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Push or Pull? Performance-Pay, Incentives, and Information

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Abstract

We study a principal-agent model wherein the agent is better informed of the prospects of the project, and the project requires both an observable and unobservable input. We characterize the optimal contracts, and explore the trade-offs between high and low-powered incentive schemes. We discuss the implications for push and pull programs used to encourage R&D activity, but our results are relevant in other contexts.

KEYWORDS: Pay for Performance, Moral Hazard, Adverse Selection, Observable Action, Principal-Agent Problem
JEL Classifications: D82, D86, O31

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1 Introduction

To what extent should incentives be tied to performance? This question is relevant in many areas, including labor markets – where it relates to the debate on salaries vs. piece rates (see, e.g. Lazear, 1986, 2000) – and innovation incentives, where it pertains to the efficacy of "push" and "pull" programs (see, e.g., Kremer, 2002). Push programs, such as research grants or R&D tax credits, subsidize inputs; payments are not contingent on results. Pull programs, such as inducement prizes, or patent buyouts, tie rewards to output.

Adverse selection (AS) and moral hazard (MH) are inherent challenges in incentive provision. Given these problems, Kremer raises the concerns that push programs may finance projects unlikely to succeed, and provide weak incentives for unobservable inputs. Indeed, the MH literature stresses the importance of performance-pay; in the canonical model,[1] compensation must be tied to output to provide an incentive for effort. Yet low-powered incentives in which compensation is weakly, or not at all tied to performance, are commonly used. In this paper, we explore trade-offs between high and low-powered incentives in a model with AS and MH. We show that performance-pay may not be optimal for all types, but is always optimal for the highest types.

We consider a principal-agent model wherein a risk-neutral funder (he; the principal) motivates a risk-neutral researcher (she; the agent) to undertake an R&D project. The outcome depends on the researcher's private type, and two essential and complementary inputs – "investment" and "effort". Investment is contractible; effort is not.[2] If she succeeds, the researcher profits by marketing the technology, but this incentive is insufficient from the funder's perspective. To motivate greater R&D activity, the funder specifies a transfer independent of performance – a "grant"[3] – and a payment for success – a "prize".

We contribute to the contracting literature under AS/MH. In many models, output is the only verifiable signal available to the principal.[4] This renders

[1] See, e.g., Grossman and Hart (1983) or Bolton and Dewatripont (2005) (Ch. 4).

[2] We use the terms "observable", "contractible", and "verifiable" interchangeably.

[3] It may happen that the grant is negative, in which case we refer to it as an entry fee.

[4] Studies close to this analysis include Lewis and Sappington (2000a,b) and Ollier and

performance-pay indispensable, as output-independent rewards will not affect marginal incentives. While it may be infeasible to monitor research effort, some inputs, such as large-scale capital investments, may be easier to verify. If so, then investment can be encouraged with rewards tied only to these expenditures. But a researcher's effort may be more productive when she has better equipment with which to work. Then, as long as there is *some* benefit to success, greater investment increases the marginal returns to effort. A similar intuition obtains in multitasking models (e.g. Hölmstrom and Milgrom, 1991; Meng and Tian, 2013). Given multiple complementary tasks, a stronger incentive on one task induces greater effort on others.

Our results reveal that the virtues of performance-pay depend on the relative strengths of AS and MH. In our model, a prize creates a strong incentive for effort, but generates costly rent for the researcher due to AS. A grant effectively limits rent, but creates only an indirect incentive for effort (by motivating investment). The virtue of the prize depends on the balance of these trade-offs. In some circumstances, the optimal prize is zero for a range of types. For high enough types, however, the prize is always strictly positive; moreover, when MH is more severe, the prize is strictly positive for all types.

We also contribute to the literature on innovation incentives under MH, which has largely focused on pull programs;[5] few studies have examined the interactions between push and pull programs taking MH into account. Maurer and Scotchmer (2003) argue that repeated interaction between grantees and grantors resolves the MH problem. Our insights complement their's, as they are relevant in a static setting. Fu et al. (2012) show that grants may facilitate greater competition in a contest between researchers with asymmetric capital endowments. We abstract from competition to focus on the role of information.

Thomas (2013). There are notable exceptions, which will be discussed.

[5]For instance, there is a large literature on optimal patent design; e.g., Gilbert and Shapiro (1990) Klemperer (1990), O'donoghue et al. (1998), Cornelli and Schankerman (1999), and Hopenhayn and Mitchell (2001). See Hall (2007) for a survey. And a literature on alternatives to intellectual property such as prizes or contracts; e.g., Wright (1983), Kremer (1998), Shavell and Van Ypersele (2001), Hopenhayn et al. (2006), Weyl and Tirole (2012), and Che et al. (2015). See Maurer and Scotchmer (2003) for an overview.

Many studies have explored trade-offs between high and low-powered incentives. Due to the "effort-substitution problem" Hölmstrom and Milgrom show the potential for "fixed-wage" contracts.[6] This fixed wage is independent of *any* signal received by the principal, while the grant in our model depends on investment, but is independent of performance. Baker (1992) shows that performance-pay may be muted if performance is weakly correlated with verifiable measures.[7] Low-powered incentives may also arise as a means of risk-sharing (see, e.g., Prendergast, 1999); we abstract from risk-sharing as all parties are risk-neutral in our model. Allowing costly monitoring of effort, Prendergast (2002) shows that performance-pay may be more beneficial if the principal is uncertain of the "correct" action an agent should take.[8]

Also related, Zhao (2008) and Chen (2010, 2012) allow for partially observable actions, but abstract from AS. In a class of models following Laffont and Tirole (1986), the agent devotes unobservable effort directed at cost reduction, then chooses an observable output. But these models tend to involve "false moral hazard".[9] Meng and Tian (2013) study a multitasking model with AS and MH, and explain why the agent may be led to specialize on certain tasks. Finally, our framework relates closely to Laffont (1995), Lewis and Sappington (2000b) and Ollier and Thomas (2013). Differentiating our model is the presence of partially observable complementary actions, and the researcher's profit motivation, which both play a critical role in our model.

[6]The effort-substitution problem arises in multitasking models when efforts are substitutes. As a result, a stronger incentive on one task reduces effort on the other task.

[7]Baker's results may be relevant in our setting if "success" is difficult to define precisely.

[8]There is also a distinct literature that examines the question of whether it is optimal to monitor input or output if it is too costly to observe both (see, Maskin and Riley, 1985).

[9]False moral hazard arises when there is a deterministic relationship between type, unobservable effort, and a contractible signal (see Laffont and Martimort, 2009, Ch. 7).

2 The Model

The Primitives

A researcher undertakes an R&D project whose outcome – success or failure – depends on her type, θ,[10] investment, $x \in \mathbb{R}_+$, and effort, $y \in [0,1]$. Investment is contractible; effort is not. Success is verifiable at no cost. θ is a random variable with CDF, F, (smooth) PDF, f, and support $\Theta = [\underline{\theta}, \overline{\theta}] \subset (0,1]$. The researcher knows the true θ; the funder knows only its distribution. For each $\theta \in \Theta$, assume $f(\theta) > 0$, and $h'(\theta) < 0$, where $h(\theta) = \frac{1-F(\theta)}{f(\theta)}$ is the inverse hazard rate.

Given x, y, and θ, the probability of success is $\theta y \rho(x)$. The function, $\rho : \mathbb{R}_+ \to [0,1]$, is twice continuously with $\rho(0) = 0$, $\rho' > 0$ and $\rho'' < 0$. Notice that investment and effort are both essential for success, and are complements. If the researcher chooses (x, y), she incurs a cost $C(x, y) = x + \psi(y)$. For ease of exposition, we let $\psi(y) = \frac{c}{\alpha} y^\alpha$, where $\alpha > 1$, but our main results may be generalized (see Section 3.4). If she succeeds, the researcher earns profit, $\pi > 0$,[11] and the funder captures $W > 0$, otherwise both receive nothing. W might represent, for example, the consumer surplus associated with the technology. Absent intervention, the payoff to a type-θ researcher who chooses (x, y), is $\Pi(x, y, \theta) = \theta y \rho(x)\pi - C(x, y)$. We focus on settings where π is "small", relative to W. This is quite natural in the context of R&D as the social value of an innovation often exceeds the value to the innovator.[12] For simplicity, assume for all $y \in [0,1]$ and $\theta \in \Theta$,

$$\frac{\partial \Pi(x, y, \theta)}{\partial x}|_{x=0} = \theta y \rho'(0)\pi - 1 < 0, \tag{1}$$

which implies $\Pi(\cdot, y, \theta)$ is strictly decreasing and $\max_{x \geq 0, y \in [0,1]} \Pi(x, y, \theta) = 0$.[13]

[10] θ may capture some characteristic of the project and/or the researcher's innate ability.

[11] π might also reflect the prospect of a future outside job opportunity, or intrinsic motivation.

[12] See, e.g., Hall et al. (2009).

[13] If $\max_{x \geq 0, y \in [0,1]} \Pi(x, y, \theta) > 0$ then the researcher's outside option is type-dependent, which could give rise to the phenomenon of "countervailing incentives" (see, e.g., Lewis and Sappington, 1989). In our model, so long as the funder induces the researcher to invest

Feasible Contracts and the Funder's Problem

The funder designs contracts to motivate R&D activity. A contract specifies a transfer independent of performance, $g \in \mathbb{R}$, a prize for success, $v \in \mathbb{R}_+$, and an investment, $x \in \mathbb{R}_+$. We interpret $g > 0$ as a grant, and $g < 0$ as an *entry fee*. Following Innes (1990) and Poblete and Spulber (2012), the funder is subject to a "free-disposal" constraint, requiring $v \geq 0$.[14] Without loss of generality, the researcher is eligible for any rewards only if she chooses the agreed-upon, x'. If she deviates from x', and chooses $x \neq x'$, her payoff is $\Pi(x, y, \theta) \leq \max_{x\geq 0, y\in[0,1]} \Pi(x, y, \theta) = 0$. So, as long as her payoff is non-negative when she chooses x', it is never optimal to deviate.

By the Revelation Principle, it suffices to consider direct mechanisms (or menus). The funder commits to a menu, $m = \{v(\theta), g(\theta), x(\theta)\}_{\theta\in\Theta}$, where $v : \Theta \to \mathbb{R}_+$ is a prize schedule, $g : \Theta \to \mathbb{R}$ is a grant/fee schedule, and $x : \Theta \to \mathbb{R}_+$ is an investment schedule. The researcher observes m, and if she participates, reports her type, $\hat{\theta}$, forming the contract, $\{v(\hat{\theta}), g(\hat{\theta}), x(\hat{\theta})\}$. The researcher then chooses her inputs, the project's outcome is realized, and transfers are made accordingly. If the researcher does not participate, both parties earn zero. We restrict attention to continuous, piecewise-differentiable prize, grant/fee, and investment schedules, but it will be shown (in the proof of Proposition 4) that this is without loss of generality.

Given (v, g, x),[15] the researcher's effort, $y^*(x, v, \theta)$, solves $\max_{y\in[0,1]}\{\theta y \rho(x)(v + \pi) - C(x, y) + g\}$. At an interior solution, $y^*(\cdot)$ is unique-valued and given by, $y^*(x, v, \theta) = \left(\frac{\theta}{c}\rho(x)(v + \pi)\right)^\beta$. Note that under our chosen cost-of-effort function, ψ, the elasticity of the researcher's effort with respect to $v + \pi$ is constant, and equal to β: $\frac{\partial y^*}{\partial(v+\pi)}\frac{v+\pi}{y^*} = \beta$. The payoff to a type-$\theta$ who reports $\hat{\theta}$ is,

more than she would otherwise, this issue does not arise (see Rietzke and Chen, 2016).

[14]This constraint ensures that a successful researcher cannot benefit by shrouding her success from the funder. This may be possible if a researcher who claims failure cannot be disproved by the funder.

[15]Where it does not cause confusion, we will liberally abuse notation, and sometimes let $v \in \mathbb{R}_+, g \in \mathbb{R}$, and $x \in \mathbb{R}_+$ denote particular prize, grant, and investment amounts.

$$u(\hat{\theta}|\theta) = \theta y^*(x(\hat{\theta}), v(\hat{\theta}), \theta)\rho(x(\hat{\theta}))(v(\hat{\theta}) + \pi) - C(x(\hat{\theta}), y^*(x(\hat{\theta}), v(\hat{\theta}), \theta)) + g(\hat{\theta})$$

Let $u(\theta) \equiv u(\theta|\theta)$, $y^*(\theta) \equiv y^*(x(\theta), v(\theta), \theta)$, and $S(x, y, \theta) = \theta y\rho(x)(W + \pi) - C(x, y)$ denote total surplus. Under truthful reporting, the funder's payoff is,

$$\phi = \int_{\underline{\theta}}^{\overline{\theta}} \Big(\theta y^*(\theta)\rho(x(\theta)) [W - v(\theta)] - g(\theta)\Big) f(\theta)d\theta = \int_{\underline{\theta}}^{\overline{\theta}} \Big[S(x(\theta), y^*(\theta), \theta) - u(\theta)\Big] f(\theta)d\theta.$$

The second expression follows from the first by replacing $g(\theta)$ by $u(\theta)$ at each θ. The funder's payoff can be interpreted as expected consumer surplus, less the expected cost of funding, or expected total surplus, less the researcher's expected rent.[16]

Individual rationality (IR) requires $u(\cdot) \geq 0$; incentive compatibility (IC) requires for all $\theta, \hat{\theta} \in \Theta$, $u(\theta) \geq u(\hat{\theta}|\theta)$. By standard techniques, IC is satisfied if and only if for (almost) all θ:[17]

$$u'(\theta) = \frac{\partial u(\hat{\theta}|\theta)}{\partial \theta}|_{\hat{\theta}=\theta} = y^*(\theta)\rho(x(\theta))(v(\theta) + \pi) \tag{IC-F}$$

and

$$\frac{d}{d\theta}\rho(x(\theta))(v(\theta) + \pi) \geq 0. \tag{IC-S}$$

Since (IC-F) implies $u' \geq 0$, IR is satisfied for all types so long as $u(\underline{\theta}) \geq 0$; since ϕ is strictly decreasing in u, this constraint binds in equilibrium. Using (IC-F) and setting $u(\underline{\theta}) = 0$: $\int_{\underline{\theta}}^{\overline{\theta}} u(\theta)f(\theta)d\theta = \int_{\underline{\theta}}^{\overline{\theta}} y^*(\theta)\rho(x(\theta))(v(\theta) + \pi)h(\theta)f(\theta)d\theta$. The funder's problem may then be expressed,[18]

[16] Our results extend to an environment where the principal also values the researcher's profit, but there is a social cost to raising funds, as in Laffont and Tirole (1986).

[17] See, e.g. Laffont and Tirole (1993) pp. 64 and 121.

[18] Since y^* is unique, we need not include y as part of the principal's strategy.

$$\max_{x,v} \int_{\underline{\theta}}^{\overline{\theta}} \{S(x(\theta), y^*(\theta), \theta) - h(\theta)y^*(\theta)\rho(x(\theta))(v(\theta) + \pi)\} f(\theta)d\theta$$
$$\text{s.t. } x(\cdot) \geq 0, \text{ (IC-S), and } v(\cdot) \geq 0. \qquad \text{[P]}$$

Let $J(x, v, \theta)$ denote the integrand above. For each θ, we assume $\max_{x,v\geq 0} J(x, v, \theta) > 0$, which holds if W is sufficiently large,[19] and implies $\max_{x,y\geq 0} S(x, y, \theta) > 0$. Also assume for some $\hat{x} > 0$, $x < [>]\hat{x}$ implies $\rho''(x)\rho(x) + \beta\rho'^2 > [<]0$, which ensures J is strictly concave in x for $x > \hat{x}$ at the optimal v.[20] Finally, we will assume throughout that the parameter's are such that $y^*(\cdot) < 1$ at the solution to [P] (this holds if, for instance, $c > W + \pi$).

3 Results

Before characterizing the optimal contracts with AS and MH, we study three benchmarks: complete information, pure MH, and pure AS. We use upper-case letters (X, V, G, etc.) to denote optimal solutions. In the environments with MH, we let $Y(\theta) \equiv y^*(X(\theta), V(\theta), \theta)$.

3.1 First-Best: Complete Information

With complete information, the funder observes the true θ, as well as x and y. The funder's problem can be written $\max_{x,y,u\geq 0} \{S(x, y, \theta) - u\}$. Our first result characterizes the solution to this problem. We let $X_{FB}(\theta)$ and $Y_{FB}(\theta)$ denote the first-best investment and effort levels (respectively).

Proposition 1. *With complete information, the optimal means of funding is any combination of a prize, $V(\theta) \geq 0$, and transfer, $G(\theta)$, satisfying, $U(\theta) = \theta\rho(X_{FB}(\theta))(V(\theta) + \pi) - C(X_{FB}(\theta), Y_{FB}(\theta)) + G(\theta) = 0$. $X_{FB}(\theta)$*

[19] Relaxing this assumption may yield an interval of low types who invest nothing (and receive no rewards from the funder), but will not change the qualitative conclusions of our analysis for the range of types investing a positive amount.

[20] For instance, if $\rho(x) = 1 - \exp(-x)$ and $\beta = 1$ then $\hat{x} = \log(1 + \beta)$.

and $Y_{FB}(\theta)$, are unique, and satisfy, $\theta Y_{FB}(\theta)\rho'(X_{FB}(\theta))(W+\pi) = 1$ and $Y_{FB}(\theta) = (\theta\rho(X_{FB}(\theta))(W+\pi))^{\beta}$. A higher type invests more and exerts greater effort (i.e., $X'_{FB}(\theta) > 0$, $Y'_{FB}(\theta) > 0$).

Proposition 1 shows that with complete information, the optimal means of funding may take the form of a pure prize ($V(\theta) > 0$ and $G(\theta) = 0$), a pure grant ($V(\theta) = 0$ and $G(\theta) = 0$) or some combination of the two. Indeed, with complete information and risk-neutrality, a grant of value, $g > 0$ is equivalent to a prize, v, with expected value $\theta y\rho(x)v = g$.

3.2 Pure Moral Hazard

With pure MH, y is unobservable by the funder, but he observes θ and x. Given θ, the funder's problem may be expressed, $\max_{x,v,u\geq 0}\{S(x, y^*(x, v, \theta), \theta) - u\}$. The next result characterizes the optimal scheme with pure MH.

Proposition 2.
With pure MH, the optimal means of funding is a prize, $V(\theta) = W$, coupled with an entry fee $G(\theta) < 0$. Moreover, $X(\theta) = X_{FB}(\theta)$, $Y(\theta) = Y_{FB}(\theta)$, and $G(\theta)$ satisfies, $U(\theta) = S(X_{FB}(\theta), Y_{FB}(\theta), \theta) + G(\theta) = 0$.

With pure MH, the optimal funding scheme takes the form of a "franchise contract" in which the agent is made a full residual claimant, and the funder extracts the researcher's rent through an entry fee.

Although the researcher's effort, $y^*(x, v, \theta) = \left(\frac{\theta}{c}\rho(x)(v+\pi)\right)^{\beta}$, is completely independent of g, it is important to emphasize that a positive grant *can* induce greater effort. Note that y^* is strictly increasing in both x and v. As investment is observable, the funder may condition the grant on this variable and elicit greater investment. Greater investment increases the marginal returns to effort and increases effort. A prize also induces greater investment (and thus operates along this indirect channel), but it operates along a direct channel as well, since it is only received in the event of success.

3.3 Pure Adverse Selection

With pure AS, both x and y are observable by the funder, but θ is observed only by the researcher. The funder's problem is given by [P] (see Section 2), except the funder also chooses y, since effort is contractible. Our next result characterizes the optimal funding scheme under pure AS.

Proposition 3. *With pure AS, the optimal means of funding is a pure grant for all types (i.e., $G(\cdot) > 0$, $V(\cdot) = 0$). Moreover,*

(i) Higher types invest more, exert greater effort, and receive larger grants, but the grant only partially reimburses costs; higher types internalize a greater cost: $C(X(\theta), Y(\theta)) > G(\theta)$ and $\frac{d}{d\theta}\left[C(X(\theta), Y(\theta)) - G(\theta)\right] > 0$.

(ii) Investment and effort are distorted below the first-best: For $\theta < \overline{\theta}$, $X(\theta) < X_{FB}(\theta)$ and $Y(\theta) < Y_{FB}(\theta)$; but there is "efficiency at the top": $X(\overline{\theta}) = X_{FB}(\overline{\theta})$ and $Y(\overline{\theta}) = Y_{FB}(\overline{\theta})$.

Proposition 3 shows that a pure-grant is optimal for all types under pure AS. To understand why the optimal prize is zero, consider a two-type version of the model: $\Theta = \{\underline{\theta}, \overline{\theta}\}$, where $\overline{\theta} > \underline{\theta}$. IC dictates, $u(\overline{\theta}) \geq u(\underline{\theta}|\overline{\theta})$; in equilibrium, this constraint binds. Let x_L, y_L, v_L, and g_L denote the investment, effort, prize, and grant (respectively) intended for the low type. One can show,

$$u(\overline{\theta}) = u(\underline{\theta}|\overline{\theta}) = (\overline{\theta} - \underline{\theta})y_L\rho(x_L)(v_L + \pi) > 0. \tag{2}$$

From (2), notice that $u(\overline{\theta})$ is strictly increasing in v_L, but does not depend on g_L. Intuitively, if a high type mimics the low type, she is more likely to succeed and receive v_L than the low type would be. Hence, the expected value of the prize, $\theta y_L\rho(x_L)v_L$, is greater for the high type. To prevent under-reporting, the high type must receive a rent to compensate. A grant, in contrast, is received independently of outcome, so its expected value is the same for both types. For this reason, the prize is a more expensive means of funding. Also from (2), notice that $u(\overline{\theta})$ is increasing in x_L and y_L. To limit the rent of higher types, investment and effort are distorted below the first-best for all but the highest type. The optimal investment/effort schedules balances the trade-off between rent-extraction and efficiency.

Although this efficiency/rent extraction trade-off is standard, we note the role played by π and free-disposal in our model. If we relax free-disposal, then the funder appropriates all of the researcher's rent by setting $v(\cdot) = -\pi$ and $g(\cdot) = C(x(\cdot)), y(\cdot))$ (see Lewis and Sappington, 2000b). But under free-disposal, the researcher must capture at least π in the event of success, which leaves an inappropriable rent. To prevent low types from overstating their ability, the optimal grant does not fully reimburse costs ($C(X,Y) > G$), and higher types must internalize a greater cost ($\frac{d}{d\theta}\left[C(X,Y) - G\right] > 0$).

3.4 Mixed Case: Adverse Selection and Moral Hazard

We now study the case of AS and MH. In what follows, for a given θ, and a fixed y, we let $X_{FB|y}(\theta)$ denote the investment that maximizes total surplus: $X_{FB|y}(\theta) = \arg\max_{x\geq 0} S(x,y,\theta)$. $X_{FB|y}(\theta)$ is the unique solution to $\theta y \rho'(X_{FB|y}(\theta))(W+\pi) = 1$. We call $X_{FB|y}$ the first-best investment, given effort; we write $X_{FB|Y}(\theta)$ when $y = Y(\theta)$. The next result provides a condition under which the optimal funding scheme is a pure grant for some types.

Proposition 4. *If $\beta\underline{\theta}W < h(\underline{\theta})\pi(1+\beta)$, then there exists $\theta_v \in (\underline{\theta}, \overline{\theta})$ satisfying $\beta\theta_v W = h(\theta_v)\pi(1+\beta)$ such that,*

(i) For $\theta \in [\underline{\theta}, \theta_v]$, the optimal means of funding is a pure grant: $V(\theta) = 0$ and $G(\theta) > 0$. Moreover, $Y(\theta) < Y_{FB}(\theta)$ and $X(\theta) < X_{FB|Y}(\theta) < X_{FB}(\theta)$. Higher types invest more, exert greater effort, and receive larger grants, but the grant only partially reimburses investment: $X'(\theta) > G'(\theta) > 0$, $Y'(\theta) > 0$, and $X(\theta) > G(\theta)$.

(ii) For $\theta \in (\theta_v, \overline{\theta}]$, the prize is strictly positive, and given by,

$$V(\theta) = \frac{\beta\theta W - h(\theta)\pi(1+\beta)}{\theta\beta + h(\theta)(1+\beta)}.$$

Moreover, for $\theta < \overline{\theta}$, $Y(\theta) < Y_{FB}(\theta)$ and $X(\theta) = X_{FB|Y}(\theta) < X_{FB}(\theta)$, but $Y(\overline{\theta}) = Y_{FB}(\overline{\theta})$ and $X(\theta) = X_{FB}(\overline{\theta})$. Higher types invest more, exert greater effort, and receive larger prizes. Finally, there is some $\theta_g \in (\theta_v, \overline{\theta})$ such that for $\theta \in (\theta_g, \overline{\theta})$, the optimal funding scheme includes an entry fee, and higher types pay larger entry fees (i.e., $G(\theta) < 0$ and $G'(\theta) < 0$).

Proposition 4 shows that the optimal prize may be zero for some range of types, $[\underline{\theta}, \theta_v]$, despite the MH problem. Recall that the prize creates a strong incentive for effort (see Section 3.2), but generates large information rent for the researcher due to AS (see Section 3.3). Under the hypothesis of Proposition 4, the AS problem dominates the MH problem for types in $[\underline{\theta}, \theta_v]$, and the prize is zero. θ_v can be interpreted as capturing the relative severity of AS and MH. Specifically, when AS is more severe, relative to MH, θ_v is greater.[21] When the prize is zero, effort is incentivized indirectly through the grant; higher types receive large grants, but internalize a greater investment cost (i.e. $\frac{d}{d\theta}[X(\theta) - G(\theta)] > 0$).

To further elucidate the trade-off between the use/non-use of a prize, let us consider an auxiliary problem. Suppose the funder wants to induce effort, $\overline{y} \in (0, 1)$, at some θ. Let $\mathcal{I}(x, y, v, \theta) = h(\theta)y\rho(x)(v + \pi)$ denote the "virtual information cost". The funder's instantaneous payoff at θ when $y^*(x, v, \theta) = \overline{y}$ is, $J = [S(x, \overline{y}, \theta) - \mathcal{I}(x, \overline{y}, v, \theta)]\, f(\theta)$. The funder chooses (x, v) to maximize J, subject to $y^*(x, v, \theta) = \overline{y}$. In this problem, the optimal (x, v) depends on two considerations: First, is the relative effectiveness of x and v to elicit effort, while limiting $\mathcal{I}$. Second, is the impact of x and v on S.

For simplicity, set $\beta = c = 1$. To see how the first consideration operates in our model, note that the combinations of (x, v) that induce $\overline{y}$ satisfy, $y^*(x, v, \theta) = \theta\rho(x)(v + \pi) = \overline{y}$, or $\rho(x)(v + \pi) = \frac{\overline{y}}{\theta}$. Then $\mathcal{I}(x, \overline{y}, v, \theta) = h(\theta)\frac{\overline{y}^2}{\theta}$, which is independent from (x, v). Thus, any (x, v) that induces $\overline{y}$ yields the same virtual information cost, $\mathcal{I}$. To see how the second consideration operates, note that by strict concavity of $S(\cdot, y, \theta)$, $S(\cdot, \overline{y}, \theta)$ is strictly increasing [decreasing] for $x < [>] X_{FB|\overline{y}}$, while S is independent from v. Jointly, these

[21] Proposition 6 in Section 4 formalizes this intuition.

two considerations imply that it is optimal for the funder to induce a maximum investment of $x = X_{FB|\overline{y}}$, and provide any residual incentive necessary for effort through the prize. If $y^*(\tilde{x}, 0, \theta) = \overline{y}$ for some $\tilde{x} \leq X_{FB|\overline{y}}$, then the funder sets $x = \tilde{x}$, $v = 0$; in this case investment/effort are induced through the grant. If $y^*(X_{FB|\overline{y}}, 0, \theta) < \overline{y}$, the funder sets $x = X_{FB|\overline{y}}$, and specifies $v > 0$ to satisfy $y^*(X_{FB|\overline{y}}, v, \theta) = \overline{y}$. If $\overline{y} = Y(\theta)$, this scheme is precisely what is described in Proposition 4. A similar argument holds for any effort-cost function, ψ.

Multiplicative separability between x, y, and θ simplifies the first consideration. For more general technologies, x and v may differ in terms of their ability to raise effort while limiting $\mathcal{I}$, and the optimal (x, v) will depend on the interplay of the two considerations in a more intricate way. In particular, the stronger is the complementarity between x and y, the cheaper it will be (in terms of limiting $\mathcal{I}$) to induce effort through x.[22]

Proposition 4 also shows that the prize is strictly positive and increasing for $\theta \in (\theta_v, \overline{\theta}]$, and for θ high enough, the scheme resembles a franchise contract that emerged under pure MH (see Section 3.2). To understand these features, first note that the marginal benefit of effort (to the principal) is strictly increasing in θ. Larger prizes are therefore more attractive for higher types, as they can elicit greater effort. Second, as discussed in Section 3.3, a prize offered to some type, θ', generates rent for slightly higher types. But for θ' close to $\overline{\theta}$, the funder is less concerned about limiting the rent of a higher type (since the researcher is very unlikely to be of such a type), and the issue of limiting rent due to AS (which tilts against prizes) vanishes, while the MH problem (which tilts in favor or prizes) does not. Still, the following example demonstrates that θ_v can be arbitrarily close to $\overline{\theta}$.

Example 1. *Let* $\theta \sim U\left[\frac{2}{3}, 1\right]$, $\rho(x) = 1 - \exp(-x)$, $c = 5$, $\pi = 1$, $W = 4$, *For any* $\alpha > 8$ *(*$\beta < \frac{1}{7}$*), it holds that* $X(\theta) > 0$, *and the hypothesis of Proposition 4 is satisfied with,* $\theta_v = \frac{\alpha\pi}{W+\alpha\pi} = \frac{\alpha}{4+\alpha}$. *See that as* $\alpha \to \infty$ *(*$\beta \to 0$*),* $\theta_v \to \overline{\theta} = 1$.

[22]In general, the nature of the interactions between the considerations can be quite complex and may vary over the distribution of types. The primary technical complication is then determining if and when (IC-S) binds.

Example 1 shows that the interval of types that receive a prize can be arbitrarily small. Intuitively, when $\alpha \to \infty$ ($\beta \to 0$), $y^*(\cdot)$ becomes less sensitive to the prize. Reducing the prize has little impact on effort, and AS becomes the dominant information problem. We also note that if, due to a limitation on the researcher's time/energy, the upper-bound on effort binds for some $\tilde{\theta} < \theta_v$ (that is, $y^* = \left(\frac{\theta}{c}\rho(X(\tilde{\theta}))\pi\right)^\beta = 1$), then for $\theta > \tilde{\theta}$, MH imposes a non-binding constraint on the funder, and pure grant funding will be optimal for all types.[23] Our next result provides a condition under which the prize is strictly positive for all types.

Proposition 5. *If $\beta\underline{\theta}W - h(\underline{\theta})\pi(1+\beta) > 0$, then the prize is strictly positive for all types, and the conclusion of Proposition 4(ii) holds with $\theta_v = \underline{\theta} \leq \theta_g$.*

Under the hypothesis of Proposition 5, the MH problem is more severe relative to AS, and performance-pay is optimal for all types. The funding scheme may still include a positive grant for low/intermediate types, but for high enough types, it includes an entry fee.

In the next section, we explore more completely the circumstances under which performance-pay is or is not utilized, but before proceeding, we mention the role of free-disposal and the particular functional form of ψ we have studied. Under the hypothesis of Proposition 5, free-disposal never binds, and has no impact. If we relax this constraint under the hypothesis of Proposition 4, it can be shown that for $\theta < \theta_v$, $G(\theta) > 0$, $X(\theta) = X_{FB|Y}(\theta)$, and $-\pi < V(\theta) = \frac{\theta\beta W - h(\theta)\pi(1+\beta)}{\theta\beta + h(\theta)(1+\beta)} < 0$. In some circumstances, this scheme can be interpreted as one in which the funder purchases an equity stake in the researcher's project.

The functional form of ψ we have studied implies a constant elasticity of effort with respect to the reward for success; however, the qualitative features of our main results hold for more general technologies. Consider any ψ with $\psi'(0) = \psi(0) = 0$, and $\psi''(y) > 0$ for $y > 0$. It can be shown that the elasticity, ϵ, of the researcher's effort with respect to $v + \pi$ depends only on $z = \theta\rho(x)(v + \pi)$. The qualitative nature of Propositions 4 and 5 hold so long

[23] In Rietzke and Chen (2016), we illustrate a similar result in a binary-effort model.

as $\epsilon'(z) \geq 0$ for all z. This condition is analogous to condition (12b) in Ollier and Thomas, and is sufficient to rule out a binding (IC-S) constraint.[24]

4 Further Analysis and Discussion

Comparative Statics

Our next result explores how θ_v depends on the parameters of the model. First, it will be useful to introduce a parameter that captures the severity of the AS problem. To this effect, let us parameterize the distribution of θ by $t \in \mathbb{R}$ such that, at each $\theta \in [\underline{\theta}, \overline{\theta})$, $\frac{\partial h(\theta;t)}{\partial t} > 0$. Greater t reflects a more severe AS problem in the following sense: Recall that higher types capture rent through their ability to mimic slightly lower types. Fix $\theta' < \overline{\theta}$, and consider the event that $\theta > \theta'$, conditional on θ being in some neighborhood of θ'. When this event is more likely, the greater is the expected rent relinquished in this neighborhood (for a given (x, v)), and can we say the AS problem is more severe. This is precisely what is captured by t: a higher t increase the likelihood of this event at each $\theta' \in [\underline{\theta}, \overline{\theta})$.

Proposition 6. *Suppose the hypothesis of Proposition 4 is satisfied. θ_v is strictly decreasing in β and W, and is strictly increasing in π and t.*

Proposition 6 reveals that pure grant funding is used for a wider range of types when: (1) the funder's value for the project is modest (i.e., W is not "too large"); (2) the researcher associates a higher value to the project (i.e., π is large); (3) the researcher's effort is less sensitive to the reward for success (i.e., β is small); and (4) the AS problem is more severe (i.e., t is large).

Point (1) holds since effort is less valuable to the funder when W is small, rendering prizes less attractive. Points (2)-(3) speak to the severity of the MH problem: When π is large, the researcher has a stronger natural incentive to exert effort. When β is small, the researcher's effort is less sensitive to the prize;

[24] When $\epsilon' < 0$, the effort of higher types is less sensitive to changes in the prize, and the funder may want to reduce the prize of higher types. In consequence, (IC-S) may bind, and bunching could arise in equilibrium.

therefore, even a large reduction in the prize results in a modest reduction in effort. Point (4) holds since AS raises the expected cost of performance-pay; the greater is this problem, the less attractive prizes become.

The complementarity between investment/effort is also important for the usefulness of the grant in our model. As a point of comparison, in multitasking models when efforts are substitutes, the principal reduces incentives on more-easily observed tasks to avoid crowding-out effort on less-easily observed tasks. By a similar logic, if investment/effort were substitutes in our model then greater investment reduces effort, rendering the grant an ineffective means of eliciting effort.

We now explore how the prize depends on the parameters of the model.

Proposition 7. *Fix θ, and suppose $V(\theta) > 0$. $V(\theta)$ is strictly increasing in β and W, and is strictly decreasing in π and t.*

Proposition 7 shows that larger prizes are utilized under antithetical circumstances to those described in Proposition 6. The intuitions follow as the counterpoint to those results.

The use and design of push and pull programs

Grace and Kyle (2009) note that there is limited evidence on how push and pull incentives work together. Our results describe precisely how these incentives can work together to help resolve AS and MH problems. Taken as a whole, our results suggest that hybrid incentive schemes, which combine push and pull elements, could be particularly effective in dealing with these information problems: A push incentive to subsidize contractible inputs and limit information rent, and a pull incentive to motivate non-contractible inputs.

Hybrid schemes are indeed used in practice. The U.S. Department of Defense, for example, subsidizes R&D expenditures of its contractors, but also uses prizes to reward successful innovations (Lichtenberg, 1988; Rogerson, 1989). In the UK, the Department for International Development provides funding for agricultural research in the form of grants, but also awards a "bonus payment" if particular performance milestones are achieved.[25] Programs in both the U.S. and EU meant to promote the development of "orphan drugs" include both R&D tax credits (push) as well as priority review vouchers (pull) (Mossialos et al., 2010). To spur the development of a meningitis C vaccine, an initiative sponsored by the UK Department of Health in 1996 offered clinical trial support (push) and advanced purchase agreements (pull) (Levine et al., 2005).

Once concern with push incentives is that they may pay for research unlikely to succeed. To deal with this issue, when the pure grant emerges in our model, higher types receive larger grants but must internalize a greater investment cost; this ensures that low types are unwilling to accept large grants.[26] Matching grants share this feature, and are commonly used in practice. For instance, the National Science Foundation "...requires that each grantee share in the cost of research projects resulting from unsolicited proposals".[27] As part of its Horizon 2020 program, the European Commission awards grants that cover a fraction of firms' costs.[28]

As policy implications, our results call for stronger pull incentives when an innovation yields a high social value, but the researcher is unable to appropriate much of the surplus and lacks other (potentially intrinsic) motivations to succeed. Moreover, greater pull incentives are called for when a researcher is highly responsive to the reward for success. Our results call for stronger push incentives when a researcher possesses superior knowledge as to the likelihood of success, but highlight that these incentives should require researchers to share in the cost of R&D. Finally, offering a choice between a push and pull incentive could be an effective means of screening: a researcher should be more

[25] `https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/517111/DFIDs-Perf-Man-Funding-Intern-Agri-Research-Centres.pdf`

[26] Maurer and Scotchmer (2003) point out that a matching grant can be an effective screening device with AS; our result also takes MH into account. Cost sharing has been advocated in other contexts for dealing with AS and MH (see, e.g., Laffont and Tirole, 1986).

[27] `https://www.nsf.gov/pubs/manuals/gpm05_131/gpm3.jsp#330`.

[28] `http://ec.europa.eu/programmes/horizon2020/en/h2020-section/sme-instrument`

willing to accept a pull incentive when success is more likely.[29]

Capital Constraints

Some push programs provide upfront funding, which may be necessary if a researcher is capital-constrained. Yet as Scotchmer (2004, Ch. 8) points out, an appropriately designed pull program should be capable of attracting financing. Indeed, this is precisely the logic behind the "Pay for Success" model run by the U.S. Department of Labor.[30] Moreover, some push programs (e.g., R&D tax credits), do not provide funding upfront. It is therefore useful to explain why push incentives might emerge naturally under MH, rather than out of necessity due to a capital constraint. Nevertheless, our results are relevant for understanding a related issue. Consider a researcher who has a strong incentive to devote her time/energy to a project (i.e. π is large and/or β is small), but is unwilling to raise the necessary capital.[31] While a pull program *could* be used to encourage investment, in our model, pure-grant funding may be optimal for a wider range of types.

Still, it is worth commenting on how a capital constraint affects our results. In an extreme case where the researcher has no access to capital, the grant must fully reimburse investment. The funder then loses a critical tool used for screening, namely, that higher types internalize a greater investment cost.[32] In this version of the model, (IC-S) inevitably binds and bunching may arise.

5 Conclusion

We have characterized the optimal contracts in a setting with AS/MH and partially observable actions. In contrast to typical findings in MH models, we showed that performance-pay may not be optimal for all types, but is always utilized for the highest types. Our results are useful for understanding the

[29] Lazear (2000) delivers a similar insight regarding performance-pay in labor markets.

[30] https://www.doleta.gov/workforce_innovation/success.cfm

[31] In our model, the unwillingness of the researcher to raise capital is captured by (1).

[32] The issue is analogous to the wealth constraint in Lewis and Sappington (2000b).

basic trade-offs caused by AS/MH between push and pull programs used to encourage R&D activity, but our results are relevant in other contexts, e.g., worker compensation.

Appendix

For clarity, we omit the arguments of functions when there is no ambiguity.

Proof of Proposition 1

The funder's payoff is strictly decreasing in u, so optimality dictates $U = 0$. The first-best investment/effort levels solve, $\max_{x,y\geq 0} S(x,y,\theta)$, where $S(x,y,\theta) = \theta\rho(x)y(W+\pi) - C(x,y)$ is total surplus. By assumption, $\max_{x,y\geq 0} S(x,y,\theta) > 0$, so the optimal investment/effort levels must satisfy the two first-order conditions given in the proposition.

We now show that X_{FB} and Y_{FB} are unique. Consider sequentially maximizing over y, then x. For a given x, the effort level, $Y(x)$, that maximizes $S(x,\cdot,\theta)$, is unique, and, when interior, is, $Y(x) = \left(\frac{\theta}{c}\rho(x)(W+\pi)\right)^{\beta}$. Then, the first-best investment level must satisfy,

$$S_x(x, Y(x), \theta)|_{x=X_{FB}(\theta)} = \frac{\theta^{\beta+1}}{c^{\beta}}\rho(X_{FB})^{\beta}\rho'(X_{FB})(W+\pi)^{\beta+1} - 1 = 0. \quad (3)$$

The second-order necessary condition requires $\rho''(X_{FB})\rho(X_{FB}) + \rho'(X_{FB})^2 \leq 0$; equivalently, $X_{FB} \geq \hat{x}$. As, $\rho^{\beta}\rho'$ is strictly decreasing for $x > \hat{x}$, there is a unique solution to (3) satisfying $X_{FB} \geq \hat{x}$; since X_{FB} is unique, Y_{FB} is unique.

Next, we show that $X_{FB} > \hat{x}$. We proceed by contradiction. As we argued above, it must be that $X_{FB} \geq \hat{x}$; so, contrary to the claim, suppose $X_{FB} = \hat{x}$. See that $S_{xx}(\cdot, Y(\cdot), \theta) > 0$ for $x < \hat{x}$. It follows that for any $x < \hat{x}$, $S_x(x, Y(x), \theta) < S_x(\hat{x}, Y(\hat{x}), \theta) = 0$, where the equality holds since the contradiction hypothesis is $X_{FB} = \hat{x}$. Then, noting that $S(0, Y(0), \theta) = 0$, it follows that $S(x, Y(x), \theta) < 0$ for all $x \leq \hat{x}$; in particular, $S(\hat{x}, Y(\hat{x}), \theta) < 0 = S(0, Y(0), \theta)$, which contradicts the hypothesis that $X_{FB} = \hat{x}$. Thus, it must be that $X_{FB} > \hat{x}$. Using $X_{FB} > \hat{x}$, the Implicit Function Theorem applied to (3) implies $X'_{FB} > 0$; it is then straightforward to confirm $Y'_{FB} > 0$.

Proof of Proposition 2

The funding scheme outlined in the Proposition gives the funder the first-best payoff of $\max_{x,y\geq 0} S(x,y,\theta)$; therefore, this is an optimal scheme. To see that it is the *unique* optimal funding scheme, note that when the researcher invests X_{FB}, then, $y^*(X_{FB}, v, \theta) = Y_{FB}$ if and only if $v = W$. It follows that any $v \neq W$ leads to strictly lower total surplus, and the funder's payoff is strictly less than $\max_{x,y\geq 0} S(x,y,\theta)$.

Proof of Proposition 3

We first ignore (IC-S); we then verify it is non-binding at the solution to the relaxed problem. As the funder's payoff is strictly decreasing in v, the optimal prize is zero. Setting $V = 0$, the point-wise first-order conditions are,

$$\theta Y \rho'(X)(W + \pi) - 1 - hY\rho'(X)\pi = 0 \tag{4}$$

and

$$Y = \left(\frac{\theta}{c}\rho(X)[W + \pi - h\pi]\right)^{\beta} \tag{5}$$

Similar arguments to those in the proof of Proposition 1 reveal that X and Y are unique, and $X > \hat{x}$. Now, fix $\theta < \overline{\theta}$; we show $X(\theta) < X_{FB}(\theta)$ and $Y(\theta) < Y_{FB}(\theta)$. Combining (4) and (5):

$$\rho(X)^{\beta}\rho'(X)\frac{[\theta(W + \pi) - h\pi]^{\beta+1}}{c^{\beta}} = 1. \tag{6}$$

As $h(\theta) > 0$ for $\theta < \overline{\theta}$, (3) and (6) imply $\rho(X)^{\beta}\rho'(X) > \rho(X_{FB})^{\beta}\rho'(X_{FB})$. But since $\rho^{\beta}\rho'$ is strictly decreasing for $x > \hat{x}$, it follows that $X < X_{FB}$. Using (5), it is then straightforward to see that $Y(\theta) < Y_{FB}(\theta)$. Finally, as $h(\overline{\theta}) = 0$, (4) and (5) imply $X(\overline{\theta}) = X_{FB}(\overline{\theta})$ and $Y(\overline{\theta}) = Y_{FB}(\overline{\theta})$.

Next, we show that (IC-S) is satisfied. When $V = 0$, (IC-S) is satisfied if $X' \geq 0$. Applying the Implicit Function Theorem to (6):

$$\rho(X)^{\beta-1}\left[\beta\rho'^2 + \rho''(X)\rho(X)\right](\theta(W+\pi) - h\pi)^{\beta+1} X' \\ + \rho(X)^{\beta}\rho'(X)(\beta+1)\left[\theta(W+\pi) - h\pi\right]^{\beta}\left[W + \pi - h'\pi\right] = 0$$

As $X > \hat{x}$, the term in square brackets on the first line above is strictly negative; moreover, the expression on the second line is strictly positive. It follows that $X' > 0$. It is then straightforward to confirm $Y' > 0$. This establishes part (ii).

We now establish part (i). (IR), $V = 0$ and expression (1) jointly imply $G > 0$. Next, $U(\underline{\theta}) = V(\underline{\theta}) = 0$ means, $X(\underline{\theta}) + \psi(Y(\underline{\theta})) - G(\underline{\theta}) = \underline{\theta}\rho(X(\underline{\theta}))\pi > 0$. Hence, $X(\underline{\theta}) + \psi(Y(\underline{\theta})) > G(\underline{\theta})$. Then, using the definition of $U(\theta)$, (IC-F) can be written:

$$X' + \psi'(Y)Y' - G' = \theta\pi\left[Y\rho'(X)X' + Y'\rho(X)\right] > 0 \tag{7}$$

We have shown $X(\underline{\theta}) + \psi(Y(\underline{\theta})) > G(\underline{\theta})$ and $X' + \psi'(Y)Y' > G'$, which means, $X + \psi(Y) > G$. Finally, we show $G' > 0$. Fix θ; (1) and (4) imply $\theta W > h\pi$. Re-writing (5), $\theta\rho(X)\pi - \psi'(Y) = -\rho(X)(\theta W - h\pi) < 0$, and re-writing (7), $X'\left[\theta Y\rho'(X)\pi - 1\right] + Y'\left[\theta\rho(X)\pi - \psi'(Y)\right] + G' = 0$. Expression (1) and $\theta\rho(X)\pi - \psi'(Y) < 0$ then imply $G' > 0$. $\square$

Proof of Proposition 4

For the moment, we ignore (IC-S); we will then show that it is satisfied at the solution to the relaxed problem. The relaxed problem amounts to pointwise maximization of J, (where J is the integrand of the problem [P]) subject to, $x \geq 0$ and $v \geq 0$. Plugging in $y^* = \left(\frac{\theta}{c}\rho(x)(v+\pi)\right)^{\beta}$ into J,

$$J(x, v, \theta) = \left(\frac{\theta}{c}\right)^{\beta}(v+\pi)^{\beta}\rho(x)^{1+\beta}\left[\theta(W+\pi) - (v+\pi)\left(\theta\frac{\beta}{1+\beta} + h\right)\right] - x$$

The first-order conditions/complementary slackness conditions are:

$$J_x = (1+\beta)\left(\frac{\theta}{c}\right)^{\beta}(V+\pi)^{\beta}\rho(X)^{\beta}\rho'(X)\left[\theta(W+\pi)-(V+\pi)\left(\theta\frac{\beta}{1+\beta}+h\right)\right]=1 \tag{8}$$

and

$$J_v = \left(\frac{\theta}{c}\right)^{\beta}\rho(X)^{1+\beta}(V+\pi)^{\beta-1}\left[\theta\beta(W+\pi)-(V+\pi)\big(\theta\beta+h(1+\beta)\big)\right]\leq 0 \tag{9}$$

$$VJ_v = 0;\quad V\geq 0$$

(9), together with the complementary slackness conditions, imply that $V(\theta) > 0$ if and only if $k(\theta) \equiv \theta\beta W - h(\theta)(1+\beta)\pi > 0$. By the hypothesis of the proposition, $k(\underline{\theta}) < 0$. As $k(\overline{\theta}) = \overline{\theta}\beta W > 0$, by continuity, there exists $\theta_v \in (\underline{\theta}, \overline{\theta})$ such that $k(\theta_v) = 0$. Moreover, $k' = \beta W - h'(1+\beta)\pi > 0$. Thus, $\theta < \theta_v$ implies $k(\theta) < 0$; $\theta > \theta_v$ implies $k(\theta) > 0$. Now, fix $\theta > \theta_v$; (9) implies, $V = \frac{\theta\beta W - h(1+\beta)\pi}{\theta\beta + h(1+\beta)}$. It is straightforward to confirm that V is differentiable and $V' > 0$; moreover, as $h(\overline{\theta}) = 0$, it holds that $V(\overline{\theta}) = W$. Plugging V into (8):

$$\theta\left(\frac{\theta}{c}\rho(X)(V+\pi)\right)^{\beta}\rho'(X)(W+\pi)=1; \tag{10}$$

equivalently, $\theta Y\rho'(X)(W+\pi) = 1$, or $X = X_{FB|Y}$. Note that the second-order necessary conditions require $J_{xx}(X,V,\theta) \leq 0$ and $J_{xx}(X,V,\theta)J_{vv}(X,V,\theta) - J_{xv}(X,V,\theta)^2 \geq 0$. It may be verified that $J_{xv}(X,V,\theta) = 0$; moreover, for any x, $J_{vv}(x,V,\theta) < 0$. Additionally, $J_{xx}(x,V,\theta) > [<]0$ for $x < [>]\hat{x}$. By the necessary second-order condition, it must be that $X \geq \hat{x}$. But by arguments similar to those made in the proof of Proposition 1, in fact, $X > \hat{x}$. For $\theta < \overline{\theta}$, $V(\theta) < W$, but $V(\overline{\theta}) = W$. Arguments similar to those made in the proof of Proposition 3 then reveal, $X(\theta) \leq X_{FB}(\theta)$ and $Y(\theta) \leq Y_{FB}(\theta)$, holding with equality only when $\theta = \overline{\theta}$. For $x > \hat{x}$, the LHS of (10) is strictly decreasing;

it follows that X is unique. Moreover, the Implicit Function Theorem implies that X is differentiable in θ and satisfies,

$$\begin{aligned}&\frac{\theta^{1+\beta}}{c^{\beta}}(V+\pi)^{\beta}\rho(X)^{\beta-1}\left[\beta\rho'^{2}+\rho(X)\rho''(X)\right]X'\\&+(1+\beta)\frac{\theta^{\beta}}{c^{\beta}}\rho(X)^{\beta}\rho'^{\beta}+\frac{\theta^{1+\beta}}{c^{\beta}}\rho(X)^{\beta}\rho'^{\beta-1}V'=0.\end{aligned}$$

As $X>\hat{x}$, the term in square brackets on the first line above is strictly negative. The two terms on the second line are both strictly positive. Thus, $X'>0$. It is then straightforward to confirm that $Y'>0$.

Next, fix $\theta<\theta_v$. $V=0$ and (8) imply, $\theta\rho'(X)Y(W+\pi)-1+\rho'(X)Yk=0$. As $k<0$, it holds, $\theta\rho'(X)Y(W+\pi)-1>0$; by strict concavity of ρ, $X<X_{FB|Y}(\theta)$. Following similar arguments as made when $\theta>\theta_v$, it can be shown that $X>\hat{x}$, and that X is unique and differentiable in θ with $X'>0$.

Finally, we explore properties of G. Using (IC-F) and the IR condition, $U(\underline{\theta})=0$, for any θ, it holds, $U(\theta)=\int_{\underline{\theta}}^{\theta}Y(t)\rho(X(t))(V(t)+\pi)dt$. It follows,

$$G(\theta)=\int_{\underline{\theta}}^{\theta}Y(t)\rho(X(t))(V(t)+\pi)dt+C(X(\theta),Y(\theta))-\theta\rho(X(\theta))Y(\theta)(V(\theta)+\pi). \tag{11}$$

We have already shown that X, V, and Y are differentiable in θ, for $\theta\in[\underline{\theta},\theta_v)\cup(\theta_v,\overline{\theta}]$; therefore G is differentiable on this set. Fix $\theta<\theta_v$; it holds, $G'=X'[1-\theta Y\rho'(X)\pi]$. $X'>0$ and (1) imply $G'>0$. Moreover, $X'-G'=\theta Y\rho'(X)\pi X'>0$. Now, as $V(\underline{\theta})=0$, expression (1), and the IR constraint, $U(\underline{\theta})=0$, imply $G(\underline{\theta})>0$. Moreover, the optimality of $Y(\underline{\theta})$ means $\underline{\theta}Y(\underline{\theta})\rho(X(\underline{\theta}))\pi-\psi(Y(\underline{\theta}))>0$. $U(\underline{\theta})=0$ then implies, $X(\underline{\theta})-G(\underline{\theta})=\underline{\theta}Y(\underline{\theta})\rho(X(\underline{\theta}))\pi-\psi(Y(\underline{\theta}))>0$. We have shown $X(\underline{\theta})>G(\underline{\theta})>0$ and for all $\theta\in[\underline{\theta},\theta_v)$, $X'(\theta)>G'(\theta)>0$; hence, $X(\theta)>G(\theta)>0$ for $\theta\in[\underline{\theta},\theta_v]$.

Next, we show that $G(\theta)<0$ for θ sufficiently close to $\overline{\theta}$. Let $S^*(\theta)=\max_{x\geq 0,y\in[0,1]}\{\theta y\rho(x)(W+\pi)-C(x,y)\}$. By the Envelope Theorem, $\frac{dS^*(\theta)}{d\theta}=Y_{FB}(\theta)\rho(X_{FB}(\theta))(W+\pi)$. It follows that,

$$S^*(\theta) = \int_{\underline{\theta}}^{\theta} Y_{FB}(t)\rho(X_{FB}(t))(W+\pi)dt + S^*(\underline{\theta}). \tag{12}$$

(IC-F) and IR imply $U(\overline{\theta}) = \int_{\underline{\theta}}^{\overline{\theta}} Y(t)\rho(X(t))(V(t)+\pi)dt$. But noting that $X(\overline{\theta}) = X_{FB}(\overline{\theta})$, $Y(\overline{\theta}) = Y_{FB}(\overline{\theta})$, and $V(\overline{\theta}) = W$, it also holds, $U(\overline{\theta}) = S^*(\overline{\theta}) + G(\overline{\theta})$. Thus, $\int_{\underline{\theta}}^{\overline{\theta}} Y(t)\rho(X(t))(V(t)+\pi)dt = S^*(\overline{\theta}) + G(\overline{\theta})$. Plugging in the expression for S^* from (12) and rearranging:

$$G(\overline{\theta}) = \int_{\underline{\theta}}^{\overline{\theta}} \left[Y(t)\rho(X(t))(V(t)+\pi) - Y_{FB}(t)\rho(X_{FB}(t))(W+\pi)\right] dt - S^*(\underline{\theta}) < 0,$$

where the inequality follows since $S^*(\underline{\theta}) > 0$, and for all $\theta < \overline{\theta}$, $X(\theta) < X_{FB}(\theta)$, $Y(\theta) < Y_{FB}(\theta)$, and $V(\theta) < W$, which means that the integrand above is strictly negative for all $t < \overline{\theta}$. As $G(\overline{\theta}) < 0$, continuity implies that $G(\theta) < 0$ for θ sufficiently close to $\overline{\theta}$. To complete the proof, we show $G'(\theta) < 0$ for θ sufficiently close to $\overline{\theta}$. For $\theta > \theta_v$, (11) implies,

$$G'(\theta) = X'(\theta)\left[1 - \theta Y(\theta)\rho'(X(\theta))(V(\theta)+\pi)\right] - \theta Y(\theta)\rho(X(\theta))V'(\theta)$$

As $Y(\overline{\theta}) = Y_{FB}(\overline{\theta})$, $X(\overline{\theta}) = X_{FB}(\overline{\theta})$, and $V(\overline{\theta}) = W$, the term in square brackets on the RHS above is equal to zero when $\theta = \overline{\theta}$. Since X' is finite, and the term in square brackets is continuous, the term, $X'(\theta)[1 - \theta Y(\theta)\rho'(X(\theta))(V(\theta)+\pi)]$, can be made arbitrarily small for θ sufficiently high. For any $\theta \in (\theta_v, \overline{\theta}]$, the term, $\theta Y(\theta)\rho(X(\theta))V'(\theta)$, is strictly positive, and bounded away from zero. Therefore $G'(\theta) < 0$ for θ sufficiently high. $\square$

Proof of Proposition 5

The proof is nearly identical to the proof of Proposition 4 in the case where $\theta > \theta_v$. We therefore omit this proof here. $\square$

Proof of Proposition 6

Fix $\theta < \overline{\theta}$ and let $L(\theta; t) = \frac{\theta}{h(\theta;t)}$. Note that $\frac{\partial L}{\partial \theta} > 0$ and $\frac{\partial L}{\partial t} < 0$. By definition, $L(\theta_v; t) = \frac{(1+\beta)\pi}{\beta W}$. Applying the Implicit Function Theorem, it is straightforward to establish the properties of θ_v given in the Proposition. $\square$

Proof of Proposition 7

By Propositions 4 and 5, when $V(\theta) > 0$ it holds, $V(\theta) = \frac{\beta\theta W - h(\theta;t)\pi(1+\beta)}{\theta\beta + h(\theta;t)(1+\beta)}$. Differentiating $V(\theta)$ with respect to β, W, π, and t the comparative statics given in the Proposition follow. $\square$

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