \beta)m_2(W + p)}{4c} + \frac{m_g^2\gamma^2(m_1 + m_g)W}{4\delta} - \frac{m_2^2\gamma^2(m_1 + m_2)W}{4\delta}].
$$
Therefore, the reduced form of the difference in social welfare (∆) is

\[ \Delta = \tilde{U} - \bar{U} = \left\{ \frac{m_1 W}{4c} (1 - \beta) - \beta \left( \frac{\theta_1}{\theta_2} - 1 \right) \right\} \left\{ \frac{m_1 W}{2} - \epsilon \frac{\beta m_1 (W + P)}{4c} + \frac{m_1 W}{4c} \right\} \\
+ \frac{W W}{4c} \left[ m_y m_y - (1 - \beta) m_2 m_2 \frac{\theta_1}{\theta_2} \right] + \left( 1 - \beta \right) m_2 W \theta_1 \frac{m_2 W}{4c} \theta_2 + \beta m_1 \left[ (1 - \beta) m_1 - \beta \right] \left[ \frac{\beta m_1^2 (1 - \beta) - \beta}{4c} \right] \\
+ \frac{m_1 W}{2} \frac{\gamma^2 W (m_y - m_2)}{4\delta} + \frac{W m_y \gamma^2 (m_y - m_2) W}{4\delta} + \frac{\gamma W m_2 \gamma (m_y - m_2) W}{4\delta}.
\]

Let \( \Delta = \tilde{U} - \bar{U} = \Delta_1 + \Delta_2 + \Delta_3 \). We start from \( \Delta_1 \) and have the following result:

\[ \Delta_1 = \left\{ \frac{m_1 W}{4c} (1 - \beta) - \beta \left( \frac{\theta_1}{\theta_2} - 1 \right) \right\} \left\{ \frac{m_1 W}{4} (1 - \beta \frac{\theta_1}{\theta_2}) \right\} > 0. \]

Next, recall that \( W + p^* = W \frac{\theta_1}{\theta_2} \).

\[ \Delta_2 = \frac{W W}{4c} \left[ m_y^2 - (1 - \beta) m_y^2 \frac{\theta_1}{\theta_2} \right] + \frac{\theta_1 (1 - \beta) m_2 W m_2 W}{4c} \frac{\theta_2}{4} + \beta m_1 \left[ \frac{\beta m_1^2 (1 - \beta) - \beta}{4c} \right] \left[ \frac{\beta m_1^2 + (1 - \beta)^2 m_2^2}{4c} \right]. \]

This term is positive, \( \Delta_2 > 0 \), when \( [m_y^2 - (1 - \beta) m_y^2 \frac{\theta_1}{\theta_2}] > 0 \) and \( (1 - \beta) - \beta > 0 \).

The last one, \( \Delta_3 \), is as follows:

\[ \Delta_3 = \frac{m_1 W}{2} \frac{\gamma^2 W (m_y - m_2)}{4\delta} + \frac{W m_y \gamma^2 (m_y - m_2) W}{4\delta} + \frac{\gamma W m_2 \gamma (m_y - m_2) W}{4\delta}. \]

Note that

\[ \frac{W m_y \gamma^2 (m_1 + m_2) W}{4\delta} = \frac{\gamma W m_y \gamma (m_1 + m_2) W}{4\delta} = \frac{W m_y \gamma^2 (m_1 + m_2) W}{4\delta}. \]

and

\[ \frac{\gamma W m_2 \gamma (m_1 + m_2) W}{4\delta} = \frac{W m_2 \gamma^2 (m_1 + m_2) W}{4\delta}. \]

Hence, \( \Delta_3 = \frac{m_1 W}{2} \frac{\gamma^2 W (m_y - m_2) W}{4\delta} + \frac{W m_y \gamma^2 (m_y - m_2) W}{4\delta} + \frac{\gamma W m_2 \gamma (m_y - m_2) W}{4\delta} > 0 \) when \( m_y - m_2 > 0 \).

Combining the effect of \( \Delta_1, \Delta_2, \) and \( \Delta_3 \), we find that total welfare in the hybrid intervention case is higher than under the indirect intervention \( \tilde{U} - \bar{U} > 0 \), when \( \beta < \frac{1}{2}, m_y > m_2 \) and \( [m_y^2 - (1 - \beta) m_y^2 \frac{\theta_1}{\theta_2}] > 0 \).

According to our previous calculation, we have the following result:

\[ \Delta_1 + \Delta_2 = \frac{W W}{4c} \frac{\beta m_1^2 (1 - \beta) m_2^2 + (1 - \beta)^2 m_2^2 [m_y^2 - m_2^2]}{\beta^2 m_2^2 + (1 - \beta)^2 m_2^2} \\
+ \frac{m_1 W}{4c} \frac{1}{\theta_2^2} \left[ (1 - 2\beta)(1 - \beta) m_2^2 \right] + \frac{W W}{4c} \frac{1}{\theta_2^2} \left[ \theta_1 (1 - \beta) \beta (1 - 2\beta) m_2^2 \right]. \]
We know $\Delta_1 + \Delta_2 < 0$ when $\beta > \frac{1}{2}, m_g^2 < m_2^2$, and $\beta m_g^2 < (1 - \beta)m_2^2$. And we also know that if $m_g < m_2$, $\Delta_3 < 0$. Therefore, $\bar{U} - U < 0$ if $\beta > \frac{1}{2}$ and $(\frac{m_g}{m_2})^2 < \left(\frac{1 - \beta}{\beta}\right)$. 

$\square$
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