Graz Economics Papers – GEP

GEP 2018–11

# Delegated Project Search

Xi Chen, Yu Chen and Xuhu Wan

May 2018

Department of Economics
Department of Public Economics
University of Graz

# Delegated Project Search*

Xi Chen† Yu Chen‡ Xuhu Wan§

This draft: May 21, 2018

**Abstract**

This paper explores a new continuous-time principal-agent problem for a firm with both moral hazard and adverse selection. Adverse selection appears at random times. The agent finds projects sequentially by exerting costly effort. Each project brings output to the firm, subject to the agent's private shocks. These serial shocks are i.i.d and independent of the arrival time of new projects and the agent's efforts. The shocks and efforts constitute the agent's asymmetric information. We provide a full characterization of optimal contracts in which moral hazard effect and adverse selection effects interact. The second-best contract with moral hazard can achieve first-best efficiency, and third-best contract with the moral hazard and adverse selection can achieve second-best efficiency under pure adverse selection, if the agent is expectably rich enough. The payment is front-loaded under pure moral hazard. When moral hazard is combined with adverse selection, the payment can be back-loaded or front-loaded, depending on the level of expectable wealth.

**Keywords**: Dynamic Contract, Continuous Time, Moral Hazard, Adverse Selection, Project Search

**JEL Classification**: C61, D82, D86, J30.

---

*This work was supported in part by the National Natural Science Foundation of China under Grant Nos. 71771118, 71471083, and 71390521; the National Social Science Foundation of China under Grant No. 15ZDB126; the Jiangsu Natural Science Foundation under Grant No. BK20151388; the Jiangsu Social Sciences Talent Project and the Six Talent Peaks Project in Jiangsu Province.

†School of Management, Nanjing University, China (doctor_chan@163.com);

‡Department of Economics, University of Graz, Austria (yu.chen@uni-graz.at);

§Department of Information Systems, Business Statistics and Operation Management, Hong Kong University of Science and Technology, Hong Kong (xuhu.wan@gmail.com).

# 1 Introduction

Businesses that involves dynamic agency often face the problem of providing incentives for agents when a continual, costly project search is delegated to agents, and the agents privately know the hidden characteristics (shocks) of the projects discovered. Although the agents constantly search for projects, all of the projects are usually discovered at random times due to significant uncertainty. For instance, an R&D department manager[1] must exert costly effort to continue searching for R&D projects, since innovation is a continuous process, and the consequent competition will be constantly intense; the projects, in contrast, normally arrive at random times. However, the project qualities (or shocks) that this manager takes to generate R&D outputs from the discovered projects are, in most circumstances, only known perfectly to himself. Clearly, executives' private shocks, in terms of adverse selection, must be considered when deciding on their incentive packages, in addition to performance measures. Moreover, moral hazard arises because he needs to exert unobserved costly effort to increase the likelihood that a new project will be found. The firm owner must design an optimal contract that can motivate the manager to truthfully reveal the quality of projects (or his private shocks) and obediently follow effort recommendations over time. Consider another example of contracting with a firm's CEO. The CEO needs to persistently make every effort to discover and develop profitable business projects, while knowing more about the production and market demand concerning all of the projects, which vary over time. To design an optimal investment and compensation scheme, the investor needs to identity this CEO's private information; otherwise, the CEO has an incentive to mis-report based on private benefits or the empire-building incentive (as in Stulz (1990)[43], Harris and Raviv (1996)[18], and Bernardo et al. (2001)[6]).

A few questions remain, however, that are important and desirable for relevant business practices. How do companies prevent managers from untruthfully releasing their private information? How does private knowledge affect managers' hidden effort on continual project search? How should the optimal compensation scheme be designed for managers over time? Can it achieve any efficiency? Motivated by these questions, in this paper we examine the optimal continuous-time contracting problem of the dynamic delegation of project search, in the presence of the repeated adverse selection that arises from repeated private shocks from the projects and the dynamic moral hazard that arises from the agent's hidden effort in project search. Moreover, the findings from this study may also shed light on other contexts in dynamic (continuous-time) optimal contracting, in addition to the dynamic delegation of project search.

The continuous-time principal-agent problem has attracted much attention recently from economic theory researchers. Sannikov (2008)[40] introduces a continuous-time principal-agent model that focuses on the dynamic properties of optimal incentive provision with moral hazard, and provides conditions under which the principal will compel the agent to retire early, depending on general outside options. Many studies have been carried out to extend Sannikov's (2008)[40] work. Most of them considers dynamic

[1] Kloyer and Helm (2008)[26] report empirical findings of such a relation.

moral hazard in different settings.[2] There are a few exceptions.[3] Sannikov (2007a)[38] investigates an optimal dynamic financing contract for a cash-constrained entrepreneur who privately knows the constant quality of the project. Zhang (2009)[53] and Tchistyi (2006)[45] investigate persistent hidden information by assuming that the private shock is a Markov switching process, and Williams (2009) studies a similar model with a hidden diffusion process. Zhu (2013)[54] explicitly derives the optimal dynamic incentive contract in a standard continuous-time agency setting in which the agent is shirking. Mason and Välimäki (2015)[32] examine dynamic incentives to complete a project that requires continuous-time effort input. Williams (2015)[51] proposes a solvable continuous time dynamic principal–agent model.

This paper examines the determinants of executive compensation with a continuous-time principal-agent model. Our model represents a significant departure from the previous studies of continuous-time agency, as described above, since they allow the agent to have repeated, independent and identically distributed (i.i.d.) private shocks,[4] in addition to dynamic project search efforts. The assumption of i.i.d. is a simplification of reality that provides a tractable framework for investigating contracting and games subject to repeated adverse selection. To the best of our knowledge, this is the *first* article to allow for i.i.d. private shocks in a continuous-time principal-agent model. We **(1)** identify the factors that cause the manager to truthfully report the values of shocks; **(2)** investigate the degree to which current and future payments motivate the agent; and **(3)** discuss how the payment is dynamically loaded to provide the incentive.

In our model, at the initial time, the principal (or firm owner; she) commits to a contract contingent on the arrival of projects and the binary values of the shocks reported by the agent (or manager; he). Such a contract specifies the payment the principal makes to the manager when a new project arrives. The agent's payment is subject to two types of uncertainty or risk. The first is the uncertainty of the project's arrival, and the second is the uncertainty of shock values. Hence the principal incentivizes the agent not only through current consumption (instantaneous payment) but also through the level and risk of future consumption. The agent has an initial reservation utility to accept the contract. We focus on the setting in which the agent's payments are bounded in our model;[5] his continuation value process takes value in a bounded set that is called "the feasible set." This is beneficial for discussing the limits of continuation values and identifying explicit boundary conditions for the Hamilton-Jacobi-Bellman optimality equation. As a result, if the agent's continuation value reaches the minimum value of the feasible set, he will have

[2] He (2008)[20] investigates the moral hazard problem with firm size change. Biais et al. (2010)[8] investigate optimal contracting when the agent's hidden effort can delay the arrival of loss. Hoffmann and Pfeil (2010)[21] and Li (2011)[30] examine problems similar to Sannikov and Skrzypacz (2010)[41] with persistent but public exogenous disturbances, which are explained as luck.

[3] Sung (2005)[44] and Cvitanic and Zhang (2007)[11] investigate optimal lump-sum contracts when the agent has hidden constant ability.

[4] Malenko (2011)[31] also applies i.i.d shocks to investigate optimal auditing and internal capital design. When the manger is not audited, the optimal contract is pooling, because in Malenko's model the manager's instantaneous utility is not dependent on shocks, as Malenko notes in the paper.

[5] This setting also has economic rationales in contracting problems. See Jewitt et al. (2008)[25] among many others.

to quit involuntarily if moral hazard is involved. If that value reaches the maximum, he will attain ownership.

An optimal contract can be described in terms of the agent's continuation value as a single state variable; that is, the manager's total future expected utility. Different from Sannikov (2008)[40], the continuation value process is contingent on the agent's report. Before he reports a new project, the continuation value is continuous in time. The value process jumps when a new project arrives and its shock is reported. This shock-dependent jump and payment must be carefully designed to reveal the true value of the shock. An important finding is that the difference of the utilities of current consumption upon different values of shocks must be proportional to the difference of jumps to induce the agent to report the shocks truthfully. Meanwhile, the agent's optimal choice of effort level is determined by the expected sum of his utility of current consumption and the jump, in which the expectation is taken with respect to the shocks.

We also find that the efficiency of the consumption distribution and incentive provision can be improved if the agent is expectably rich enough; that is, his continuation value is sufficiently high. This is because in our setting, the cost of compensating the agent for his participation is proportional to the cost of giving the agent value through promised utility, which is identical to the continuation value if the new project does not arrive at the moment.[6] Typically, the second-best contract is optimal under pure moral hazard, and the third-best contract, under both moral hazard and adverse selection, can achieve second-best efficiency under pure adverse selection, if the agent's continuation value is sufficiently large. We define the set of these continuation values as the "efficient domain," and the complement of the efficient domain in the feasible set is called "inefficient domain."

In our mixed model with both moral hazard and adverse selection, the growth rate of promised continuation value is negative on the inefficient domain and zero on the efficient domain. Hence on the inefficient domain, the agent's continuation value will keep moving to the value, which triggers the principal to fire the agent. However, if a new project arrives, the continuation value will jump to higher value. If the value is large, the continuation value can jump into the efficient domain when the project is accompanied by a low shock, in which case it will remain unchanged until a new project arrives. This is different from the dynamics of the second-best contract under pure moral hazard, in which the agent's value cannot move into the efficient domain if the agent's initial reservation is located on the inefficient domain.

The *drift* of the agent's continuation value is related to the allocation of payments over time. This value can have an *upward drift*, which means the agent's current payment is small relative to his expected future payoff. In this case, we say that the agent's payment is *back-loaded*. It is *front-loaded* if the agent's value has a downward drift, which means the agent's current payment is large relative to his expected future payoff. As a comparison, the drift of the continuation value is zero if the principal and the agent have symmetric information. Under pure moral hazard, the payment is front-loaded, and the drift (net of the agent's private benefit) is gradually reduced to zero when the agent becomes expectably rich (with higher continuation values), since the moral hazard

[6] Otherwise, it is called *transitional utility*.

effect decreases as the continuation value rises. When contracting under repeated adverse selection and dynamic moral hazard, the moral hazard effect becomes dominated by the adverse selection effect, and the payment can be back-loaded when the agent become expectably richer on the inefficient domain. The drift (net of the agent's private benefit) on the efficient domain is gradually reduced and becomes negative, and hence the payment is front-loaded again if the agent's continuation value is sufficiently large.

Our novel results are due to the technical advantages of continuous-time methods over discrete-time methods. Using a discrete time multi-period model, Green (1987), Thomas and Worrall (1990), and Atkeson and Lucas (1992) investigate optimal contracting with i.i.d. private shocks, and Albanesi and Sleet (2006) examine optimal taxation with i.i.d private information. Fernandes and Phelan (2000) provide a recursive formulation for repeated agency with persistent private shocks. Chassang (2013)[10] studies calibrated incentive contracts in a discrete-time agency problem that includes limited liability, moral hazard, and adverse selection. Sadzik and Stacchetti (2015)[37] analyze discrete-time principal–agent models with short period lengths and random shocks.

By contrast, **(1)**, as Sannikov (2008) notes, continuous time leads to a much simpler computational procedure for finding the optimal contract by solving an ordinary differential equation (ODE). This simplification stems from the martingale representation of the agent's expected utility over time. Thus we can obtain new and dynamic insights into incentive provision and optimal contracting. For example, it allows much easier comparison of the impacts of instantaneous payments and future payments on the agent's incentives. **(2)**, the continuous-time model highlights many features of long-term contracts, including the agent's retirement and ownership transfer between the principal and the agent. And **(3)**, the continuous-time model setting is well motivated in our paper, since we assume that the agent successfully finds the projects at random times. Hence, it is natural to apply continuous-time approaches in the context of continual project search.

Our paper is also closely related to the literature of delegated search, in which the principal delegates the search process to the agent. In Armstrong and Vickers (2010)[4], a case in which an agent must invest effort to discover a project for a principal is examined. Lewis (2012)[28] develops a theory of delegated search for the best alternative with pure moral hazard by treating the agent's search as a continuous-time process per se. Ulbricht (2016)[48] studies optimal delegated search with adverse selection and moral hazard. Compared to their studies, our paper focuses more on repeated delegation of project search. We do not specify projects with different characteristics or alternatives. The agent needs to search for projects constantly. We focus on the case in which projects may be discovered at any random time. Only the probabilities of the projects' arriving at a particular time matters. We emphasize the agency relationship more than the traditional search process; this is an important aspect which previous literature does not explicitly address. In our setting, we can better highlight the dynamic incentive for long-term delegation of project search. Hence, this paper is relatively closer to a traditional dynamic principal-agent problem.

The remainder of the paper is organized as follows. Section 2 explains the model's setting and formulates the principal's problem. Section 3 describes the instantaneous conditions for the incentive compatibility constraint. Section 4 presents the optimality equation with boundary conditions that characterizes the optimal contract problem. Section 5 characterizes the dynamics of optimal contracts, and Section 6 concludes. Proofs of the main results are given in the appendix.

# 2 The Model

Our model setup extends the discrete-time constrained efficient allocation model with one-sided private information, which is similar to that in Atkeson and Lucas (1992)[5] by including dynamic moral hazard and random arrival of new projects.[7]

Under the static setting, the agent's utility function from consumption is $\theta u(c)$, where $u$ is a strictly concave real-valued function from his consumption[8] $c \in [0, \infty)$ satisfying $u(0) = 0$, and $\theta$ is a random variable representing the preference shock, which is privately observed by the agent. We denote $v(\cdot)$ as the inverse of the marginal utility function $u'(\cdot)$ and $U(\cdot)$ as the inverse of $u(\cdot)$. For simplicity and to highlight the dynamics of optimal contracts, we focus on preference shocks with binary values: $\xi_H > \xi_L > 0$ and $P(\theta = \xi_i) = p_i$, $i = L, H$ and $\sum_{i=L,H} p_i = 1$. For convenience, we denote the expected shock value by $\bar{\xi} = p_H \xi_H + p_L \xi_L$. [9]

Our model focuses on the continuous time case. The time horizon is infinite; that is, time $t \in [0, \infty)$. The principal (firm owner) hires an agent (manager) to manage the firm. By exerting effort, the agent affects the probability with which a new project arrives: A higher effort increases the probability $\Lambda_t dt$ that a new project will arrive during $(t, t + dt]$. For simplicity, we also consider the binary effort of the agent at each period $t$, in terms of $\Lambda_t = \lambda - \Delta > 0$ representing low effort, and $\Lambda_t = \lambda$ representing high effort, with $\Delta > 0$. We say that the agent shirks (works) if the low (high) effort is chosen.

To model the cost of effort, we adopt the same convention as Holmstrom and Tirole (1997)[22]: If the agent shirks at time $t$, that is, if $\Lambda_t = \lambda - \Delta$, he obtains a *private benefit* $B > 0$; by contrast, if the agent exerts effort at time $t$, that is, if $\Lambda_t = \lambda$, he obtains no private benefit. This formulation is also similar to one in Biais, Mariotti, Rochet, and Villeneuve (2010)[8]. Thus, the agent's total utility is the sum of his utility of consumption and his private benefit. Furthermore, we assume that the principal hires the agent because he has the talent to exert a high level of effort.

From each new project indexed by $n \in \mathbb{N}$, the agent can generate output $Y \in (0, \infty)$.[10]

[7] In Atkeson and Lucas (1992)[5], the i.i.d. endowment arrives after a fixed time interval, which cannot be trivially extended to a continuous-time version by letting the time interval go to zero, because there are few stochastic processes that are i.i.d over continuous time. The increment of Brownian motion is a typical one, which Demarzo and Sannikov (2006)[12] apply to investigate the hidden saving problem. The restriction for such an application is that it requires the manager to be risk neutral. A major contribution of this paper is that we assume that the endowment arrives after a random time interval and that the shocks accompanying each endowment are i.i.d..

[8] We assume that the agent has no saving behavior.

[9] However, our model is readily extended to the case with multiple or continuous types.

[10] For simplicity, we assume the outputs for all projects are identical to be $Y$. Our analysis is readily extended, however, to the situation in which project $n$ may generate $Y_n \in (0, \infty)$.

We denote the mutually-observable arrival time of the $n_{th}$ project by $\tau_n$ and the shock of the $n_{th}$ project by $\theta_n$. Then, the agent's *(time-discounted) expected utility* is

$$rE\left\{\sum_{n=1}^{\infty} e^{-r\tau_n}\theta_n u\left(c_{\tau_n}\right) + \int_0^{\infty} e^{-rs}1_{[\Lambda_s=\lambda-\Delta]}Bds|\left\{\tau_n,\theta_n\right\}_n\right\}, \tag{1}$$

with a discount rate $r > 0$. The rate $r$ in front of the expectation normalizes the total payoffs to the same scale as the stage payoffs. $h_{t^-} = \{\tau_n, \theta_n\}_{\tau_n<t}$ is the real history of project arrivals and the values of the shocks. Let $c_{\tau_n}$ denote *the payment transferred from the principal to the agent for his consumption* at the time $\tau_n$.

We assume that the arrivals of projects are publicly observed.[11] Note that shock $\{\theta_n\}_{n=1}^{\infty}$ and the effort exerted in the project search constitute the agent's asymmetric information, which is unobservable to the principal. Hence, the principal's optimal contracting problem is subject to the repeated adverse selection that arises from hidden shocks and the dynamic moral hazard that arises from the agent's hidden search efforts. When a new project is obtained, the agent needs to decide which shock value to report to the principal. One reporting strategy for the agent is denoted by $\hat{\sigma} = \left\{\hat{\theta}_n\right\}_{n=1}^{\infty}$. Allow $\hat{h}_{t^-} = \left\{\tau_n, \hat{\theta}_n\right\}_{\tau_n<t}$ to denote the history of the arrival time of projects and the agent's reports, and denote the truth-telling strategy by $\sigma^* = \{\theta_n\}_{n=1}^{\infty}$. Let $c_n(\hat{h}_{\tau_n^-}, \hat{\theta}_n)$ (short for $c_{\tau_n}(\hat{\theta}_n)$ with a little abuse of notation) denote the payment transferred from the principal to the agent, given his report $\hat{\theta}_n$ of the values of the shocks at time $\tau_n$ and history $\hat{h}_{t^-}$. The payment stream $\{c_{\tau_n}(\hat{\theta}_n)\}_{n\geq 1}$ is contingent on the previous history and the report of the current value of the shock, which is nonnegative and upper bounded by the output of each project.

Since the principal fully commits to the contract, the revelation principle helps us restrict our attention to the truth-telling direct mechanism. Thus, the principal's problem **[P]** is to choose an optimal contract (stream) $M = \left\{\{c_{\tau_n}\}_{n\geq 1}, \{\Lambda_t\}_{t\geq 0}\right\}$[12] to maximize her (time-discounted) expected profit:

$$rE\left\{\sum_{n=1}^{\infty} e^{-r\tau_n}\left[Y - c_{\tau_n}(\theta_n)\right] | \left\{\tau_n,\theta_n\right\}_n\right\},$$

subject to the *global incentive compatibility* (GIC) condition (2) and *individual rationality (IR) condition* (3) as follows:

$$\left(\sigma^*, \{\Lambda_t\}_{t\geq 0}\right) \in \arg\max_{\hat{\sigma},\left\{\hat{\Lambda}_t\right\}_{t\geq 0}} rE\left\{\sum_{n=1}^{\infty} e^{-r\tau_n}\theta_n u\left(c_{\tau_n}(\hat{\theta}_n)\right) + \int_0^{\infty} e^{-rs}1_{[\hat{\Lambda}_s=\lambda-\Delta]}Bds\right\} \tag{2}$$

[11]If the arrivals of projects are also the private information of the manager, then the manager may hide or overreport the arrivals of projects. Following our discussion in the next section, we show that it is not difficult to derive the conditions under which the manager truthfully reports the arrivals of projects.

[12]Here $\Lambda_t$ denotes the principal's action recommendations for the agent.

$$rE\left\{\sum_{n=1}^{\infty} e^{-r\tau_n}\theta_n u\left(c_{\tau_n}(\theta_n)\right) + \int_0^{\infty} e^{-rs}1_{[\Lambda_s=\lambda-\Delta]}Bds\right\} \geq \underline{w}. \tag{3}$$

The principal has the same discount rate as the agent, and $\underline{w}$ is the agent's reservation utility at time $t = 0$. The contract stream $M$ is said to be incentive compatible if the condition (2) holds, and individual rational if the condition (3) holds.

# 3 The Agent' s Continuation Value

## 3.1 Basics

We now unify the discrete-time circumstance into a continuous-time circumstance for analytical tractability to allow for random arrival discrete time by introducing the agent's continuation value. Denote the number of projects found in time interval $[0, t]$ by $N_t$. We replace $\theta_n$ with $\theta_t$, $t \geq 0$, where $\theta_t$ represents the shock value if the project arrives at time $t$. Compared to $c_n(\hat{h}_{\tau_n^-}, \hat{\theta}_n)$ or $c_{\tau_n}(\hat{\theta}_n)$, we let $c_t(\hat{h}_{t^-}, \widehat{\theta_t})$ (short for $c_t(\widehat{\theta_t})$ with a little abuse of notation) represent the payment transferred at time $t$ if a new project accompanied by shock $\theta_t$ arrives at time $t$. Its value is contingent on $\hat{h}_{t^-}$, the history of the arrival time of projects and the agent's reports of shock values, and $\hat{\theta}_t$, the agent's current report of shock value if a new project arrives at time $t$. Then the agent's expected utility can be reformulated as

$$rE\left\{\int_0^{\infty} e^{-rs}\theta_s u\big(c_s(\widehat{\theta}_s)\big)dN_s + \int_0^{\infty} e^{-rs}1_{[\Lambda_s=\lambda-\Delta]}Bds|\left\{\tau_n, \theta_n\right\}_n\right\}.$$

As there is repeated private information, the optimal contract can be history-dependent, which is well-known following Townsend (1982)[47].[13] In our problem, the agent's continuation value $W_t$, along with the shock he reports, is the instrument by which the incentive is provided. Therefore, a general optimal contract can be equivalently written in terms of continuation value $W_t$ and reports $\left\{\hat{\theta}_t\right\}_{t\geq 0}$. Note that $\hat{\theta}_t = \theta_t$ for all $t$ when the optimal contract satisfies incentive compatibility.

**Definition 1.** *If the agent chooses the truth-telling type-reporting strategy and exerts the effort desired by the principal after time t, the agent's* ***continuation value*** $W_t$ *is the total utility (wealth) that the principal expects the agent to derive from the future at time t . Specifically,*

$$W_t = e^{rt}rE_t\left\{\int_t^{\infty} e^{-rs}\left[\theta_s u\big(c_s(\theta_s)\big)dN_s + 1_{[\Lambda_s=\lambda-\Delta]}Bds\right]|h_{t^-}, \theta_t\right\}. \tag{4}$$

Note that the value of $W_t$ is not continuous in time, but is indeed right-continuous with the left limit.[14] We define $W_{t^-} = \lim_{s\uparrow t} W_s$. When there is a new project arriving

[13] Townsend solves for the optimal contract by extending the set of state variables to include the continuation value. The continuous-time analogous method is first investigated by Sannikov (2008)[40].

[14] This is also mentioned in Zhang (2009)[53].

at time $t$, $W_t$ is the continuation value dependent on the shock type reported up to time $t$, which is also called "*transitional utility*," and $W_{t^-}$ is the continuation value before the shock value at time $t$ is reported, which is called "*promised utility*."

Thus, we have the following representation for the continuation value process.

**Lemma 1.** *Given an incentive-compatible contract* $\mathcal{M} = \left\{c_t(\theta_t), \Lambda_t\right\}_{t\geq 0}$,[15] *there exists a* $h_{t^-}$*- predictable process* $\{\mathcal{J}_t(\theta_t)\}_{t\geq 0}$, *such that at any time* $t$, *the evolution of the manager's continuation value process* $W_t$ *is given by*

$$dW_t = \underbrace{r\left[W_{t^-} - 1_{[\Lambda_t=\lambda-\Delta]}B - \Lambda_t \sum_{i=L,H} p_i \xi_i u\left(c_t\left(\xi_i\right)\right)\right]dt}_{Drift\ in\ the\ Continuation\ Value} + \underbrace{r\mathcal{J}_t(\theta_t)}_{Jump} dN_t \tag{5}$$
$$- r\left[\Lambda_t \sum_{i=L,H} p_i \mathcal{J}_t(\xi_i)\right] dt.$$

*Moreover,* $\tilde{N}_t = r\int_0^t \mathcal{J}_s(\theta_s)dN_s - r\int_0^t \left\{\Lambda_s \sum_{i=L,H} p_i \mathcal{J}_s(\xi_i)\right\} ds$ *is a martingale.*

We call $r\left[W_{t^-} - 1_{[\Lambda_t=\lambda-\Delta]}B - \Lambda_t \sum_{i=L,H} p_i \xi_i u\left(c_t\left(\xi_i\right)\right)\right]$ *the drift of the continuation value* $W_t$. It is related to the allocation of payments over time. We call $r\mathcal{J}_t(\theta_t)$ *the jump of the continuation value* $W_t$. Lemma 1 implies that the drift of the continuation value is related to the allocation of payments over time, which is *not* contingent on the shock.

**Definition 2.** *Payments are* ***back-loaded (front-loaded)*** *if the agent's continuation value* $W_t$ *has an upward (downward) drift;* that is, the drift net of the agent's private benefit $W_{t^-} - \Lambda_t \sum_{i=L,H} p_i \xi_i u\left(c_t\left(\xi_i\right)\right)$ is positive (negative).

The agent's *expected instantaneous utility* at time $t$ is $\Lambda_t \sum_{i=L,H} p_i \xi_i u\left(c_t\left(\xi_i\right)\right)$ for the period $[t, t+dt]$, and $W_{t^-}$ is the promised utility during $[t, \infty)$. If the drift is negative (positive), the instantaneous expected utility is more (less) than that for the resting period of the contract. This means that current payment is more (less) than the average payment in the future.

Note that from Equation (5) the agent's continuation value can be rewritten as

$$dW_t = \underbrace{r\left[W_{t^-} - 1_{[\Lambda_t=\lambda-\Delta]}B - \Lambda_t \sum_{i=L,H} p_i\left(\xi_i u\left(c_t\left(\xi_i\right)\right) + \mathcal{J}_t(\xi_i)\right)\right]}_{Growth\ rate\ of\ promised\ utility} dt + r\mathcal{J}_t(\theta_t)dN_t$$

and we know that $W_t = W_{t^-} + r\mathcal{J}_t(\theta_t)$ is the transitional utility if a new project arrives at time $t$. We call $r\left[W_{t^-} - 1_{[\Lambda_t=\lambda-\Delta]}B - \Lambda_t \sum_{i=L,H} p_i\left(\xi_i u\left(c_t\left(\xi_i\right)\right) + \mathcal{J}_t(\xi_i)\right)\right]$ *the growth rate of promised utility* $W_{t^-}$. Clearly, the growth rate is equal to the drift of continuation value only when $\sum_{i=L,H} p_i J_t(\xi_i) = 0$. The growth rate is another important

[15] Under $\mathcal{M}$, the agent will truthfully report the shock value and exert the desired effort over time.

characterization of the pattern of incentive provision in addition to the drift of continuation value. It also characterizes the dynamics of the continuation value when there is no new project.

**Definition 3.** *An optimal contract is* ***weakly stationary*** *at some real value $w$ if the growth rate of the agent's continuation value is zero when $W_t = w$. In addition, if the jump is zero, then the optimal contract is said to be* ***stationary*** *at $w$.*

The agent may lie about the shocks and may also exert effort that is not desirable for the principal. Contingent upon his reports $\{\hat{h}_{t^-}, \widehat{\theta}_t\}_{t\geq 0}$, the continuation value process takes the following form:

$$dW_t = r\left[W_{t^-} - 1_{[\Lambda_t=\lambda-\Delta]}B - \Lambda_t \sum_{i=L,H} p_i\xi_i u\left(c_t\left(\xi_i\right)\right)\right]dt + r\mathcal{J}_t(\hat{\theta}_t)dN_t \qquad (6)$$
$$-r\left[\Lambda_t \sum_{i=L,H} p_i\mathcal{J}_t(\xi_i)\right]dt.$$

It is worth noting that $\left\{W_t, \hat{\theta}_t\right\}_{t\geq 0}$ is fully determined by the sequence $\left\{c_t\left(\hat{\theta}_t\right), \mathcal{J}_t\left(\hat{\theta}_t\right), \Lambda_t\right\}_{t\geq 0}$ , where $\{\Lambda_t\}_{t\geq\infty}$ in representation (6) is the probabilities of the arrival of new projects desired by the principal within the $\left\{\hat{\Lambda}_t\right\}_{t\geq\infty}$ pre-offered by the agent.

**Remark 1.** $\left\{c_t(\hat{\theta}_t), \mathcal{J}_t(\hat{\theta}_t)\right\}_{t\geq 0}$ *may be dependent on the entire history of the agent's reports. With our setup, however, the entire history can be replaced by the continuation value.*

## 3.2 Reformulation of the Global Incentive Compatibility Condition

It will be useful to derive the conditions in terms of $\left\{c_t\left(\hat{\theta}_t\right), \mathcal{J}_t\left(\hat{\theta}_t\right), \Lambda_t\right\}_{t\geq 0}$ equivalent to the GIC condition (2). First, we explore the desired conditions in the pure adverse selection case; that is, the agent's effort of project search can be observed and dictated. Given any time $u$, let $u^-$ denote the left limit of integral as $t$ approaches $u$ from the left. The *accumulated gain over* $[0, u)$ is defined by

$$e^{-ru}\hat{G}_{u^-} = r\int_0^{u^-} e^{-rs}\theta_s u\big(c_s(\widehat{\theta}_s)\big)dN_s + e^{-ru}W_{u^-},$$

where $W_t$ follows the dynamics of (6) for $t \in [0, u)$, and $W_{u^-}$ denotes the left limit of $W_t$ as $t$ approaches $u$ from the left. Thus, if the agent truthfully reports the shocks on $[u, \infty)$, his expected total gain by time $u$ is $e^{-ru}\hat{G}_{u^-}$, regardless of whether he reports the true shocks before time $u$. In other words, his historical performance has no impact on his expected future gain as long as $\hat{G}_{u^-}$ is fixed. Hence, the agent's incentive on $[u, \infty)$ is not affected by his previous report.

Furthermore, assume that the agent gets a new project at time $u$, which is accompanied by the real shock $\theta_u = \theta^*$. Define $\tau_u^*$ as the time of the next project's arrival. Suppose the agent reports the shock value as $\theta'$ at time $u$ and truthfully reports the shocks on $[\tau_u^*, \infty)$. If $\theta' = \theta^*$, then the *expected gain at time $u$* is

$$e^{-ru}\hat{G}_{u^-} + re^{-ru}\left(\theta^* u(c_u(\theta^*)) + \mathcal{J}_u(\theta^*)\right).$$

If $\theta' \neq \theta^*$, the *private benefit expected at time $u$* is

$$e^{-ru}\hat{G}_{u^-} + re^{-ru}\left(\theta^* u(c_u(\theta')) + \mathcal{J}_u(\theta')\right)$$

Hence, to prevent mis-reporting, we must have $\theta^* u(c_u(\theta')) + \mathcal{J}_u(\theta') \leq \theta^* u(c_u(\theta^*)) + \mathcal{J}_u(\theta^*)$, that is, for all $t \geq 0$

$$\xi_L\left[u(c_t(\xi_H)) - u(c_t(\xi_L))\right] \leq \mathcal{J}_t(\xi_L) - \mathcal{J}_t(\xi_H), \quad (7)$$

$$\xi_H\left[u(c_t(\xi_H)) - u(c_t(\xi_L))\right] \geq \mathcal{J}_t(\xi_L) - \mathcal{J}_t(\xi_H).^{16} \quad (8)$$

In a continuous-time setting, the global condition (2) needed to prevent repeated adverse selection is reduced to the instantaneous conditions (7) and (8), which is consistent with the recursive formulation of the discrete-time multi-period model in which the continuation value $W_t$ replaces $\mathcal{J}_t$ as the instrument for incentive provision.[17] (7) and (8) imply that $c_t(\xi_H) \geq c_t(\xi_L)$ and $\mathcal{J}_t(\xi_L) \geq \mathcal{J}_t(\xi_H)$; that is, the principal must pay higher payment and promise lower transitional utility if the new project arrives accompanied by a high value of preference shock.

Second, we explore the desired conditions in the pure moral hazard case. Since Sannikov (2008)[40], it has been a standard procedure to find the instantaneous condition for the continuous-time modeling of pure moral hazard. Details can be found in Sannikov (2008)[40] and Biais et al. (2010)[8]. Assume the agent truthfully reports the value of shocks, and the agent' s problem is then reduced to a pure moral hazard problem. Then, we necessarily have a constraint $\Lambda_t = \lambda$, which is equivalent to

$$\Delta \sum_{i=L,H} p_i\left[\xi_i u(c_t(\xi_i)) + \mathcal{J}_t(\xi_i)\right] \geq B, \quad (9)$$

where the left-hand side denotes the *additional expected gain of the agent if he works*, and $B$ is the agent's *private benefit if he shirks*. Hence, given that the inequality in (9) holds for almost all $t$, the agent will work through time. Different from Sannikov (2008)[40], there is no jump, and the instantaneous payment $c_t(\theta)$ in our setting provide the agent with the incentive to work directly.

In sum, (7) and (8) are the conditions for pure adverse selection, and (9) is the condition for pure moral hazard. In general, if adverse selection is mixed with moral hazard, then the incentive conditions should be stronger than the combination of (7), (8) and (9). However, in our setting, we can show that these conditions are also sufficient for (2). We conclude the above discussion of our first main result as follows.

[17]For further details, see Bolton and Dewatripont (2005).

**Proposition 1.** *The global incentive compatibility condition (2) holds if and only if (7), (8), and (9) are satisfied.*

We make the following assumption, which implies that the agent will work–i.e., $\Lambda_{t^-} = \Lambda_t$–if he owns the firm. There is a certain additional value of expected gain if he keeps working for the rest of his lifetime. Intuitively speaking, $B$ cannot be too large.

**Assumption 1.** $\Delta\bar{\xi}u(Y) > B$.

$W_{t^-}$ is still able to summarize the past history completely with i.i.d private shocks in our setting. The intuition is straightforward: Proposition 1 states that the agent's incentives remain unchanged if we replace the contract in terms of a continuation value $W_t$ with a different contract with the same continuation value. This implies that one can instead consider $(c_t(\theta_t), J_t(\theta_t))_{t\geq 0}$ as the control variables in the principal's (contracting) problem previously presented in **[P]** without loss of generality.

The main reason that the equivalence underlying Proposition 1 holds is that in our setup the private shock is assumed to be *i.i.d.* If the shock is persistent over time–for example, the agent's chances of having a high shock for the new project is large given that his previous shock value is high[18]–then this argument will fail. $W_{t^-}$ must be contingent on the value of the previous shock, which we denote by $W^{\theta}_{t^-}$. Moreover, the transitional utility process $W^{\theta}_{t^-} + r\mathcal{J}_t(\theta')$ is required to provide an incentive, where $\theta'$ is the shock value of the next project. Alternatively, if there are $N$ kinds of values for the shock, then the principal needs to design a contract based on $N$ state processes, which renders the principal's problem extremely difficult. The i.i.d. private shock model is a simplification of reality, but it is a good benchmark for investigating the repeated adverse selection problem and its application in a variety of fields. The continuous-time model we propose in this paper provides a tractable framework for this stream of research.

# 4 Derivation of Optimal Contracts

In this section, based on Proposition 1, we further derive optimal contracts that induce the agent to work throughout the contracting period. This is in line with most of the literature on the principal-agent model, which offers more precise insights into how the repeated adverse selection affects the agent's incentive under moral hazard. Then it is obvious from (4) that the agent's continuation value process can take value in $[0, w^H]$, where $w^H = \lambda \sum_{i=L,H} p_i \xi_i u(Y)$, which denotes the feasible set for the agent's continuation value. The principal can take over management and exert low effort if he fails to incentivize the agent to work due to Assumption **??**.

We next derive the optimality equations for characterizing value functions under four model settings: symmetric information (first best), pure adverse selection (second best), pure moral hazard (second best), and a mixed model with both moral hazard and adverse selection (third best). In the first-best setting, efforts and shock values are both observable by the principal. The principal can force the agent to work and pay the agent based on her

[18] See Zhang (2009)[53] for details.

observation of shock values. In the pure adverse selection setting, only efforts over time are observable–and therefore enforceable–by the principal. In the pure adverse selection setting, only shock values are observable. In the mixed setting, neither efforts nor shock values are observable.

We denote the principal's *(optimal) value functions* under the first-best setting, pure adverse selection, pure moral hazard, and mixed model by $F^1(W), F^{2a}(W), F^{2m}(W), F^3(W)$.

First, we identify boundary conditions for value functions. Figure 2 illustrates the different value functions in different settings. When there exists no moral hazard (i.e., the

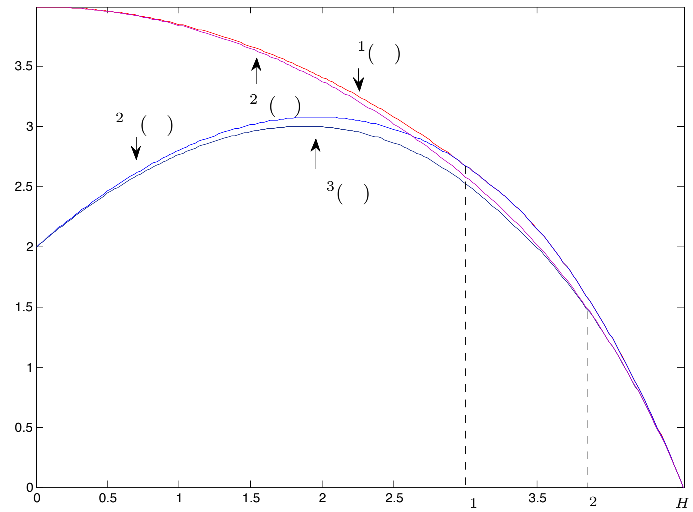


Figure 1: Value functions

first-best situation or pure adverse selection), the principal can force the agent to work, and the agent's continuation value reaches 0. Hence, the agent keeps working with no payment because of the commitment to the long-term contract. Under moral hazard or in the mixed model, however, the principal has to take over management, and the agent gets 0 payment (equivalently, is fired) thereafter, because the agent is too expectably poor to motivate. When the agent's continuation value reaches the largest possible value $w^H$, the principal must transfer ownership of the firm to the agent because of the commitment to the long-term relationship and the agent is self-incentivized. We conclude boundary conditions for $F^1(W), F^{2a}(W), F^{2m}(W), F^3(W)$ below.

From the agent's expected utility, we know that the agent's continuation value must take values in $[0, w^H]$. When $W_\tau = 0$, it is clear that the agent's payment $c_t(\theta_t) = \mathcal{J}_t(\theta_t) = 0$ for $t \geq \tau$. For the principal, we have $F^1(0) = \lambda Y$ under the first best setting or pure adverse selection. Therefore, we have two corresponding boundary conditions:

$$(BC_1)\ F^1(0) = \lambda Y, F^1(w^H) = 0;$$
$$(BC_{2a})\ F^{2a}(0) = \lambda Y, F^{2a}(w^H) = 0;$$

Under pure moral hazard, the principal cannot force the agent to work, and hence the agent is fired and the principal takes over management. Hence $F^{2m}(0) = (\lambda - \Delta)Y$ because of Assumption **??**. Therefore, we have one corresponding boundary condition

$$(BC_{2m})\ F^{2m}(0) = (\lambda - \Delta)Y, F^{2m}(w^H) = 0;$$

Similarly, $F^3(0) = (\lambda - \Delta)Y$. For the principal, we have $F^1(w^H) = F^{2a}(w^H) = 0$. By part (a) of Assumption 1, the agent will work under moral hazard at $t \geq \tau$, hence $F^{2m}(w^H) = F^3(w^H) = 0$. Thus, we have one corresponding boundary condition

$$(BC_3)\ F^3(0) = (\lambda - \Delta)Y, F^3(w^H) = 0.$$

Because the principal discounts the future at rate $r$, her *expected flow of value* at time $t$ is given by

$$rF^i\left(W_{t^-}\right), \tag{10}$$

where $i = 1, 2a, 2m, 3$. This should be equal to the sum of the expected instantaneous cash flow and the expected rate of change in her continuation value, since $F^i$ is the principal's value function for case $i$. The *expected instantaneous cash flow* is

$$r\lambda\left[Y - \sum_{j=L,H} p_j c_t\left(\xi_j\right)\right]. \tag{11}$$

This represents the expected output from the project less the agent's compensation. To evaluate the *expected rate of change in the principal's continuation value*, we employ the dynamics (6) of the agent's continuation value and apply Ito's formula for the jump processes that yield this expected rate of change:

$$\begin{aligned} & rF_W^i\left(W_{t^-}\right)\left(W_{t^-}dt - \lambda\sum_{j=L,H} p_j\xi_j u\left(c_t\left(\xi_j\right)\right)\right) \\ & - rF_W^i\left(W_{t^-}\right)\left(\lambda\sum_{j=L,H} p_j\mathcal{J}_t(\xi_j)\right) + \lambda\sum_{j=L,H} p_j\left(F^i\left(W_{t^-} + r\mathcal{J}_t(\xi_j)\right) - F^i\left(W_{t^-}\right)\right). \end{aligned} \tag{12}$$

The first and second terms arise because of the drift in $W_{t^-}$ and the compensated process of the jump component. The third term reflects the possibility of jumps in the agent's continuation value due to the arrival of new projects.

Thus, adding (11) and (12) and equating them with (10), we find that the principal's optimal value functions $F^1(W), F^{2a}(W), F^{2m}(W), F^3(W)$ together with choice variables $\mathcal{C} = (c_j, \mathcal{J}_j)_{j=L,H}$ satisfy the Hamilton-Jacobi-Bellman **(HJB)** optimality equation:

$$\begin{aligned} rF^i\left(W\right) = \max_{\mathcal{C}\in\Upsilon^i(W)} r\lambda\left[Y - \sum_{j=L,H} p_j c_j\right] + rF_W^i\left(W\right)\left(W - \lambda\sum_{j=L,H} p_j\xi_j u\left(c_j\right)\right) \\ + \lambda\left(\sum_{j=L,H} p_j\left(F^i\left(W + r\mathcal{J}_j\right) - rF_W^i(W)\mathcal{J}_j\right) - F^i\left(W\right)\right), \end{aligned} \tag{13}$$

under boundary conditions $(BC_i)$, $i = 1, 2a, 2m, 3$, where we denote $c_j = c_t(\xi_j), J_j = J_t(\xi_j)$ to simplify the notation. We will use $(c_j, \mathcal{J}_j)_{j=L,H}$ and $(c_t(\theta_t), \mathcal{J}_t(\theta_t))$ interchangeably. The choice variables $\mathcal{C} = (c_j, \mathcal{J}_j)_{j=L,H}$ take values in *corresponding incentive compatible set* $\Upsilon^i(W)$ under our four model settings, where $i = 1, 2a, 2m, 3$. This equivalently reformulates the optimal contracting problem in four different model settings.

We now derive the optimal contract by concluding the properties of each value function $F^i(W)$ and identifying its corresponding incentive compatible set $\Upsilon^i(W)$ below.

**Proposition 2.** $F^1(W), F^{2a}(W)$ *are strictly concave and monotonically decreasing,*

$$\Upsilon^1(W) = \left\{(c_i, \mathcal{J}_i)_{i=L,H} | (c_i, W + r\mathcal{J}_i) \in [0, Y] \times [0, w^H]\right\},$$

$$\Upsilon^{2a}(W) = \left\{(c_i, \mathcal{J}_i)_{i=L,H} | (c_i, W + r\mathcal{J}_i) \in [0, Y] \times [0, w^H], \mathcal{J}_L = \mathcal{J}_H + \xi_L [u(c_H) - u(c_L)], \ c_H \geq c_L\right\}.$$

*If* $F^{2m}(W), F^3(W)$ *are strictly concave,*[19] *then*

$$\Upsilon^{2m}(W) = \left\{(c_i, \mathcal{J}_i)_{i=L,H} | (c_i, W + r\mathcal{J}_i) \in [0, Y] \times [0, w^H], \Delta \sum_{j=L,H} p_j [\xi_j u(c_j) + \mathcal{J}_j] \geq B\right\},$$

$$\Upsilon^3(W) = \Upsilon^{2a}(W) \cap \Upsilon^{2m}(W).$$

According to Proposition 2, intuitively, for pure adverse selection, the principal's value function is concave; also, it is more costly if the (8) is binding, which introduces a higher variance for transitional utility. Moreover, it is surprising to see that (7) is still binding in the third-best contract even if moral hazard is also involved. Basically, the incentive condition (9) for moral hazard only adds the constraint on the mean of transitional utility. Hence, being binding of (7) minimizes the risk introduced by the transitional utility.

Let us see how Equation (13) and Proposition 2 imply the constraints that $C$ should satisfy for optimality in the four different model settings. In the first-best setting, we directly have that the optimal choice of consumption $c_j$ maximizes

$$-c_j - F^i_W(W)\xi_j u(c_j), \ j = L, H, \tag{14}$$

and separately optimal transitional utility $\mathcal{J}_j$ maximizes

$$F^i(W + r\mathcal{J}_j) - rF^i_W(W)\mathcal{J}_j, \tag{15}$$

where $F^i_W(W)$ denotes the first-order derivative (margin) of value function $F^i(W)$. The agent's consumption and the transitional utility increase in $W$ in view of $F^i_W(W)$. $\frac{1}{\xi_j u'(c_j)}$ and $-F^i_W(W)$ are the marginal cost of giving the agent continuation value through current consumption and through his promised utility, respectively. $F^i(W) - F^i(W + r\mathcal{J}_j)$ denotes the cost of giving value through transitional utility, and $-F^i_W(W + r\mathcal{J}_j)$ denotes the marginal cost to deliver the value to the agent through transitional utility. Those marginal

[19]It is beyond the range of this article to prove the concavity of value functions with moral hazard, which can be accomplished by showing that the equilibrium payoff set of the principal and the agent must be convex.

costs must be equal for any value of preference shock in the first-best contract if interior solutions are optimal.

However, under asymmetric information, all of these marginal costs must be distorted to provide the incentive. By proposition 2, *the* pure adverse selection model is subject to

$$\mathcal{J}_L = \mathcal{J}_H + \xi_L(u(c_H) - u(c_L)), \tag{16}$$

and the pure moral hazard model is subject to

$$\Delta \sum_{j=L,H} p_j \left[\xi_j u(c_j) + \mathcal{J}_j\right] \geq B, \tag{17}$$

and the third-best contract is subject to (16) and (17). If (17) is binding, for every possible value of preference shock the marginal costs through current consumption and through transitional utility should be larger than the marginal cost through promised utility. Hence consumption and transitional utility $(c_j, W + rJ_j)_{j=L,H}$ should be higher than those in the first-best setting with the same marginal cost through promised utility.

Moreover, $r\lambda Y$ is the expected flow of output if the agent is incentivized to work.

$$-r\lambda F^i_W(W) \sum_{j=L,H} p_j \left(\xi_j u\left(c_j\right) + \mathcal{J}_j\right)$$

is not only proportional to the cost of giving the agent value through promised utility, but is also the cost of compensating the agent for his effort because of $\lambda$ in this term. This implies that the efficiency of allocation and incentive provision will be improved if moral hazard is involved and the agent's continuation value is high (Figure 1).

Golosov, Kocherlakota, and Tsyvinski (2003)[16] adopt a finite-period discrete-time model with a general structure of private information and show that the reciprocal of marginal utility satisfies the reciprocal Euler equation (REE), which implies a long-run convergence of the agent's payment. First, by martingale convergence theorem, we know the payment converges along almost every sample path. Second, the marginal utility is a submartingale that implies that the payment is always front-loaded in the second best setting (Albanesi and Armenter (2012)[1]). With a continuous-time pure moral hazard model, Sannikov (2008)[40] shows that the margin of the principal's value is a martingale, to which the reciprocal of marginal utility equals if it is negative. However in Sannikov (2008)[40], the margin of the principal's value is positive when the agent's value is low, and therefore the agent is still incentivized even if the consumption is back-loaded.

In our setting, as we discuss in Section 5, under moral hazard, the marginal cost through current consumption is aligned with the marginal cost through transitional utility, and is larger than $-F^{2m}_W(W)$, even when $-F^{2m}_W(W) < 0$; hence the consumption is front-loaded. On the other hand, under pure adverse selection, the payment is back-loaded when the agent's value is low, even if the principal's value function is monotonically decreasing. This is because the adverse selection effect becomes stronger as expectable wealth increases. When adverse selection is mixed with moral hazard, the payment must be high to provide for the agent the incentive to work, and hence the payment is front-loaded even if the agent's value is low. However, as the agent gets expectably richer, the incentive condition (17) is not binding, and since there is no constraint on the expected current consumption, the monotone change of consumption slows and becomes backloaded. More detailed discussion can be found in Section 5.

In our setting, the margin of value function $F_W^i(W_t)$ is a martingale on the "continuation domain" which is defined as follows.

**Definition 4.** *Continuation domain ($\mathcal{CD}$) is a subset of $(w^L, w^H)$ such that at optimality, $0 < w + r\mathcal{J}_i < w^H$, for $i = L, H$.*

This implies that given $W_{t^-} \in \mathcal{CD}$, the contract will not be terminated at time $t$ even if a new project arrives at time $t$, because $W_{t^-} + r\mathcal{J}_i \in (w^L, w^H)$. Inside of $\mathcal{CD}$, the contract space $\Upsilon^i(W)_{i=1,2a,2m,3}$ is not contingent on $W$, and therefore we can apply envelope theorem and have

$$0 = rF_{WW}^i(W)\left(W - \lambda \sum_{j=L,H} p_j\left(\xi_j u\left(c_j\right) + \mathcal{J}_i\right)\right) + \lambda\left(\sum_{j=L,H} p_j F_W^i\left(W + r\mathcal{J}_j\right) - F_W^i(W)\right). \tag{18}$$

Applying Ito's formula, we have

$$\begin{aligned} dF_W^i(W_t) &= rF_{WW}^i(W_{t^-})\left(W_{t^-} - \lambda \sum_{j=L,H} p_j\left(\xi_j u\left(c_t(\xi_j)\right) + \mathcal{J}_t(\xi_j)\right)\right) dt \\ &\quad + \left[F_W^i\left(W_{t^-} + r\mathcal{J}_t(\theta_t)\right) - F_W^i(W_{t^-})\right] dN_t. \end{aligned}$$

By (18), we can see that the margin of the principal's value function is a martingale before $W_t$ moves outside the $CD$:

$$\begin{aligned} dF_W^i(W_t) &= \left[F_W^i\left(W_{t^-} + \mathcal{J}_t(\theta_t)\right) - F_W^i(W_{t^-})\right] dN_t \\ &\quad -\lambda\left(\sum_{j=L,H} p_j F_W^i\left(W_{t^-} + r\mathcal{J}_t(\xi_j)\right) - F_W^i(W_{t^-})\right) dt, \\ \text{where } W_{t^-} &\in \mathcal{CD}. \end{aligned}$$

In Section 5, we will prove that $(0, w^H)$ is a continuation domain under all different settings of asymmetric information. Hence $F_W^i(W)$ is a martingale before the continuation value reaches 0 or $w^H$. Transitional utility will increase when the agent becomes expectably richer; that is, his continuation value becomes larger. However, if the agent's value is less than $w^H$, the principal can always find better payment and transitional utility to avoid transferring ownership to the agent, because the principal's value achieves minimum at $w^H$. The continuation value can reach 0 if moral hazard is involved.

In Section 5, we will show that the agent's value is weakly stationary on $(0, w^H)$ under pure adverse selection. Then we have

$$\sum_{j=L,H} p_j F_w^{2a}\left(W_{t^-} + r\mathcal{J}_t(\xi_j)\right) = F_W^{2a}(W_{t^-}).$$

In the pure moral hazard model, the jump in continuation value is not contingent on the shock value, and $\mathcal{J}_L = \mathcal{J}_H$. The marginal cost of delivering the value through transitional utility is higher than the marginal cost of delivering the value through promised utility, and

$$-\sum_{j=L,H} p_j F_W^{2m}(W + r\mathcal{J}_j) > -F_W^{2m}(W) \tag{19}$$

because of (17).

In the mixed model, although the drift and growth rate are negative when the agent is expectably poor, the agent is likely to be expectably richer if she finds a project accompanied by a low preference shock, because the principal randomizes the jump of continuation value in order to provide incentive for truth-telling. Hence apart from the pure moral hazard model, significantly, the agent can be expectably richer. If the agent's continuation value is sufficiently high, his payment can be either back-loaded or front-loaded.

# 5 Characterization of Optimal Contracts

In this section, we characterize optimal contracts in details, and describe the dynamics of long-term contracts.

## 5.1 First-best Benchmark

In the first-best setting, the principal can force the agent to work and pay the agent based on her observation of shock values. From optimality equation (13), we know that the jump should be zero, i.e., $\mathcal{J}_L = \mathcal{J}_H = 0$. From (18), $W = \lambda \sum_{j=L,H} p_j \xi_j u(c_j)$. By combining these, we have the following results.

**Proposition 3.** *In the first-best setting, the agent's continuation value will remain unchanged: $W_t = \underline{w}$ for all $t \geq 0$, with constant payment $c_H^*$ and $c_L^*$ given by*

$$\lambda \sum_{i=L,H} p_i \xi_i u(c_i^*) = \underline{w}, \text{ and}$$

$$\xi_H u'(c_H^*) = \xi_L u'(c_L^*), \ for \ \underline{w} \leq w_c^1,$$

$$c_H^* = Y, \ for \ \underline{w} > w_c^1,$$

*where $w_c^1 = \lambda[p_L \xi_L u(v(\frac{\xi_H}{\xi_L} u'(Y))) + p_H \xi_H u(Y)]$. The principal's value $F^1(\underline{w}) = \lambda(Y - \sum_{i=L,H} p_i c_i^*)$.*

Hence, continuation value $W_t$ has zero growth rate. Moreover, we know that $\mathcal{J}_i = 0$. Therefore, in the first-best setting the optimal sharing rule is stationary throughout time by Definition 3: Given the initial promised utility $\underline{w} \in [0, w^H]$, the agent's and the principal's respective continuation values will remain as $\underline{w}$ and $F^1(\underline{w})$ for ever, regardless of whether a new project arrives.

By the first-order condition with respect to $c_j$ on (14), $\frac{1}{\xi_j u'(c_j)}$ must be equal to $-F^1_W(W)$ for $W < w^1_c$. This implies that the instantaneous payment has to be higher if the agent is expectably richer, since both $c_H$ and $c_L$ are increasing as $W$ increases. Without moral hazard or adverse selection, the payment is neither front-loaded nor back-loaded, since the drift is always equal to 0.

We measure the efficiency of the contract by defining the *insurance wedge* and *transition wedge* discussed in Albanesi and Sleet (2006). The insurance wedge is computed by $\frac{\xi_H u_c(c_H)}{\xi_L u_c(c_L)} - 1$, which measures the consumption smoothing that is implied by the optimal contract. The transition wedge $E\left[\frac{\textit{transition utility}}{\textit{promised utility}}\right] - 1 = \sum_{i=L.H} p_i \frac{r\mathcal{J}_i}{W}$, measures the intertemporal smoothness of consumption. It is clear that in the first-best setting, both wedges are equal to zero. The deviation of the wedges from zero, represents the strength of the distortion of allocation under asymmetric information.

## 5.2 Pure Adverse Selection Model

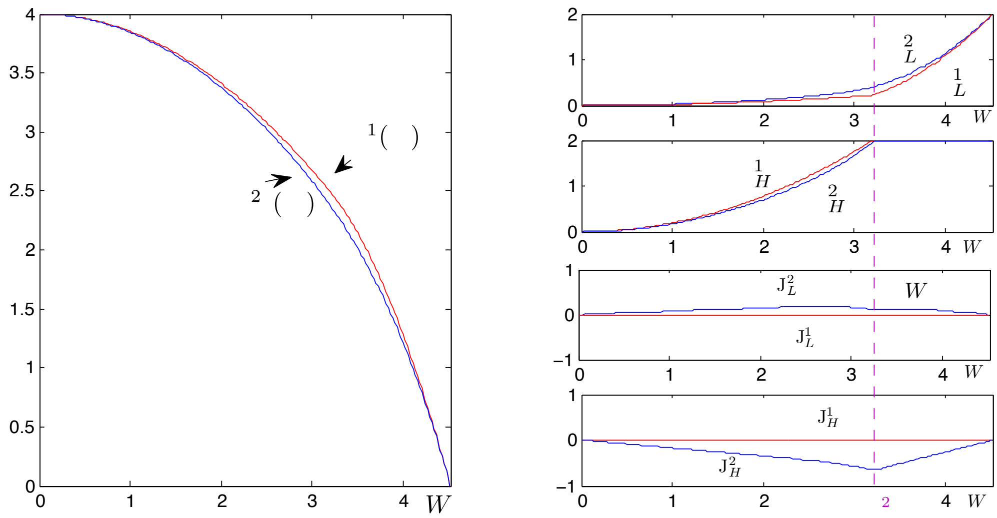


Figure 2: Value function and optimal contract for the pure averse selection model. $u(c) = \sqrt{c}$, $\xi_L = 1, \xi_H = 3$, $P(\xi_H) = 0.3$, $r = 1.8$, $\Delta = 1.0$, $B = 1.5$, $\lambda = 2$. $(c^i_j, \mathcal{J}^i_j)^{i=1,2a}_{j=L,H}$ are the optimal payment and the jump of continuation value process in the first-best setting and under pure adverse selection. $w^{2a}_c$ is the smallest continuation value at which $c^{2a}_H$ reaches the upper Bound $Y$.

Now we consider the pure adverse selection model. Figure 2 illustrates the comparison between the first-best and second-best contracts under pure adverse selection. Naturally, $F^1(w)$ dominates $F^{2a}(w)$; that is, given any $w$, $F^1(w) > F^{2a}(w)$ except coincidence at two ends of $[0, w^H]$. The insurance wedge (Figure 3) is bigger than 0.

In general analysis, we plug (16) into optimality equation (13), and have the following properties for the optimal contract.

**Proposition 4.** *In the pure adverse selection setting,* $(0, w^H)$ *is the continuation domain on which* $0 < c_L < Y$*,* $\mathcal{J}_L > 0 > \mathcal{J}_H$ *and transitional utility* $0 < W + r\mathcal{J}_i < w^H$*. On* $[0, w^H]$*, the dynamics of the agent's and the principal's continuation values are characterized by*

$$dW_t = r\mathcal{J}_t(\theta_t)dN_t,\ dF_W^{2a}(W_t) = \left[F_W^{2a}(W_{t^-} + \mathcal{J}_t(\theta_t)) - F_W^{2a}(W_{t^-})\right]dN_t. \tag{20}$$

*Moreover, there exists* $w_c^{2a} \in (0, w^H)$ *such that with the optimal contract, on* $[0, w_c^{2a}]$*,* $c_L, c_H, w{+}r\mathcal{J}_L, w{+}r\mathcal{J}_H$*, and* $\mathcal{J}_L{-}\mathcal{J}_H$ *all increase with* $w$*, and on* $(w_c^{2a}, w^H]$*,* $c_L, w{+}r\mathcal{J}_L$*, and* $w + r\mathcal{J}_H$ *all increase with* $w$*, but* $c_H = Y$*, and* $\mathcal{J}_L - \mathcal{J}_H$ *decreases with* $w$*.*

From Proposition 4, because $(0, w^H)$ is the continuation domain, on which both the transitional utility and $c_L$ do not bind with their bounds, we can apply first-order conditions (on (14) with respect to $c_L$, $\mathcal{J}_L$, and $\mathcal{J}_H$) and have

$$\frac{1}{\xi_H u'(c_H)} = -(1 - \frac{\xi_L}{\xi_H})F_W^{2a}(w) - \frac{\xi_L}{\xi_H}F_W^{2a}(w + r\mathcal{J}_H),\ for\ w \leq w_c^{2a}, \tag{21}$$

$$\frac{1}{\xi_L u'(c_L)} = -F_W^{2a}(w + r\mathcal{J}_L), \tag{22}$$

$$-F_W^{2a}(w) = -\sum_{j=L,H} p_j F_W^{2a}(w + r\mathcal{J}_j). \tag{23}$$

$\frac{1}{\xi_j u'(c_j)}$ is the marginal costs of giving the agent value through current consumption if the shock value is $\xi_j$. Different from the first-best setting, $\frac{1}{\xi_H u'(c_H)} < -F_W^{2a}(W) < \frac{1}{\xi_L u'(c_L)}$. The insurance wedge $\frac{\xi_H u'(c_H)}{\xi_L u'(c_L)} - 1 > 0$, although $c_H > c_L$ in the continuation domain.

If the agent's value is increasing and less than $w_c^{2a}$, there is more leeway for punishment for lying, since $J_L - J_H$ increases; if the agent's value is more than $w_c^{2a}$, the punishment decreases to avoid transferring the ownership. Moreover, we can observe a distortion from the first-best benchmark caused by *adverse selection effect*: The payment is higher (lower) when the shock value is low (high) than that in the first-best contract, given the same expected current consumption level. This is due to the pure adverse selection constraint (16). Compared with the first-best setting, the difference between $c_H$ and $c_L$ is additionally constrained by (16) (in fact, we will show $J_L > 0 > J_H$), and such a difference is smaller than that in the first best setting. More specifically, $c_L^{2a} > c_L^1$ and $c_H^{2a} < c_H^1$.[20] This implies that the private shocks prevent the separation in information.

Accordingly, in the pure adverse selection model, the drift (net of the agent's private benefit) is equal to $r\left[\Lambda_t \sum_{i=L,H} p_i \mathcal{J}_t(\xi_i)\right]$. It is negative when $\omega$ is low, and turns positive when $\omega$ is high, since $J_L - J_H$ first increases and then decreases. It may even become negative again when $\omega$ is sufficiently high. This fact renders the payments first front-loaded and then back-loaded (and even front-loaded again eventually). In addition, the drift is increasing in absolute value when the agent is expectably poor (on $[0, w_c^{2a}]$), whereas the

[20] This is also illustrated in Figure 3.

drift is decreasing in absolute values when he becomes expectably rich (after $w_c^{2a}$). When the agent is sufficiently expectably rich, the drift may even become positive and negative again later.

Different from Golosov, Kocherlakota, and Tsyvinski (2003), in which consumption is front-loaded, in pure adverse selection, as shown in Figure 3, the payment can be back-loaded when the agent is expectably poor ($w < w_d^{2a}$) and front-loaded when the agent gets expectably richer ($w > w_d^{2a}$). Note that $\sum_{j=L,H} p_j \xi_j u(c_j) = \bar{\xi} u(c_L) + p_H \xi_H (u(c_H) - u(c_L))$. Thus, in optimum, $(c_H, c_L)$ must be distorted, and $u(c_H) - u(c_L)$ is smaller than that in the first-best setting because of the adverse selection effect. Meanwhile, $u(c_L)$ has to be larger than the first-best payment for low shock. When the agent's continuation value is small, the adverse selection effect is strong and expected instantaneous utility is smaller than that in the first-best setting. When the agent's value is more than $w_d^{2a}$, the payment is front-loaded because the adverse selection effect is gradually reduced until the agent's value is more than $w_c^{2a}$, where $c_H$ remains unchanged and the adverse selection effect is gradually reduced.

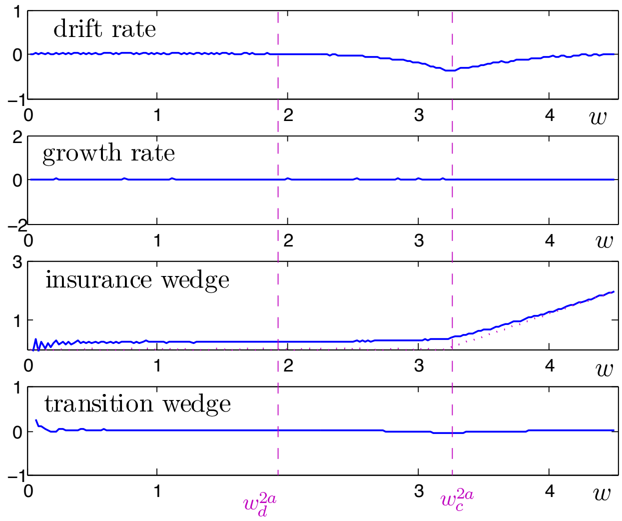


Figure 3: Drift, growth rate and wedges for the pure adverse selection model. The dotline in the third graph is the insurance wedge of the first-best contract. The insurance wedge of the first-best contract is positive because $c_H$ is binding with $Y$ when the continuation value is sufficiently high. $w_d^{2a}$ is the smallest continuation value at which the drift rate is negative.

Finally, by Definition 3, the optimal contract is weakly stationary on continuation domain $(0, w^H)$. The continuation value of the agent will drop (increase) if the new project arrives accompanied by a high (low) shock. Since $0 < w + r\mathcal{J}_i < w^H$ on $\mathcal{CD}$ and the growth rate is zero, $W_t$ will remain in $\mathcal{CD}$ forever and will not hit 0 or $w^H$ if the agent's initial reservation $\underline{w}$ is in $\mathcal{CD}$. Then the contract will not be terminated: When the agent's continuation value is small ($< w_d^{2a}$), the drift of the continuation value is positive and the payment is back-loaded; this means, over the long run, that the continuation value will increase. If the agent's continuation value is sufficiently large, the drift of the continuation is negative and the payment is front-loaded. In the long run, the agent's continuation value will decrease.

### 5.3 Pure Moral Hazard Model

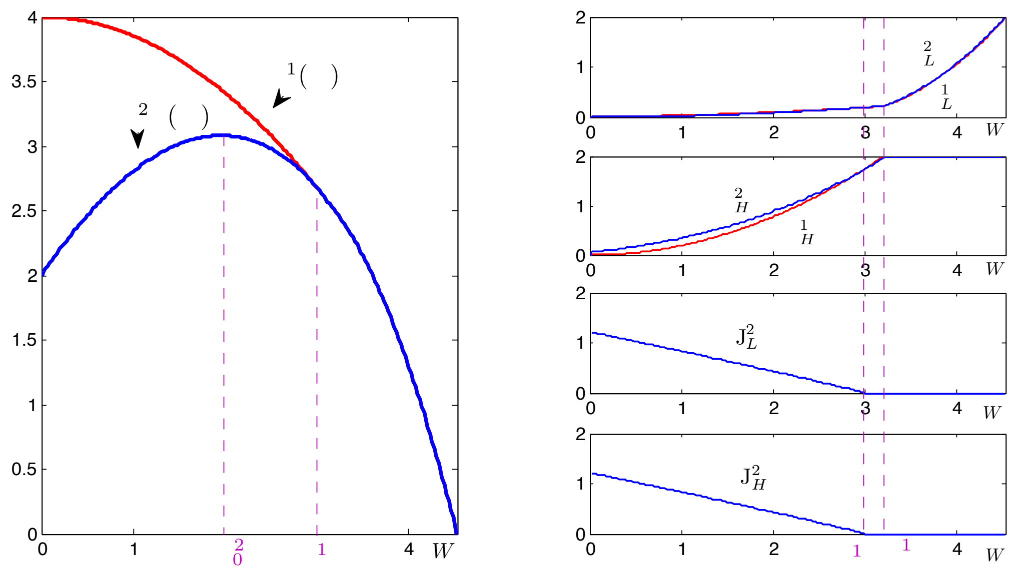

Figure 4: Comparison of optimal contract under the first-best and the second-best setting with pure moral hazard. $u(c) = \sqrt{c}$, $\xi_L = 1, \xi_H = 3$, $P(\xi_H) = 0.3$, $r = 1.8$, $\Delta = 1.0$, $B = 1.5$, $\lambda = 2$. $(c_j^i, \mathcal{J}_j^i)_{j=L,H}^{i=1,2m}$ is the optimal payment and the jump in continuation value process in the first-best and pure moral hazard models. $w_0^{2m}$ is the agent's continuation value at which the principal's value reaches the maximum.

Now we consider the pure moral hazard model. Figure 4 illustrates the comparison between optimal contracts under the first best and second best contracts with pure moral hazard. Different from Sannikov (2008) and other continuous-time moral hazard models, the agent's payment stream directly provides incentive for the agent to work. This is because payment frequency is contingent on the agent's effort, and the cost of compensating the agent for his effort is proportional to the cost of giving the agent value through promised utility. When the agent's promised utility is low, the promised instantaneous payment and transitional utility upon a new project's arrival are combined to incentivize the agent. The variance of transitional utility is minimized to be zero to reduce the principal's risk associated with the uncertainty of preference shock, because only the mean of transitional utility matters in the provision of incentive. If the agent's promised utility is high enough, instantaneous payment can alone provide incentives, and the jump in the continuation value process is reduced to zero; the risk associated with the uncertainty of

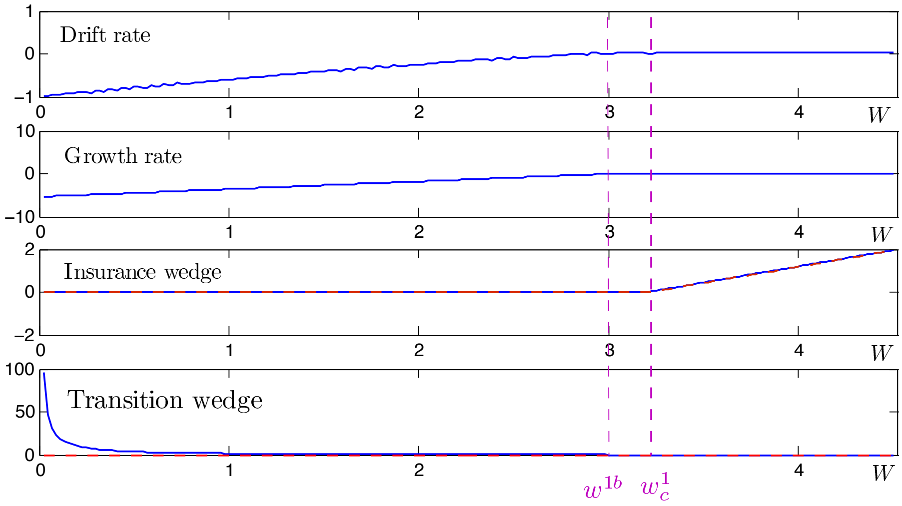


Figure 5: Drift, growth rate, and wedges for the pure moral hazard model.

the arrival of new projects is also reduced to zero. Hence the contract stream becomes stationary when $W \geq w^{1b}$ for some critical value $w^{1b}$, and the principal's value $F^{2m}(W)$ coincides with $F^1(W)$. We define $(0, w^{1b})$ ($(w^{1b}, w^H)$) as the *inefficient* (*efficient*) *domain.*

At $w^H$, $c_L = c_H = Y$ and $\mathcal{J}_L = \mathcal{J}_H = 0$. (17) is not binding by Assumption 1. Then there is $w^{1b} < w^H$ such that on $(w^{1b}, w^H]$, (17) is not binding, and also $F^{2m}(W) = F^1(W)$ and $\mathcal{J}_H = \mathcal{J}_L = 0$. In other words, $w^{1b}$ is the largest continuation value of the agent at which (17) is binding and

$$w^{1b} = \frac{\lambda}{\Delta} B.$$

Under Assumption 1, $w_c^1 > \frac{\lambda}{\Delta} B$ by Proposition 3, where $w_c^1$ is the smallest value at which the first-best and second-best $c_H$ under moral hazard are binding with $Y$. We have the following properties for optimal contracts.

**Proposition 5.** *In the pure moral hazard setting,* $(0, w^H)$ *is the continuation domain. On* $(0, w^{1b})$*, payment* $(c_L, c_H)$ *and transitional utility increase as* $W$ *increases, where* $0 < c_L, c_H < Y$, $w^{1b} > w + r\mathcal{J}_L = w + r\mathcal{J}_H > w_0^{2m}$. *On the inefficient domain, the dynamics of the agent's and the principal's continuation values are characterized by*

$$dW_t = r(W_{t^-} - \frac{\lambda}{\Delta} B)dt + r\mathcal{J}_t dN_t,^{21} \tag{24}$$

$$dF_W^{2m}(W_t) = \left[F_W^{2m}(W_{t^-} + \mathcal{J}_t) - F_W^{2m}(W_{t^-})\right][dN_t - \lambda dt], \tag{25}$$

*with optimal payment satisfying*

$$\frac{1}{\xi_j u_c(c_j)} = -F_W^{2m}(W + r\mathcal{J}_L) = -F_W^{2m}(W + r\mathcal{J}_H) > -F_W^{2m}(W). \tag{26}$$

[21] $\mathcal{J}_t = \mathcal{J}_L = \mathcal{J}_H$ which is contingent on $W$.

*and*

$$\mathcal{J}_L = \mathcal{J}_H = \frac{B}{\Delta} - \sum_{j=L,H} p_j \xi_j u\left(c_j\right), \tag{27}$$

*for $j = L, H$. On the efficient domain, the optimal contract is the same as that in the first best setting.*

Proposition 5 states that on the inefficient domain, the payment and jump in the agent's value must be higher than those in the first-best setting due to (17), Moreover, the moral hazard incentive compatible condition is binding only on the efficient domain. This implies that the optimal compensation and transitional utility in the first-best setting must be distorted. More specifically, there exists a moral hazard effect: In the pure moral hazard model, the drift is negative, since $r(W_{t^-} - \frac{\lambda}{\Delta} B)$ should be negative on the inefficient domain. This renders the payment front-loaded. But this moral hazard effect becomes weaker as $w$ increases, since the payment and the transitional utility increase. The drift will reduce to 0 after reaching the efficient domain. In other words, the jump in the agent's value is gradually reduced, because the moral hazard effect decreases to zero as $w$ increases. On the efficient domain, there is no moral hazard effect, the payment continues to increase, and the incentive is provided only by the agent's instantaneous payment.

According to Proposition 5, due to the incentive compatibility condition (17), the marginal costs of giving the agent value through current consumption must be aligned with the marginal cost of giving value through transitional utility, and $\mathcal{J}_H = \mathcal{J}_L > 0$ if (17) is binding. Two important facts follow: First, the allocation of consumption is not distorted across different preference shocks and the insurance wedge remains zero. However, consumption under moral hazard is improved compared to that in the first-best setting, and must be front-loaded. Second, as the agent's value grows, it becomes more expensive to deliver the agent's value through transitional utility, since the moral hazard effect on the jump size of the agent's value becomes weaker as $w$ increases.

As illustrated by Figure 6, the agent's value cannot jump from the inefficient domain to the efficient domain even if there is a new project, because it is more costly to incentivize the agent with high transitional utility if the agent is expectably rich. Moreover, under condition (17),

$$r\left[W - \lambda \sum_{j=L,H} p_j \xi_j u\left(c_j\right)\right] = r\left[\underbrace{W + \lambda \sum_{j=L,H} p_j \mathcal{J}_j}_{\text{expected transitional utility upon new project}} - w^{1b}\right]. \tag{28}$$

The transitional utility gradually increases on the inefficient domain, which implies that the payment is front-loaded more heavily when the agent's value is low.

On the inefficient domain, the total expected gain $\lambda \sum_{j=L,H} p_j \left[\xi_j u(c_j) + \mathcal{J}_j\right]$ has to be $w^{1b}$. Therefore, the growth rate must be negative, which is also the result of (26). The marginal cost through transitional utility must also be larger than that through promised utility. Then the agent's value will decrease quickly if there is no new project, and the

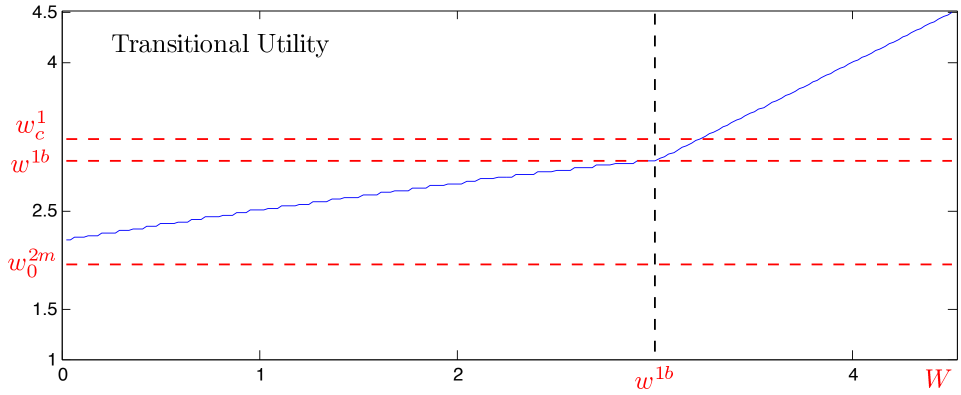


Figure 6: Transitional utility.

agent may be fired if he cannot find a project for a long time. Once he finds a new project, however, his value will jump to a higher value than $w_0^{2m}$ but never enter into the efficient domain, at which first-best efficiency is achieved (see Figure 6). Hence on the one hand, even if the growth rate of the agent's continuation value is negative, the agent is still fully incentivized to work because his transitional utility will immediately jump to a high value, and he will get higher instantaneous payment once he gets a new project. On the other hand, the agent can never reach first-best efficiency if $\underline{w} < w^{1b}$. If $\underline{w} > w^{1b}$, then the agent's continuation value will remain unchanged, and first-best efficiency is achieved.

## 5.4 Mixed Model with Moral Hazard and Adverse Selection

Now we consider the mixed model setting in which both moral hazard and adverse selection squarely exist. Figure 9 illustrates the comparison between third-best contracts under the mixed model and the other cases. From the pure moral hazard model, we know that if the agent's continuation value is high enough, only instantaneous payment is required to provide for the agent incentive to work and the optimal contract is stationary. However, the stationary contract in the first-best setting does not satisfy the truth-telling condition. Hence under both moral hazard and adverse selection, optimal contracts must still satisfy truth-telling condition (16) even if incentive compatibility condition (17) may not be binding. Hence $F^3(w)$ may coincide with the value function $F^{2a}(w)$ of the adverse selection model if the agent is expectably rich enough ($W \geq w^{2b}$), as shown in Figure **??**. That is, the third-best contract can achieve second-best efficiency with adverse selection if the agent is expectably rich enough. Define $w_0^3$ to be the agent's continuation value at which the principal's value reaches the maximum in the third-best contract, and $w^{2b}$ to be the smallest value at which $F^3(w) = F^{2a}(w)$.

We may infer that the incentive compatibility condition (17) will be binding unless $W > w^{2b}$. However, it is surprising to see that (17) is not binding, even if $F^{2m}(W) \neq F^3(W)$. To see this, suppose that on $(w^{1b}, w^H)$, we only consider truth-telling condition (16). From the pure adverse selection model, we know that the growth rate of the agent's

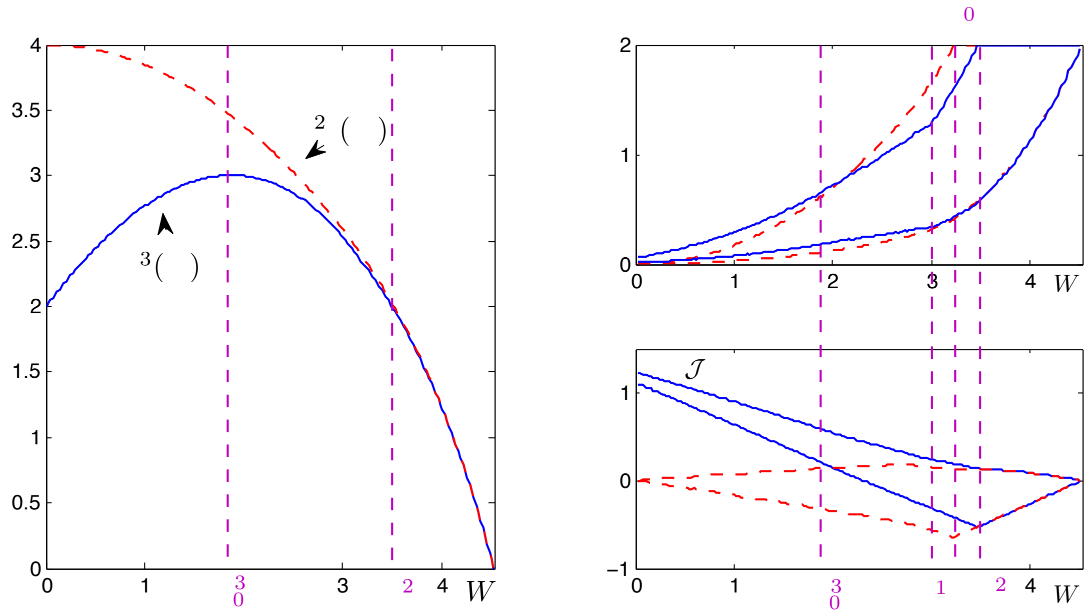


Figure 7: Comparison of optimal contract under pure adverse selection and mixed model. $u(c) = \sqrt{c}$, $\xi_L = 1, \xi_H = 3$, $P(\xi_H) = 0.3$, $r = 1.8$, $\Delta = 1.0$, $B = 1.5$, $\lambda = 2$. $(c_i^j, \mathcal{J}_i^j)_{j=2a,3}^{i=L,H}$ is the optimal payment and the jump of continuation value process in the third-best contract and the second-best contract with adverse selection. $w_0^3$ is the agent's continuation value at which the principal's value reaches the maximum in the third-best contract. $w^{2b}$ is the smallest value at which $F^3(w) = F^{2a}(w)$.

value is zero; that is, for $w \in (w^{1b}, w^H)$, we have

$$w = \lambda \sum_{j=L,H} p_j \left[\xi_j u(c_j) + \mathcal{J}_j\right] > w^{1b}.$$

This means that even on $(w^{1b}, w^{2b})$, (17) is redundant. Hence from (21)-(23),

$$\frac{1}{\xi_H u'(c_H)} = -(1 - \frac{\xi_L}{\xi_H})F_W^3(w) - \frac{\xi_L}{\xi_H}F_W^3(w + r\mathcal{J}_H)\,, \;\; for \; w^{1b} < w < w^{2b}, \tag{29}$$

$$\frac{1}{\xi_L u'(c_L)} = -F_W^3(w + r\mathcal{J}_L)\,, \;\; for \; w > w^{1b}, \tag{30}$$

$$-F_W^3(w) = -\sum_{j=L,H} p_j F_W^3(w + r\mathcal{J}_j)\,, \;\; for \; w > w^{1b}. \tag{31}$$

On $(0, w^{1b})$, (17) is binding, and hence the growth rate is negative. Consistent with the pure moral hazard model, we define $(0, w^{1b})$ as the inefficient domain and $(w^{1b}, w^H)$ as the efficient domain. On the efficient domain, the allocation of consumption and incentive provision achieve second-best efficiency under adverse selection.

Proposition 6 shows that $(0, w^H)$ is the continuation domain, and we know that (17) is binding on the inefficient domain; hence we can apply the first order condition to obtain[22]

$$\frac{1}{\xi_H u'(c_H)} = -\left(1 - (p_H + p_L \frac{\xi_L}{\xi_H})\right) F^3_W (w + r\mathcal{J}_L) - \left(p_H + p_L \frac{\xi_L}{\xi_H}\right) F^3_W (w + r\mathcal{J}_H), \tag{32}$$

$$\frac{1}{\xi_L u'(c_L)} = -F^3_W (w + r\mathcal{J}_L), \tag{33}$$

on the inefficient domain. Hence $\xi_H u'(c_H) > \xi_L u'(c_L)$ and the insurance wedge is positive. Moreover, from (18), because the growth rate of the agent's value is negative, we have

$$-F^3_W (w) < -\sum_{j=L,H} p_j F^3_W (w + r\mathcal{J}_j). \tag{34}$$

In conclusion, we have the following result.

**Proposition 6.** *In the mixed setting,* $(0, w^H)$ *is the continuation domain. On* $(0, w^{1b})$*, payment* $(c_L, c_H)$ *and transitional utility increase as* $W$ *increases, with* $0 < c_L, c_H < Y$*,* $W + r\mathcal{J}_L > W + r\mathcal{J}_H > w^3_0$*. On the inefficient domain [*$(0, w^{1b})$*?], the dynamics of the agent's and the principal's continuation values are characterized by*

$$dW_t = r(W_{t^-} - \frac{\lambda}{\Delta} B)dt + r\mathcal{J}_t(\theta_t)dN_t, \tag{35}$$

$$dF^3_W (W_t) = \left[F^3_W (W_{t^-} + \mathcal{J}_t(\theta_t)) - F^3_W (W_{t^-})\right] dN_t - \lambda \sum_{j=L,H} p_j \mathcal{J}_t(\xi_j)dt. \tag{36}$$

*with the optimal contract satisfying (32), (33) and (34). On the efficient domain,*

$$dW_t = r\mathcal{J}_t(\theta_t)dN_t, \ dF^3_W (W_t) = \left[F^3_W (W_{t^-} + \mathcal{J}_t(\theta_t)) - F^3_W (W_{t^-})\right] dN_t. \tag{37}$$

*and the optimal contract satisfies (29), (30), and (31).*

On the efficient domain, the moral hazard effect is removed; therefore, the optimal contract stream can be weakly stationary. Meanwhile, on the inefficient domain, (17) is binding and (16) holds, and we have

$$\mathcal{J}_L = \frac{B}{\Delta} - \xi_L u(c_L) - p_H [\xi_H - \xi_L] u(c_H) \ \textit{or} \ \frac{B}{\Delta} - \bar{\xi} u(c_H) + \xi_L (u(c_H) - u(c_L)), \tag{38}$$

$$\mathcal{J}_H = \frac{B}{\Delta} - \bar{\xi} u(c_H). \tag{39}$$

Hence from (32) and (33), the payment $(c_i)_{i=L,H}$ and transitional utility increase as the agent's value increases (Figures **??** and 11 ). On the other hand, from (38) and (39), we know that the jump decreases as the continuation value increases because the moral

[22] Details can be found in the proof of Proposition 6

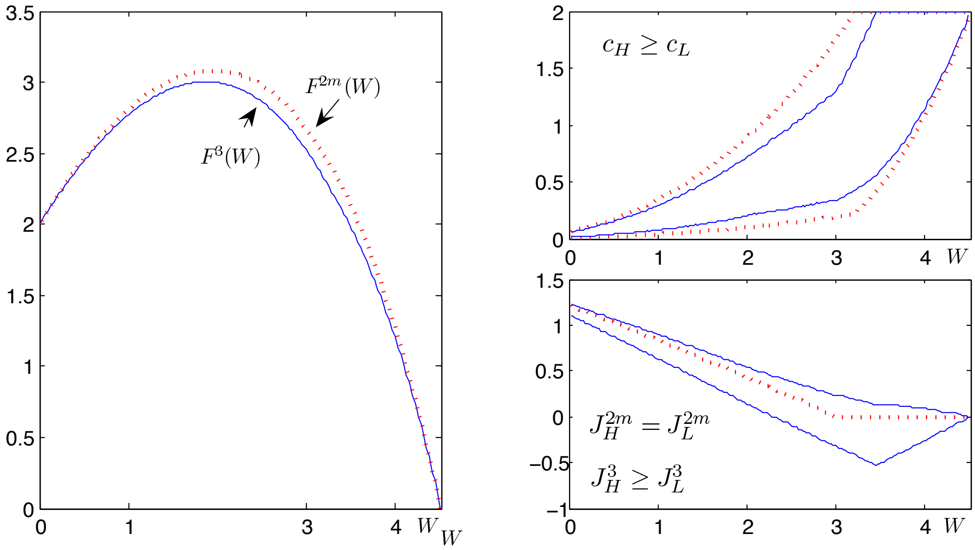


Figure 8: Comparison of optimal contract between moral hazard and mixed models. $u(c) = \sqrt{c}$, $\xi_L = 1, \xi_H = 3$, $P(\xi_H) = 0.3$, $r = 1.8$, $\Delta = 1.0$, $B = 1.5$, $\lambda = 2$. $(c_i, \mathcal{J}_i^j)_{j=2m,3}^{i=L,H}$ is the optimal payment and the jump of continuation value process in the third-best contract and the second-best Contract with the moral hazard effect.

hazard effect decreases as $w$ increases; that is, the instantaneous payment plays a larger role in incentivizing the agent to work than the jump in the continuation value when the agent becomes expectably richer (Figure **??**). Meanwhile, $\mathcal{J}_L - \mathcal{J}_H = (u(c_H) - u(c_L))$ is gradually increasing until $c_H = Y$ because the adverse selection effect becomes weaker as $w$ increases; that is, the principal can punish the agent more for lying if he is expectably richer. Considering the interplay between the moral hazard and adverse selection effects, the optimal payment of the agent may be front-loaded or back-loaded.

When the agent's value is near zero, the expected instantaneous utility must be higher than that under pure adverse selection (See Figure **??** for illustration) to incentivize the agent to work, and the payment has to be front-loaded. Moreover, we find that the payment is front-loaded even more heavily than that under pure moral hazard, since the adverse selection effect enhances it. Note that

$$\sum_{j=L,H} p_j \xi_j u(c_j) = \bar{\xi} u(c_L) + p_H \xi_H (u(c_H) - u(c_L)). \tag{40}$$

The payment has to be distorted, and $u(c_H) - u(c_L)$ is smaller in the mixed model than that under pure moral hazard, in order to lower the agency cost to enforce truth-telling. On the other hand, higher payment for low shock must be delivered to incentivize the agent to work.

When the agent's value is gradually increasing, the adverse selection effect begins to dominate the moral hazard effect in the sense that the payment is more heavily distorted than the increment of the payment for the low shock (right panels in Figures **??** and 8

). As $w$ increases, the moral hazard effect becomes weaker and weaker, and the adverse selection effect emerges to drive the payment back-loaded. Note that the payment is even more distorted than that under pure adverse selection because of the existence of moral hazard. Hence the drift of the agent's value keeps increasing, and reaches maximum at $w^{1b}$ (Figure 10). As a result, the payment is back-loaded near $w^{1b}$ on both inefficient and efficient domains.

On $[w^{1b}, w^{2b}]$, the moral hazard effect disappears, and the jumps in the continuation value are still higher than those under pure adverse selection; hence they gradually decrease in order to achieve second-best efficiency at $w^{2b}$. Note that the adverse selection effect will also become weaker and weaker in driving the payments back-loaded as $w$ keeps rising. Meanwhile, the growth rate of the agent's value is zero, and hence the drift is proportional to the expected jump $\sum_{j=L,H} p_j J_j$ and decreases until the agent's value reaches $w^{2b}$ and the drift reaches the negative minimum on the efficient domain. Eventually, the adverse selection effect will eventually drive the payments front-loaded again as $w$ keeps rising.

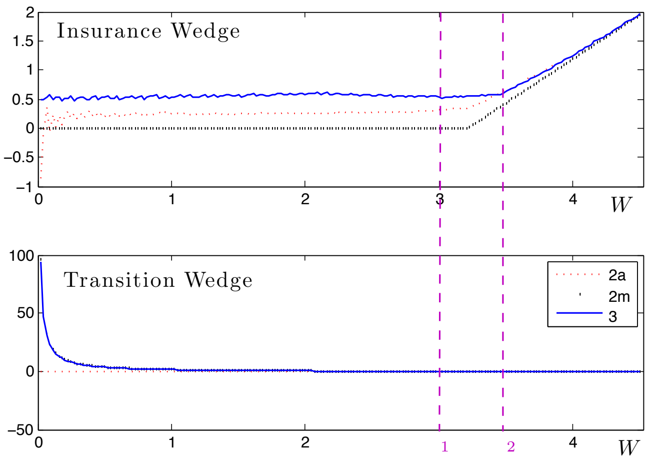


Figure 9: Comparison of wedges among the pure adverse selection, pure moral hazard and mixed models.

We conclude this section with discussion of the dynamics of the third best contract. The growth rate of $W_t$ is zero on the efficient domain and negative on the inefficient domain. Hence $W_t$ will end at 0 if it starts from the inefficient domain and the agent does not find a new project for a long time. However, if the agent finds a new project with low preference shock, his continuation value will jump to a higher level of values, as shown in Figure 11. The transitional utility accompanied by a high shock remains within the inefficient domain if the agent's current promised utility is on the inefficient domain, in order to maintain a difference from $w + r\mathcal{J}_L$ for the incentive for truth-telling. This is different from the case in the pure moral hazard model. Hence we claim that the possibility of migration from the inefficient domain to the efficient domain in the third-best contract is due to variation in the adverse selection effect with continuation values,

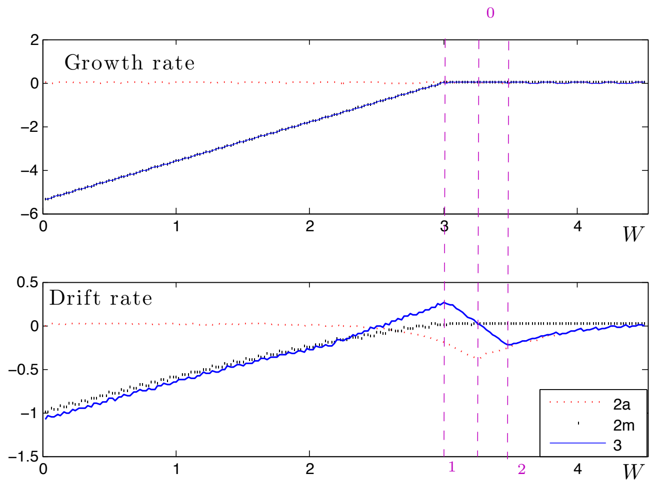


Figure 10: Comparison of drift and growth rate of the agent's value among three models.

for which the principal has to randomize transitional utility; also, the risk of transitional utility upon preference shock must be larger when the agent becomes expectably richer. Finally, on the efficient domain, consumption is first back-loaded because the efficiency of consumption allocation is improved compared with the situation on the inefficient domain. It is then front-loaded (Figure 10), because efficiency decays as the wealth level of the agent increases, which is similar to the pattern under pure adverse selection.

# 6 Conclusion

This paper develops a new benchmark for investigating optimal contracts when repeated adverse selection and dynamic moral hazard coexist. Contracts in continuous time are conveniently characterized by the agent's continuation value. The provision of incentives, the agent's efforts, and the revelation of the agent's private information all depend on the payment and the jump in the continuation value, both of which are contingent on the agent's reported values of preference shock. The i.i.d. assumption on private shocks is not realistic in many circumstances. However, it greatly simplifies characterization of the optimal contract and yields deeper insights into incentive provision under repeated adverse selection.

The cost of compensating the agent for his effort is proportional to the costs of giving the agent value through promised utility, and hence a prominent feature of our model is that the efficiency of consumption allocation and incentive provision can be improved if the agent's value is sufficiently large. The principal's value function of the moral hazard model coincides with that in the first-best setting and the third-best value function coincides with that of the pure adverse selection model in the efficient domain. The payment in the pure moral hazard model must be front-loaded to incentivize the agent to work. However, if moral hazard is mixed with adverse selection, the payment can be back-loaded because the adverse selection effect begins to dominate the moral hazard effect when the agent

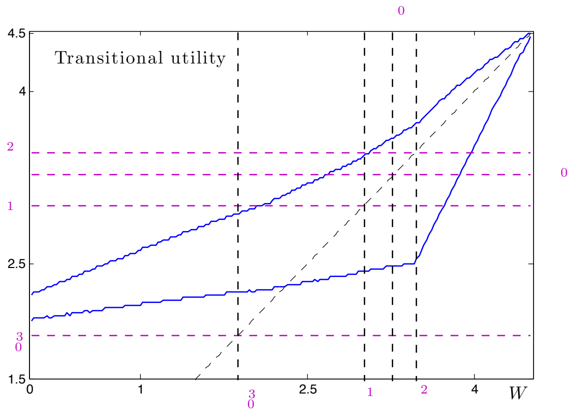


Figure 11: Dynamics of transitional utility. $w^{0b}$ is the value at which the drift of the agent's value changes the sign.

becomes richer.

In the pure adverse selection model, the agent's continuation value does not change until a new project arrives. If the new project is accompanied by a high (low) value shock, the agent is paid more (less), but his transition utility drops (increases) from the current continuation value. Moreover, consumption will be back-loaded if the agent's value is lower and become front-loaded if the agent's value is sufficiently high. Under pure moral hazard, if the continuation value starts with a low value, it will keep decreasing until a new project is found by the agent. However, the drift of the continuation remains to be negative, which implies that the payment is always front-loaded and consumption will converge to 0 in the long run. Under both moral hazard and adverse selection, the growth rate of the agent's value is negative (zero) in the efficient (inefficient) domain. In the inefficient domain, the agent's value can jump to the efficient domain if a new project is found accompanied by a low preference shock. Moreover, on the inefficient domain, the consumption is front-loaded.

Bertrand and Mullainathan (2001) find that a CEO's payment increases in response to favorable exogenous shocks. Our mixed model shows that even if the shock needs to be reported by the agent himself, the agent still gets a higher payment if the shock value is high. Moreover, the jump in continuation value is related to empirical studies of the active adjustment of agency compensation packages in response to swings in firm performance. Our model implies that the agent's future compensation package can be adjusted not only because the agent is expected to bring more profit to the firm, but also because of incentive provision for the current period. Zhang (2008) empirically finds that some newly appointed CEOs (i.e., those with tenure of three years or less) are dismissed while others are not. Our mixed model predicts that new employees (lower reservation) will get fired quickly if they cannot find new projects. Once they find new projects, their values enter the efficient domain, and it will take longer to fire those with high value.

# References


[1] Albanesi, S., & Armenter, R. (2012). Intertemporal distortions in the second best. *Review of Economic Studies*, 79(4), 1271-1307.

[2] Albanesi, S., & Sleet, C. (2006). Dynamic optimal taxation with private information. *Review of Economic Studies*, 73(1), 1-30.

[3] Allen, F. (1983). Credit rationing and payment incentives. *Review of Economic Studies*, 50(4), 639-646.

[4] Armstrong, M., & Vickers, J. (2010). A model of delegated project choice. *Econometrica*, 78(1), 213-244.

[5] Atkeson, A., & Lucas Jr, R. E. (1992). On efficient distribution with private information. *Review of Economic Studies*, 59(3), 427-453.

[6] Bernardo, A. E., Cai, H., & Luo, J. (2001). Capital budgeting and compensation with asymmetric information and moral hazard. *Journal of Financial Economics*, 61(3), 311-344.

[7] Bertrand, M., & Mullainathan, S. (2001). Are CEOs rewarded for luck? The ones without principals are. *Quarterly Journal of Economics*, 116(3), 901-932.

[8] Biais, B., Mariotti, T., Rochet, J. C., & Villeneuve, S. (2010). Large risks, limited liability, and dynamic moral hazard. *Econometrica*, 78(1), 73-118.

[9] Bolton, P., & Dewatripont, M. (2005). Contract theory. MIT press.

[10] Chassang, S. (2013). Calibrated incentive contracts. *Econometrica*, 81(5), 1935-1971.

[11] Cvitanić, J., & Zhang, J. (2007). Optimal compensation with adverse selection and dynamic actions. *Mathematics and Financial Economics*, 1(1), 21-55.

[12] DeMarzo, P. M., & Sannikov, Y. (2006). Optimal Security Design and Dynamic Capital Structure in a Continuous-Time Agency Model. *Journal of Finance*, 61(6), 2681-2724.

[13] Fernandes, A., & Phelan, C. (2000). A recursive formulation for repeated agency with history dependence. *Journal of Economic Theory*, 91(2), 223-247.

[14] Gârleanu, N., & Pedersen, L. H. (2003). Adverse selection and the required return. *Review of Financial Studies*, 17(3), 643-665.

[15] Green, E.J. (1987). Lending and the smoothing of uninsurable income. In E.C. Prescott and N. Wallace (eds.), *Contractual Arrangement for Intertemporal Trade,* 3-25. Minneapolis: University of Minnesota Press.

[16] Golosov, M., Kocherlakota, N., & Tsyvinski, A. (2003). Optimal indirect and capital taxation. *Review of Economic Studies*, 70(3), 569-587.

[17] Harris, M., Kriebel, C. H., & Raviv, A. (1982). Asymmetric information, incentives and intrafirm resource allocation. *Management Science*, 28(6), 604-620.

[18] Harris, M., & Raviv, A. (1996). The capital budgeting process: Incentives and information. The Journal of Finance, 51(4), 1139-1174.

[19] Hart, O. (2009). Hold-up, asset ownership, and reference points. *Quarterly Journal of Economics*, 124(1), 267-300.

[20] He, Z. (2008). Optimal executive compensation when firm size follows geometric brownian motion. *Review of Financial Studies*, 22(2), 859-892.

[21] Hoffmann, F., & Pfeil, S. (2010). Reward for luck in a dynamic agency model. *Review of Financial Studies*, 23(9), 3329-3345.

[22] Holmstrom, B., & Tirole, J. (1997). Financial intermediation, loanable funds, and the real sector. *Quarterly Journal of Economics*, 112(3), 663-691.

[23] Jacod, J. and Shiryaev, A.N. (2003). Limit theorems for stochastic processes. Berlin, *Springer-Verlag*, second edition.

[24] Jenter, D., & Kanaan, F. (2015). CEO turnover and relative performance evaluation. *Journal of Finance*, 70(5), 2155-2184.

[25] Jewitt, I., Kadan, O., & Swinkels, J. M. (2008). Moral hazard with bounded payments. *Journal of Economic Theory*, 143(1), 59-82.

[26] Kloyer, M. and Helm, R. (2008). Contractual design of contract R&D: State-of-the-Art of Empirical Research. *Technology and Innovation Management*, 151-165.

[27] Kocherlakota, N. R. (2010). The new dynamic public finance. Princeton University Press.

[28] Lewis, T. R. (2012). A theory of delegated search for the best alternative. *RAND Journal of Economics*, 43(3), 391-416.

[29] Last, F. and Brandt A. (2003). Marked point processes on the real line: the dynamic approach. New York: *Springer-Verlag.*

[30] Li, R. (2011). Dynamic agency with persistent exogenous shocks. *Working paper*, University of Wisconsin# Madison.

[31] Malenko, A. (2011). Optimal design of internal capital markets. *Working Paper*, Massachusetts Institute of Technology.

[32] Mason, R., & Välimäki, J. (2015). Getting it done: dynamic incentives to complete a project. *Journal of the European Economic Association*, 13(1), 62-97.

[33] Mirrlees, J. A. (1971). An exploration in the theory of optimum income taxation. *Review of Economic Studies*, 38(2), 175-208.

[34] Øksendal, B. K. and Sulem, A. (2010). Applied stochastic control of jump diffusions,"*Springer-Verlag*, 2nd Edition.

[35] Ramsey, F. P. (1928). A mathematical theory of saving. *Economic Journal*, 38(152), 543-559.

[36] Rogerson, W. P. (1985). Repeated moral hazard. *Econometrica*, 69-76.

[37] Sadzik, T., & Stacchetti, E. (2015). Agency models with frequent actions. *Econometrica*, 83(1), 193-237.

[38] Sannikov, Y. (2007a). Agency problems, screening and increasing credit lines. Working Paper.

[39] Sannikov, Y. (2007b). Games with imperfectly observable actions in continuous time. *Econometrica*, 75(5), 1285-1329.

[40] Sannikov, Y. (2008). A continuous-time version of the principal-agent problem. *Review of Economic Studies*, 75(3), 957-984.

[41] Sannikov, Y., & Skrzypacz, A. (2010). The role of information in repeated games with frequent actions. Econometrica, 78(3), 847-882.

[42] Stiglitz, J. E., & Weiss, A. (1983). Incentive effects of terminations: Applications to the credit and labor markets. *American Economic Review*, 73(5), 912-927.

[43] Stulz, R. (1990). Managerial discretion and optimal financing policies. *Journal of financial Economics*, 26(1), 3-27.

[44] Sung, J. (2005). Optimal contracts under adverse selection and moral hazard: a continuous-time approach. *Review of Financial Studies*, 18(3), 1021-1073.

[45] Tchistyi, A. (2005). Security design with correlated hidden cash flows: The optimality of performance pricing. Working Paper.

[46] Thomas, J., & Worrall, T. (1990). Income fluctuation and asymmetric information: An example of a repeated principal-agent problem. *Journal of Economic Theory*, 51(2), 367-390.

[47] Townsend, R. M. (1982). Optimal multiperiod contracts and the gain from enduring relationships under private information. *Journal of political Economy*, 90(6), 1166-1186.

[48] Ulbricht, R. (2016). Optimal delegated search with adverse selection and moral hazard. *Theoretical Economics*, 11(1), 253-278.

[49] Vath, V. L., Pham, H., & Villeneuve, S. (2008). A mixed singular/switching control problem for a dividend policy with reversible technology investment. *Annals of Applied Probability*, 18(3), 1164-1200.

[50] Williams, N. (2011). Persistent private information. *Econometrica*, 79(4), 1233-1275.

[51] Williams, N. (2015). A solvable continuous time dynamic principal–agent model. *Journal of Economic Theory*, 159, 989-1015.

[52] Zhang, Y. (2008). Information asymmetry and the dismissal of newly appointed CEOs: An empirical investigation. *Strategic Management Journal*, 29(8), 859-872.

[53] Zhang, Y. (2009). Dynamic contracting with persistent shocks. *Journal of Economic Theory*, 144(2), 635-675.

[54] Zhu, J. Y. (2012). Optimal contracts with shirking. *Review of Economic Studies*, 80(2), 812-839.

# Appendix

## A Proof of Lemma 1

Define the gain process $G_t$ by

$$e^{-rt}G_t := E\left\{\int_0^\infty re^{-rs}\theta_s u\left(c_s(\theta_s)\right)dN_s + \int_0^\infty re^{-rs}1_{[\Lambda_s=\lambda-\Delta]}Bds|h_{t^-},\theta_t\right\},$$

and $e^{-rt}G_t$ is a martingale. On one side, by the definition of the continuation value, we have

$$e^{-rt}G_t = e^{-rt}W_t + \int_0^t re^{-rs}\theta_s u\left(c_s(\theta_s)\right)dN_s + \int_0^t re^{-rs}1_{[\Lambda_s=\lambda-\Delta]}Bds.$$

On the other side, by the martingale representation theorem for a marked point process (Last and Brandt 2003[29]) , there exists a predictable $\gamma_t\left(\theta_t\right)$ and

$$de^{-rt}G_t = re^{-rt}\gamma_t(\theta_t)dN_t - re^{-rt}\Lambda_t\sum_{i=L,H}p_i\gamma_t(\xi_i)dt.$$

Define $\mathcal{J}_t(\theta_t) = \gamma_t(\theta_t) - \theta_t u\left(c_t\left(\theta_t\right)\right)$, and applying Ito's formula we obtain the representation form of $W_t$. Moreover, $\tilde{N}_t = r\int_0^t J_s(\theta_s)dN_s - r\int_0^t\left\{\Lambda_s\sum_{i=L,H}p_i\mathcal{J}_s(\xi_i)\right\}ds$ is a martingale.

## B Proof of Proposition 1

It has been proved in the main text that (7), (8), and (9) necessarily hold if (GIC) is true. We only need to prove the sufficiency. Consider an arbitrary reporting strategy $\hat{\sigma}$ and effort $\hat{\Lambda}_t$. Define $e^{-rt}\hat{G}_t = \int_0^t re^{-rs}\theta_s u\left(c_s\left(\hat{\theta}_s\right)\right)dN_s + \int_0^t re^{-rs}1_{[\hat{\Lambda}_s=\lambda-\Delta]}Bds + e^{-rt}\hat{W}_t$ with $\hat{W}_0 := w$, where $\hat{W}_t$ is the continuation value process contingent on the agent's report and desirable effort $\{\Lambda_t\}_{t\geq 0}$. Note that

$$e^{-rt}\hat{G}_t = w + e^{-rt}K_t + \int_0^t re^{-rs}\left(\theta_s u(c_s(\theta_s)) + \mathcal{J}_s(\theta_s)\right)dN_s - \int_0^t re^{-rs}\hat{\lambda}_s\sum_{i=L,H}\left(\xi_i u(c_s(\xi_i)) + \mathcal{J}_s(\xi_i)\right)ds$$

where

$$e^{-rt}K_t := \int_0^t re^{-rs}\left\{\left[1_{[\hat{\Lambda}_s=\lambda-\Delta]}B - 1_{[\Lambda_s=\lambda-\Delta]}B\right] - \left[\Lambda_s - \hat{\Lambda}_s\right]\sum_{i=L,H}\left(\xi_i u(c_s(\xi_i)) + \mathcal{J}_s(\xi_i)\right)\right\}ds$$
$$+ \int_0^t re^{-rs}\left[\theta_s u\left(c_s\left(\hat{\theta}_s\right)\right) + \mathcal{J}_s(\hat{\theta}_s) - \theta_s u\left(c_s\left(\theta_s\right)\right) - \mathcal{J}_s(\theta_s)\right]dN_s$$

The first and second parts of the integration of $e^{-rt}K_t$ are negative because of (7), (8), and (9). Hence, $e^{-rt}\hat{G}_t$ is a supermartingale. Note that $\{c_t\}$ are bounded from below, hence $e^{-rt}\hat{G}_t$ is a supermartingale bounded from below. Hence, we add

$$\int_0^\infty re^{-rs}\theta_s u\left(c_s(\hat{\theta}_s)\right)dN_s + \int_0^\infty re^{-rs}1_{[\hat{\Lambda}_s=\lambda-\Delta]}Bds$$

as the last element of the supermartingale $e^{-rt}\hat{G}_t$ by the martingale convergence theorem (Last and Brandt 2003[29]). Therefore,

$$w = \hat{G}_0 \geq E\left[\int_0^\infty re^{-rs}\theta_s u\left(c_s(\hat{\theta}_s)\right)dN_s + \int_0^\infty re^{-rs}1_{[\hat{\Lambda}_s=\lambda-\Delta]}Bds\right].$$

Hence, the truth-telling strategy $\sigma^*$ with $\{\Lambda_t\}_{t\geq 0}$ is at least as good as any alternative strategy $\hat{\sigma}$ with effort plan $\left\{\hat{\Lambda}_t\right\}_{t\geq 0}$.

# C Proof of Proposition 2

We prove that $F^{2a}(W)$ is strictly concave and monotone, and $\Upsilon^3(W) = \Upsilon^{2a}(W) \cap \Upsilon^{2m}(W)$. Other parts can be derived similarly.

## C.1 $F^{2a}(W)$ is concave and monotone

Given two initial values $w_1, w_2 \in [0, w^H]$, $w_1 > w_2$ of the agent's continuation value processes, we need to prove

$$F^{2a}(w^\chi) > \chi F^{2a}(w_1) + (1-\chi)F^{2a}(w_2).$$

where $w^\chi = \chi w_1 + (1-\chi)w_2$, where $0 < \chi < 1$.

We assume that $(\mathcal{J}_t^i(\theta_t), c_t^i(\theta_t))_{i=1,2}$ is the optimal incentive compatible contract for the agent with initial value $w_i$ and

$$dW_t^i = rW_{t^-}^i dt + r\mathcal{J}_t^i(\theta_t)dN_t - r\lambda\sum_{j=L,H}p_j\left\{\xi_j u(c_t^i(\xi_j)) + \mathcal{J}_t^i(\xi_j)\right\}dt, W_0^i = w_i.$$

Under pure adverse selection, $\left(c_t^j(\xi_j), \mathcal{J}_t^j(\xi_j)\right)_{j=L,H}$ satisfy (7), (8), $0 \leq W_t^i + r\mathcal{J}_t^i(\xi_j) \leq w_H$. Under pure adverse selection, high effort can be dictated. If $W_t^i$ hits 0, then $c_v^i(\theta_j) = \mathcal{J}_v^i(\xi_j) = 0$, for $v \geq t$. If $W_t^i$ hits $w^H$, $c_v^i(\theta_j) = Y$, $\mathcal{J}_v^i(\xi_j) = 0$ for $v \geq t$. We construct $W_t^3$ with $(c_t^3(\xi_j), \mathcal{J}_t^3(\xi_j))$ satisfying

$$dW_t^3 = rW_{t^-}^3 dt + r\mathcal{J}_t^3(\theta_t)dN_t - r\lambda\sum_{j=L,H}p_j\left\{\xi_j u(c_t^3(\xi_j)) + \mathcal{J}_t^3(\xi_j)\right\}dt, W_0^i = w^\chi$$

where $c_t^3(\theta_t)$ is defined by

$$u\left(c_t^3(\theta_t)\right) = \chi u\left(c_t^1(\theta_t)\right) + (1-\chi)u\left(c_t^2(\theta_t)\right).$$

Then, $c_t^3(\theta_t) \leq \chi c_t^1(\theta_t) + (1-\chi) c_t^2(\theta_t)$. Moreover, the equality holds only when $c_t^1(\theta_t) = c_t^2(\theta_t)$, because $u(c)$ is strictly concave. Also, let $\mathcal{J}_t^3(\xi_j) = \chi \mathcal{J}_t^1(\xi_j) + (1-\chi)\mathcal{J}_t^2(\xi_j)$. Hence $(c_t^3(\xi_j), \mathcal{J}_t^3(\xi_j))$ satisfy (7), (8) and $0 \leq W_t^i + r\mathcal{J}_t^3(\xi_j) \leq w_H$. $W_t^3 = \chi W_t^1 + (1-\chi) W_t^2$. Then, from the definition of the value function $F^{2a}(w)$, we know that $F^{2a}(w^\chi) \geq \chi F^{2a}(w_1) + (1-\chi) F^{2a}(w_2)$. Moreover, the equality holds only when $c_t^1(\theta) = c_t^2(\theta)$ because $u(c)$ is strictly concave, which implies the strict concavity of $F^{2a}(W)$. For monotonicity, let $0 < w_1 < w_2 < w_H$; we define $W_t^2$ as previously, and we let $c_t^1(\theta_t) = c_t^2(\theta_t), \mathcal{J}_t^1(\theta_t) = \mathcal{J}_t^2(\theta_t)$. Hence $W_t^1 < W_t^2$, before $W_t^1$ or $W_t^2$ hits 0 or $w_H$. Moreover, we know that $F^{2a}(w) \in [F^{2a}(w_H), F^{2a}(0)]$. So $W_t^1$ will hit $w^H$ earlier than $W_t^2$, and we have monotonicity for $F^{2a}(w)$.

### C.2 $\Upsilon^3(W) = \Upsilon^{2a}(W) \cap \Upsilon^{2m}(W)$

Next, we want to prove $\Upsilon^3(W) = \Upsilon^{2a}(W) \cap \Upsilon^{2m}(W)$, which is to prove

$$\mathcal{J}_L = \mathcal{J}_H + \xi_L [u(c_H) - u(c_L)] \tag{41}$$

under the constraint $\Delta \sum_{j=L,H} p_j [\xi_j u(c_j) + \mathcal{J}_j] \geq B$. Obviously, from (7) and (8), we have $c_H \geq c_L$ at optimality. If $c_H = c_L$, by (7) and (8), then (41) holds. Suppose $c_H > c_L$. We only need to show $\xi_L [u(c_H) - u(c_L)] \leq \mathcal{J}_L - \mathcal{J}_H$ must be binding at optimality. Suppose not; then at $w$, the optimal $(c_i, \mathcal{J}_i)$ satisfies

$$\xi_L [u(c_H) - u(c_L)] < \mathcal{J}_L - \mathcal{J}_H. \tag{42}$$

Thus, we have $W + r\mathcal{J}_L > 0$; otherwise, $W + r\mathcal{J}_H < 0$. With sufficiently small $\delta$, define $\mathcal{J}_L^\delta = \mathcal{J}_L - \delta, \mathcal{J}_H^\delta = \mathcal{J}_H + \frac{p_L}{p_H}\delta$. We still have $\xi_L [u(c_H) - u(c_L)] < \mathcal{J}_L^\delta - \mathcal{J}_H^\delta$ , and $\Delta \sum_{j=L,H} p_j \left[\xi_j u(c_j) + \mathcal{J}_j^\delta\right] = \Delta \sum_{j=L,H} p_j [\xi_j u(c_j) + \mathcal{J}_j] \geq B$. The optimality of $(c_j, \mathcal{J}_j)$ implies that there exists no $\delta > 0$, which makes the right-hand side of (13) larger given $c_H$ and $c_L$. It implies

$$F_W^3 \left(W + r\mathcal{J}_H^\delta\right) - F_W^3 \left(W + r\mathcal{J}_L^\delta\right) |_{\delta=0} \leq 0.$$

With the assumption of strict concavity, we have $\mathcal{J}_H \geq \mathcal{J}_L$, which contradicts (42) and $c_H > c_L$. Then, (41) holds.

# D Proof of Proposition 4

Plugging in the corresponding constraints shown by Proposition 2 on $[0, w^H]$, the dynamics of the agent's and the principal's continuation values are given as in Proposition 4.

### D.1 $F_W^{2a}(0) = 0$, $F_W^{2a}\left(w^H\right) \leq -\frac{1}{\bar{\xi} u'(Y)}$.

On the RHS of optimality equation (13), we take a pooling payment $c_L = c_H = b$ and $\mathcal{J}_i = 0$. Then we have

$$\left[F^{2a}(W) - W F_W^{2a}(W)\right] \geq \max_{0 \leq b \leq Y} \lambda [Y - b] - \lambda F_W^{2a}(W) \bar{\xi} u(b). \tag{43}$$

Suppose $F_W^{2a}(0^+) \neq 0$. By Proposition 2, $F_W^{2a}(0^+) < 0$. Let $W \searrow 0^+$, from (43) we have

$$0 \geq \max_{0 \leq b \leq Y} -b - F_W^{2a}\left(0^+\right)\bar{\xi}u\left(b\right). \tag{44}$$

Note that with $b' = 0$, $-b' - F_W^{2a}\left(0\right)\bar{\xi}u\left(b'\right) = 0$. Given $F_W^{2a}\left(0^+\right) < 0$, there exists $b^* > 0$ such that $-b^* - F_W^{2a}\left(0\right)\bar{\xi}u\left(b^*\right) > 0$, which contradicts (44). Hence we prove $F_W^{2a}(0^+) = 0$. On the other hand, if we let $W \nearrow w^H$ in (43), then we have

$$w^H F_W^{2a}\left(w^H\right) \leq \min_b \lambda\left[b - Y\right] + \lambda F_W^{2a}\left(w^H\right)\bar{\xi}u\left(b\right).$$

When $b' = Y$,

$$w^H F_W^{2a}\left(w^H\right) = \lambda\left[b' - Y\right] + \lambda F_W^{2a}\left(w^H\right)\bar{\xi}u\left(b'\right).$$

Hence, $Y$ achieves the minimum for $0 \leq b \leq Y$. Then we have $1 + F_W^{2a}\left(w^H\right)\bar{\theta}u'\left(Y\right) \leq 0$, that is,

$$F_W^{2a}\left(w^H\right) \leq -\frac{1}{\bar{\xi}u'\left(Y\right)}.$$

## D.2 $r\left[W - \lambda\bar{\xi}u\left(c_H\right)\right] + \lambda\left[W - \mathcal{T}_H\right] = 0.$

IC constraint (16) can be transformed into

$$\mathcal{T}_L = \mathcal{T}_H + r\xi_L(u(c_H) - u(c_L)), u(c_H) \geq u(c_L) \tag{45}$$

where $\mathcal{T}_i = W + r\mathcal{J}_i$ is the transitional problem. (13) can be transformed into

$$(r+\lambda)F^{2a}\left(W\right) - (r+\lambda)F_W^{2a}\left(W\right)W = \max_{\mathcal{C}\in\Upsilon^i(W)} r\lambda\left[Y - \sum_{j=L,H} p_j c_j\right] - r\lambda F_W^{2a}\left(W\right)\sum_{j=L,H} p_j\xi_j u\left(c_j\right) \tag{46}$$

$$+\lambda\left(\sum_{j=L,H} p_j\left(F^{2a}\left(\mathcal{T}_j\right) - F_W^{2a}(W)\mathcal{T}_j\right)\right).$$

At $W = 0$, the optimal contract is $\mathcal{T}_j = c_j = 0$ because $F_W\left(0\right) = 0$. At $W = w^H$, $\mathcal{T}_j = w^H$ and $c_j = Y$ can be verified as the optimal solution. Denote $u_j = u\left(c_j\right)$ and $U\left(.\right)$ as the inverse function of $u\left(.\right)$, which is convex and increasing. Denote $u_H = \delta + u_L$ We plug (45) into (43) and (46) becomes

$$\begin{aligned}
&(r+\lambda)F^{2a}\left(W\right) - (r+\lambda)F_W^{2a}\left(W\right)W \\
&= \max_{u_L,\mathcal{T}_H,\delta} r\lambda\left[Y - p_H U\left(u_L + \delta\right) - p_L U\left(u_L\right)\right] - \lambda F_W^{2a}\left(W\right)\left[\mathcal{T}_H + r\bar{\xi}u_L + r\bar{\xi}\delta\right] \\
&+ \lambda\left(p_L F^{2a}\left(\mathcal{T}_H + r\xi_L\delta\right) + p_H F^{2a}\left(\mathcal{T}_H\right)\right),
\end{aligned} \tag{47}$$

subject to

$$\mathcal{T}_H \geq 0, \mathcal{T}_H + r\xi_L\delta \leq w_H,\ \delta \geq 0, u_L \geq 0,\ \ \delta + u_L \leq u\left(Y\right).$$

$-rp_H U(u_L+\delta) - rp_L U(u_L) + p_L F^{2a}(\mathcal{T}_H + r\xi_L\delta) + p_H F^{2a}(\mathcal{T}_H)$ are jointly concave on $(u_L, \mathcal{T}_H, \delta)$, $-\lambda F^{2a}_W(W)$ increases as $W$ increases; hence optimal $(u_L, \mathcal{T}_H, \delta)$ increase given that $\mathcal{T}_H + r\xi_L\delta < w^H$ and $\delta + u_L < u(Y)$, which means that $(u_L, u_H, \mathcal{T}_L, \mathcal{T}_H)$ increase as $W$ increases. Applying envelope theorem on (47) we have

$$r\left[W - \lambda\bar{\xi}u(c_H)\right] + \lambda\left[W - \mathcal{T}_H\right] = 0 \tag{48}$$

The reason we can apply envelope theorem without Lagrange multipliers is that the conditions of all the control variables in (47) are not dependent on the state variable $W$. Note that (48) implies that the growth rate is zero for all $W$.

## D.3 The optimal contract is not pooling on $\left(0, w^H\right)$

Suppose pooling contract $c_H^* = c_L^* = c^*$ is optimal at $W^* \in \left(0, w^H\right)$. We have $\mathcal{T}_L^* = \mathcal{T}_H^* = W^*$. Since the growth rate is proved to be zero in the last step, we have $W^* = \lambda\bar{\xi}u(c^*)$. Because $0 < W^* < w^H$, $0 < c^* < Y$. From the optimality of pooling payment at $W^*$, we can derive two facts. First, given $\mathcal{T}_H^* = W^*$, $c_L^* = c^*$, we define $u(\hat{c}_H) = u(c^*+\delta)$ and $\hat{\mathcal{T}}_L = \mathcal{T}_H^* + r\xi_L\left[u(c^*+\delta) - u(c^*)\right]$, $\delta \geq 0$. With sufficiently small $\delta > 0$, $0 < u(\hat{c}_H) < u(Y)$ and $0 < \hat{\mathcal{T}}_L < w^H$. Then optimality of the pooling contract implies that the optimal value for $\delta$ is 0. Plug $(\mathcal{T}_H^*, \hat{\mathcal{T}}_L, c_L^*, \hat{c}_H)$ into optimality equation (13), and RHS is reduced to

$$\max_{\delta \geq 0} -rp_H(c^*+\delta) - rF^{2a}_W(W)\bar{\xi}u(c^*+\delta) + p_L F^{2a}\left(W^* + r\xi_L\left[u(c^*+\delta) - u(c^*)\right]\right)$$

If $\delta = 0$ is optimal, taking the first order condition with respect to $\delta$ and taking value 0 for $\delta$, we have

$$-1 - \xi_H F^{2a}_W(W^*)\,u'(c^*) \leq 0 \tag{49}$$

The second fact is given the pooling payment is optimal, we have

$$-1 - \bar{\xi}F^{2a}_W(W^*)u'(c^*) = 0; \tag{50}$$

because $0 < c^* < Y$. Then (50) contradicts (49), because $\xi_H > \bar{\xi}$ and $F^{2a}_W(W^*) < 0$. Hence, $c_H$ is not pooling with $c_L$. From our proof, it is clear that the strict monotonicity of the value function is a crucial condition for the non-pooling payment.

## D.4 The optimal payment with low preference shock is not zero on $\left(0, w^H\right)$

Suppose $c_L^* = 0$, hence $c_H^* \neq 0$ because on $\left(0, w^H\right)$ the optimal contract is not pooling. Then

$$r\xi_L\left[u(c_H^*) - u(c_L^*)\right] = \mathcal{T}_L^* - \mathcal{T}_H^* < r\xi_H\left[u(c_H^*) - u(c_L^*)\right]$$

We keep $\mathcal{T}_L^*, \mathcal{T}_H^*$ and improve $c_L^*$ by small amount such that

$$r\xi_L\left[u(c_H^*) - u(c_L^*+\delta)\right] < \mathcal{T}_L^* - \mathcal{T}_H^* < r\xi_H\left[u(c_H^*) - u(c_L^*+\delta)\right].$$

Sufficiently small $\delta > 0$ makes the RHS of (47) larger, because $F^{2a}_W(W^*) < 0$ on $(0, w^H)$ and contradicts $c_L^* = 0$.

## D.5 $0 < \mathcal{T}_H^* < W < \mathcal{T}_L^* < w^H$ for $W \in \left(0, w^H\right)$

First prove $\mathcal{T}_L^* \neq w^H$ and $\mathcal{T}_H^* \neq 0$. If at $w^* \in (0, w^H)$, the optimal $\mathcal{T}_L^* = w^H$, then from (47), the principal can always be better off by lowering $\delta$ unless $\delta = 0$. However if $\delta = 0$, the optimal payment at $w^*$ is pooling, which contradicts D.3.

Suppose that at some $w^* \in (0, w^H)$, the optimal $\mathcal{T}_H^* = 0$. From (47) we know that $u_L^*$, $\delta^*$ and $\mathcal{T}_H^*$ are increasing as $W$ increases before $u_L^* + \delta$ reaches $u(Y)$ (we just show that on $(0, w^H)$, $\mathcal{T}_L^* < w^H$ ). After $u_L^* + \delta^*$ reaches $u(Y)$, we can see that $\mathcal{T}_H^*, \mathcal{T}_L^*, c_L^*$ continues to increase and $\delta^*$ decreases. From D.2, we know that at $w^* = 0$, $c_H^* = c_L^* = 0$ and $\mathcal{T}_H^* = \mathcal{T}_L^* = 0$. Hence for any $w^* \in \left(0, w^H\right)$, $\mathcal{T}_H^* > 0$. Then at any $w^* \in \left(0, w^H\right)$, the optimal transition utility $0 < \mathcal{T}_i^* < w^H$. Hence $\left(0, w^H\right)$ is the continuation domain. Because the growth rate is zero, we have $\sum_i p_i F_{w^*}(\mathcal{T}_i^*) = F_W(w^*)$ from (18) and $\mathcal{T}_H < w^* < \mathcal{T}_L$.

Plugging in the corresponding constraints shown by Proposition 2, the dynamics of the agent's and the principal's continuation values are given as in Proposition 5.

On $(0, w^{1b})$, RHS of (13) is reduced to

$$\max_{c_i, \mathcal{T}_i} -r\lambda \sum_{j=L,H} p_j u(c_j) + \lambda \sum_{j=L,H} p_j F^{2m}(\mathcal{T}_j)$$

subject to

$$\Delta \sum_{j=L,H} p_j \left[r\xi_j u(c_j) + \mathcal{T}_j\right] \geq rB + \Delta W. \tag{51}$$

where $\mathcal{T}_j$ is the transitional utility for $j = L, H$. The monotonicity of contracts is obvious and $\left(0, w^H\right)$ is the continuation domain, which can be investigated in detail by applying the method of Proposition 4 and the fact that $F^{2m}(w) = F^1(w)$ if $w > w^{1b}$.Then on $\left(0, w^{1b}\right)$ there exists Lagrange multiplier $\delta(w) > 0$ contingent on $w$,

$$-\lambda F_W^{2m}(\mathcal{T}_j) = \delta(w), r\lambda = r\delta(w)\xi_j u_c(c_j).$$

Hence $\mathcal{T}_H = \mathcal{T}_L := \mathcal{T}$, $\xi_j u_c(c_j) = -\frac{1}{F_W^{2m}(\mathcal{T})}$. Then $F_W^{2m}(\mathcal{T}) < 0$ and $\mathcal{T} \geq w_0^{2m}$. Moreover, by the monotonicity of the optimal contract, on $(0, w^{1b})$, $\sum_{j=L,H} p_j \xi_j u(c_j)$ increases to $\frac{B}{\Delta}$ and $\{\mathcal{T}_j\}_{j=L,H}$ increase to $w^{1b}$.

Finally, on the inefficient domain, (51) is binding. Hence the growth rate is negative and (19) holds true from (18), and we have (26).

# E Proof of Proposition 6

Plugging in the corresponding constraints shown by Proposition 2, the dynamics of the agent's and the principal's continuation values are given as in Proposition 6.

The maximization problem on RHS of (13) can be reduced to

$$\max_{\mathcal{C}\in\Upsilon^i(W)} - r\lambda \sum_{j=L,H} p_j c_j] + rF^3_W(W)\left(W - \lambda \sum_{j=L,H} p_j \xi_j u(c_j)\right) \tag{52}$$
$$+ \lambda \left(\sum_{j=L,H} p_j \left(F^3(\mathcal{T}_j) - rF^3_W(W)\mathcal{T}_j\right)\right),$$

subject to (51) and

$$\mathcal{T}_L = \mathcal{T}_H + r\xi_L(u(c_H) - u(c_L)), u(c_H) \geq u(c_L). \tag{53}$$

Then the proof of the monotonicity of $(c_i, \mathcal{T}_i)$ is similar to that of Proposition 4. If $w$ is large enough, $F^3(w) = F^{2a}(w)$. Then $\mathcal{T}_L < w^H$, $c_L < Y$ if $w < w^H$ and hence $(0, w^H)$ is the continuation domain.

When $w > w^{1b}$ we can ignore condition (51), because with only (53), the growth rate of $W$ is zero by Proposition 4. Then $\lambda \sum_{j=L,H} p_j [\xi_j u(c_j) + \mathcal{J}_j] = w > w^{1b} = \frac{\lambda}{\Delta}B$. Note that for $w \in [w^{1b}, w^{2b})$, only (53) holds, although $F^3(w) < F^{2a}(w)$. On $(0, w^{1b})$, RHS of (52) is reduced to

$$\max_{c_i,\mathcal{T}_i} -r\lambda \sum_{j=L,H} p_j u(c_i) + \lambda \sum_{j=L,H} p_j F^3(\mathcal{T}_j)$$

subject to $\Delta \sum_{j=L,H} p_j [r\xi_j u(c_j) + \mathcal{T}_j] \geq rB + \Delta W$ and (53). Thus, on $\left(0, w^{1b}\right)$ there exists a Lagrange multiplier $\delta(W) > 0$ contingent on $w$, and then the problem is reduced to

$$\max_{u_L,\kappa,\mathcal{T}_H} -r[p_L U(u_L) + p_H U(u_L + \kappa)] + r\bar{\xi}\delta(W)[u_L + \kappa]$$
$$+ p_L F^3(\mathcal{T}_H + r\xi_L\kappa) + p_H F^3(\mathcal{T}_H) + \delta(W)\mathcal{T}_H,$$

where $U$ is the inverse function of $u$ and $u_j = u(c_j)$, $\kappa = u(c_H) - u(c_L)$. Note that $w^{1b} < w^{2m}_c < w^3_c$; hence $0 < c_L, c_H < Y$ and $0 < \mathcal{T}_L, \mathcal{T}_H < w^H$. First-order conditions yield

$$p_L \frac{1}{u'(c_L)} = -\xi_L p_L F^3_W(\mathcal{T}_L),$$
$$p_H \frac{1}{u'(c_H)} = \bar{\xi}\delta(W) + \xi_L p_L F^3_W(\mathcal{T}_L),$$
$$\delta(W) = -p_L F^3_W(\mathcal{T}_L) - p_H F^3_W(\mathcal{T}_H).$$

Hence

$$\frac{1}{\xi_H u'(c_H)} = -\left(p_L - p_L\frac{\xi_L}{\xi_H}\right) F^3_W(\mathcal{T}_L) - \left(p_H + p_L\frac{\xi_L}{\xi_H}\right) F^3_W(\mathcal{T}_H),$$
$$\frac{1}{\xi_L u'(c_L)} = -F^3_W(\mathcal{T}_L).$$

## Graz Economics Papers

For full list see:
http://ideas.repec.org/s/grz/wpaper.html
Address: Department of Economics, University of Graz,
Universitätsstraße 15/F4, A-8010 Graz

---

11–2018 **Xi Chen, Yu Chen and Xuhu Wan**: Delegated Project Search

10–2018 **Stefan Nabernegg, Birgit Bednar-Friedl, Pablo Muñoz, Michaela Titz and Johanna Vogel**: National policies for global emission reductions: Effectiveness of carbon emission reductions in international supply chains

09–2018 **Jonas Dovern and Hans Manner**: Order Invariant Tests for Proper Calibration of Multivariate Density Forecasts

08–2018 **Ioannis Kyriakou, Parastoo Mousavi, Jens Perch Nielsen and Michael Scholz**: Choice of Benchmark When Forecasting Long-term Stock Returns

07–2018 **Joern Kleinert**: Globalization Effects on the Distribution of Income

06–2018 **Nian Yang, Jun Yang and Yu Chen**: Contracting in a Continuous-Time Model with Three-Sided Moral Hazard and Cost Synergies

05–2018 **Christoph Kuzmics and Daniel Rodenburger**: A case of evolutionary stable attainable equilibrium in the lab

04–2018 **Robert J. Hill, Alicia N. Rambaldi, and Michael Scholz**: Higher Frequency Hedonic Property Price Indices: A State Space Approach

03–2018 **Reza Hajargasht, Robert J. Hill, D. S. Prasada Rao, and Sriram Shankar**: Spatial Chaining in International Comparisons of Prices and Real Incomes

02–2018 **Christoph Zwick**: On the origin of current account deficits in the Euro area periphery: A DSGE perspective

01–2018 **Michael Greinecker and Christopher Kah**: Pairwise stable matching in large economies

15–2017 **Florian Brugger and Jörn Kleinert**: The strong increase of Austrian government debt in the Kreisky era: Austro-Keynesianism or just stubborn forecast errors?

14–2017 **Jakob Mayer, Gabriel Bachner and Karl W. Steininger**: Macroeconomic implications of switching to process-emission-free iron and steel production in Europe

13–2017 **Andreas Darmann, Julia Grundner and Christian Klamler**: Consensus in the 2015 Provincial Parliament Election in Styria, Austria: Voting Rules,Outcomes, and the Condorcet Paradox

12–2017 **Robert J. Hill, Miriam Steurer and Sofie R. Waltl**: Owner Occupied Housing in the CPI and Its Impact On Monetary Policy During Housing Booms and Busts

11–2017 **Philipp Kohlgruber and Christoph Kuzmics**: The distribution of article quality and inefficiencies in the market for scientific journals

10–2017 **Maximilian Goedl**: The Sovereign-Bank Interaction in the Eurozone Crisis

09–2017 **Florian Herold and Christoph Kuzmics**: The evolution of taking roles

08–2017 **Evangelos V. Dioikitopoulos, Stephen J. Turnovsky and Ronald Wendner**: Dynamic Status Effects, Savings, and Income Inequality

07–2017 **Bo Chen, Yu Chen and David Rietzke**: Simple Contracts under Observable and Hidden Actions

06–2017 **Stefan Borsky, Andrea Leiter and Michael Pfaffermayr**: Product Quality and Sustainability: The Effect of International Environmental Agreements on Bilateral Trade

05–2017 **Yadira Mori Clement and Birgit Bednar-Friedl**: Do Clean Development Mechanism projects generate local employment? Testing for sectoral effects across Brazilian municipalities

04–2017 **Stefan Borsky, Alexej Parchomenko**: Identifying Phosphorus Hot Spots: A spatial analysis of the phosphorus balance as a result of manure application

03–2017 **Yu Chen, Yu Wang, Bonwoo Koo**: Open Source and Competition Strategy Under Network Effects

02–2017 **Florian Brugger**: The Effect of Foreign and Domestic Demand on U.S. Treasury Yields

01–2017 **Yu Chen**: On the Equivalence of Bilateral and Collective Mechanism Design

13–2016 **Jan de Haan, Rens Hendriks, Michael Scholz**: A Comparison of Weighted Time-Product Dummy and Time Dummy Hedonic Indexes

12–2016 **Katja Kalkschmied**: Complementary Institutional Elements and Economic Outcomes

11–2016 **Dieter Balkenborg, Josef Hofbauer, Christoph Kuzmics**: The refined best reply correspondence and backward induction

10–2016 **Karl W. Steininger, Wolf D. Grossmann, Iris Grossmann**: The economic value of dispatchable solar electricity: a Post-Paris evaluation

09–2016 **Sofie R. Waltl**: Estimating aggregate quantile-specific gross rental yields for residential housing in Sydney

08–2016 **Karl Farmer**: Financial integration and house price dynamics in equilibrium modeling of intra-EMU and global trade imbalances

07–2016 **Gabriel Bachner, Birgit Bednar-Friedl**: Counterbalancing the Effects of Climate Change Adaptation on Public Budgets: Factor Taxes, Transfers, or Foreign Lending?

06–2016 **Margareta Kreimer, Ricardo Mora**: Segregated Integration: Recent Trends in the Austrian Gender Division of Labor

05–2016 **Karl Farmer, Bogdan Mihaiescu**: Credit constraints and differential growth in equilibrium modeling of EMU and global trade imbalances

04–2016 **Daniel Eckert, Frederik Herzberg**: The birth of social choice theory from the spirit of mathematical logic: Arrow's theorem in the framework of model theory

03–2016 **Christoph Kuzmics, Jan-Henrik Steg**: On public good provision mechanisms with dominant strategies and balanced budget

02–2016 **Florian Gauer, Christoph Kuzmics**: Cognitive Empathy in Conflict Situations

01–2016 **Thomas Aronsson, Olof Johansson-Stenman, Ronald Wendner**: Redistribution Through Charity, and Optimal Taxation when People are Concerned with Social Status

09–2015 **Sofie R. Waltl**: Variation across price segments and locations: A comprehensive quantile regression analysis of the Sydney housing market

08–2015 **Karl Farmer, Bogdan Mihaiescu**: Pension systems and financial constraints in a three-country OLG model of intra-EMU and global trade imbalances

07–2015 **Javier López Prol, Karl W. Steininger**: Photovoltaic self-consumption regulation in Spain: profitability analysis and alternative regulation schemes.

06–2015 **Miriam Steurer, Caroline Bayr**: Quantifying Urban Sprawl using Land Use Data

05–2015 **Porfirio Guevara, Robert J. Hill, Michael Scholz**: Hedonic Indexes for Public and Private Housing in Costa Rica

04–2015 **Sofie R. Waltl**: A Hedonic House Price Index in Continuous Time

03–2015 **Ronald Wendner**: Do Positional Preferences for Wealth and Consumption Cause Inter-Temporal Distortions?

02–2015 **Maximilian Gödl, Christoph Zwick**: Stochastic Stability of Public Debt: The Case of Austria

01–2015 **Gabriel Bachner**: Land Transport Systems under Climate Change: A Macroeconomic Assessment of Adaptation Measures for the Case of Austria