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Abstract

We study optimal effort and compensation in a continuous-time model with three-sided moral hazard and cost synergies. One agent exerts initial effort to start the project; the other two agents exert ongoing effort to manage it. The project generates cash flow at a fixed rate over its lifespan; cash flow stops if a failure occurs. The three agents' efforts jointly determine the probability of the project's survival and thus its expected cash flows. We model cost synergies between the two agents exerting ongoing effort as one's effort reduces the other's cost of effort. In the optimal contract, the timing of payments reflects the timing of efforts as well as cost synergies across agents. The agent exerting upfront effort claims all cash flows prior to a predetermined cutoff date, and the two agents exerting ongoing effort divide all subsequent cash flows. Delaying payments motivate these two agents to work hard throughout. Between them, the agent with greater degree of moral hazard and bigger impact on reducing the other agent's cost claims a larger fraction of the cash flow. Our study sheds light on a broad set of contracting problems, such as compensation plans in startups and profit sharing among business partners.

KEYWORDS: optimal contracting, moral hazard in teams, cost synergies, continuous-time model

JEL CLASSIFICATION: C61, D86, J31, M52

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1 Introduction

Long-term business projects often face the challenge of providing dynamic incentives for different agents whose efforts jointly affect the project's success. The corresponding contracting problems may also involve synergies among agents, such as the potential for one agent's effort to reduce other agents' costs of effort. This paper examines optimal effort and compensation in a continuous-time model with three-sided moral hazard in the presence of cost synergies.

To highlight the interaction between moral hazard and cost synergies, we start with a model of two-sided moral hazard. Specifically, two agents (e.g., senior and junior partners at a law firm) work on a project that generates cash flow at a fixed rate over its lifespan; cash flow stops if a failure occurs. The agents exert ongoing efforts that jointly affect the probability of the project's survival. Each agent's effort is costly and unobservable to the other agent. A novel feature of our model is the presence of cost synergies. We model *Cost Synergies* as the reduction in the junior partner's cost stemming from the senior partner's effort, which we term the senior partner's *Influence*, and vice versa. To focus on incentives, we assume that all agents have limited liabilities and linear utilities. Agents are indifferent between receiving a dollar today and receiving it tomorrow.

In the optimal contract, Agent 1 (the senior partner) and Agent 2 (the junior partner) split the project's cash flow proportionally at each point in time over the project's lifespan or until a failure occurs. The moral hazard and cost synergies jointly determine each agent's share. We identify two cases. In the first case, we assume that Agents 1 and 2 face equally severe moral hazard. If the two agents also have the same influence on each other's cost of effort, they split the cash flows equally at all times. Otherwise, the agent with a relatively greater influence receives a larger fraction of the cash flows, and in an extreme case, the entire cash flows. For example, if Agent 1 has a positive influence and Agent 2 has no influence, Agent 1 receives higher payments compared to the model without cost synergies.

In the second case, both moral hazard severity and influence differ between the two agents. They still split the cash flows proportionally, where the fraction is jointly determined by moral hazard and cost synergies. If cost synergies do not exist, an agent's share of cash flow is increasing in the severity of his moral hazard and decreasing in the severity of the other agent's moral hazard. Furthermore, if Agent 1's influence exceeds the ratio of Agent 2's moral hazard to Agent 1's moral hazard, Agent 1 claims all cash flows. Agent 2 exerts effort that reduces Agent 1's cost of effort and increases Agent 1's optimal effort level. Agent 2 receives no payments and incurs zero cost, because Agent 1's influence eliminates it. In both cases of the two-sided model, cost synergies not only alter the allocation of cash flows, but also improve the expected social surplus.

Next, we study a model with three-sided moral hazard. In addition to the two agents who exert ongoing efforts, we add Agent 0, who exerts effort at the outset to launch the project. With the introduction of an agent who exerts upfront effort, our model becomes applicable to contracting problems involving multiple agents with different timing of effort, such as a startup with an entrepreneur, a chief executive officer (CEO), and a chief technology officer (CTO). In the optimal contract, Agent 0 (the entrepreneur) claims all cash flows before a predetermined cutoff date; after that date, Agent 1 (the CEO) and Agent 2 (the CTO) divide the cash flows. Thus, Agent 0 is penalized only for early failures, which he holds most responsibility for. At the same time, deferring payments motivates Agents 1 and 2 to exert efforts throughout the project's lifespan. Before the cutoff date, the two agents work hard to keep the project alive to ensure that they can collect payments in future. After the cutoff date, they exert efforts to avoid failure because they own the business and bear the variation of cash flows. The payment to Agent 0 is larger (i.e., the cutoff date is later) if the agent's moral hazard is more severe than that of Agents 1 and 2. There are several patterns for how Agents 1 and 2 divide cash flows after the cutoff date.

We consider three potential cases. In the first case, we assume that Agents 1 and 2 are symmetric: the ratio of Agent 1's influence to Agent 2's influence equals the ratio of Agent 1's unit cost of effort to Agent 2's unit cost of effort, and their moral hazard is equally severe. The optimal contract stipulates that Agents 1 and 2 divide the cash flows equally at all times after the cutoff date (i.e., after Agent 0 is fully compensated). Relative to the case without cost synergies, the cutoff date is earlier and the collective payments for Agents 1 and 2 are larger. Moreover, cost synergies also improve the expected social surplus relative to the case without cost synergies.

In the second case, we assume that Agent 1's influence exceeds the ratio of Agent 2's moral hazard to Agent 1's moral hazard. In the optimal contract, Agent 0 claims all cash flows before a known cutoff date, Agent 1 claims all subsequent cash flows, and Agent 2 receives no payments. The cutoff date is later if the moral hazard of Agent 0 is more severe than that of Agents 1 and 2. The cutoff date is earlier if Agent 1 or 2's influence is larger. Agent 2 is willing to participate without receiving any payments because Agent 1's influence reduces his cost of effort to zero. In the final and general cases, Agent 0 still collects all cash flows before a known cutoff date. Agents 1 and 2 alternate in receiving payments after the cutoff date. The lengths of these periods of alternating payments depend on the agents' moral hazard and cost synergies between them.

This study sheds light on a broad set of contracting problems in economics and finance. The two-sided model can help us explain profit sharing among business partners. For instance, in a law firm, partners are the main players. Junior partners and senior partners split profits proportionally according to seniority. In general, a senior partner has more influence than a junior partner does (for instance, a senior partner has the reputation to procure business and the ability to mentor junior partners). Hence, the senior partner will claim a larger fraction of the profits.

Similarly, the three-sided moral hazard model with cost synergies helps us understand startups' compensation plans. For example, the entrepreneur, who has the business plan and other unique resources, exerts effort at the outset to launch the business. Later, the CEO and CTO exert ongoing efforts to manage the daily operations. The CEO's experience allows him to manage the business more effectively. The CTO's effort improves the firm's information technology, reducing the CEO's cost of effort. The relative severity of moral hazard and cost synergies jointly determine how the cash flows are divided among the three agents over time. The entrepreneur collects all cash flows early on, while the CEO and CTO divide the subsequent cash flows. The entrepreneur receives more cash flows if his moral hazard is more severe than those of the CEO and the CTO. The division of cash flows between the CEO and CTO depends on the interaction of their moral hazard and cost synergies: the more influential agent receives a larger share of the cash flows.

Another example is on developing video games. The content creator produces the basic content at the outset. Then, the product development manager (PDM) and the marketing and sales manager (MSM) industrialize the content and make the video game profitable. The PDM's effort can improve the quality and the attractiveness of the video game, and thus reduce the MSM's cost of effort on marketing. Meanwhile, the MSM's effort to acquire information on consumer preference (e.g., via conducting consumer survey) helps the PDM improve product design, and thus reduces the PDM's cost of effort. In most cases, the content creator first collects all revenues from selling a predetermined number of video games. The management team shares the revenues afterward. Moreover, the management team will start collecting the proceeds earlier if their cost synergies are greater, because an efficient management team can raise the likelihood of the project's success and increase the expected revenue for all.

1.1 Related Literature

This paper contributes to two strands of literature by examining the optimal contract for a continuous-time model with three-sided moral hazard and cost synergies among agents. We extend static multi-agent models with interaction among agents by considering dynamics; furthermore, we extend dynamic multi-agent models by introducing cost synergies.

Most research on the principal-agent problem with interactions among multiple agents focuses on the design of optimal incentive contracts in a static setting. For instance, Kandel and Lazear (1992) model peer pressure as a function of all agents' efforts. They show that the equilibrium effort with peer pressure is higher than it would be without peer pressure. The principal-agent model of Winter (2010) asserts that if technology is complementary across agents, the observability of agents' efforts increases the probability of the project's success. The more transparent each agent's effort is among peers, the less costly it is for the principal to provide incentives. Such peer effects are confirmed empirically by Falk and Ichino (2006) and Mas and Moretti (2009).

This paper differs from the above antecedents in three aspects. First, a dynamic model enables us to address the timing of agents' effort and rewards, which cannot be anticipated by static models. Second, we model the interaction among agents through cost synergies rather than peer effects; that is, one agent's effort reduces another agents' costs of effort. Third, the interaction modeled by cost synergies does not rely on the observability of agents' efforts, because the level of one agent's optimal effort depends only on his expectation of other agents' optimal efforts.

Edmans et al. (2013) introduce cost synergies in a static principal-agent model with multiple agents where one agent's effort reduces the others' marginal costs of effort. The synergy between a pair of agents is the sum of their influence parameters. They derive optimal contracts for the two- and three-agent cases, in which total payments are increasing in synergy, and the agent with a larger influence receives greater payments. For the three-agent model, if one synergy component is strictly larger than the sum of the other two, the model collapses into a two-agent model. The agent with the least influence makes no effort and receives no cash flow.

This paper differs from Edmans et al. (2013) in three aspects. First, we build a dynamic model, which reflects changes in agents' optimal effort and payment over time, such as deferred compensation. Second, Edmans et al. (2013) focus on the effect of cost synergies, fixing the moral hazard severity. We model how the interaction between moral hazard and cost synergies affects optimal efforts and payment allocation. Third, while Edmans et al. (2013) model synergies across agents in a more general form (i.e., the function of agents' influence), we model synergies more transparently: one agent's effort reduces the other agent's cost of effort.

A growing literature examines intertemporal incentive provisions in continuous-time models. For instance, Holmström and Milgrom (1987), Schättler and Sung (1993), and Sung (1995) obtain an optimal contract which is linear in output under different model settings. Dybvig and Lutz (1993) study a two-sided moral hazard problem in a product warranty context. One agent (a producer) exerts upfront effort that determines the durability of the product; the other agent (a consumer) exerts ongoing effort to maintain the product. Yang (2010) extends Dybvig and Lutz (1993) by introducing a third agent exerting effort over time. This three-sided model sheds light on a broader set of contracting problems involving multiple agents with different timelines for exerting effort and reaping rewards. However, these continuous-time models do not account for the interaction among agents. In contrast, this paper bridges the gap by modeling the interplay between moral hazard and cost synergies in a three-sided model. In the presence of cost synergies, one agent takes into account the other agent's optimal effort explicitly when determining his own effort level.

Another closely related paper is Georgiadis (2015), who develops a dynamic model that a group of agents collaborate to complete a project. A pay-off is generated only upon completion. Georgiadis (2015) discusses dynamic incentives and focuses on the optimal team size. In contrast, we model the timing of agents' effort and cost synergies across agents. This paper's payment scheme reflects the timing of efforts as well as cost synergies across agents, while the back-loading compensation scheme in Georgiadis (2015) applies to different situations in the real world.

The rest of the paper is organized as follows. In Section 2, we present a continuous-time model with two-sided moral hazard and cost synergies. We discuss how moral hazard and cost synergies affect incentives. In Section 3, we present a three-sided moral hazard model with cost synergies. We derive the optimal contracts for the symmetric and general cases in Sections 4 and 5, respectively. Section 6 concludes the paper. Appendix A includes all proofs.

2 Two-Sided Moral Hazard Model with Cost Synergies

To highlight the effects of cost synergies, moral hazard in teams, and their interaction on optimal contracting, we first study a continuous-time model with two-sided moral hazard and cost synergies.

2.1 The Model

A project spanning the period from time 0 to time T generates cash flow at a fixed rate b so long as two agents continue to manage it successfully. At the outset of the project, these agents agree on a compensation contract: Agent 1 receives $c_1(t)$, a measurable function from $[0, T]$ to $[0, b]$, and Agent 2 collects the remaining cash flows, $b - c_1(t)$. The agents divide the cash flow completely at each moment, and no agent receives more than the available total cash flow. This reflects the assumption that each agent is protected by limited liability and makes no additional investment at any moment during the project's life. This pay-as-you-go setting is consistent with real-world scenarios such as profit sharing among business partners.

Given the incentive contract above, Agents 1 and 2 exert ongoing effort $e_1(t)$ and $e_2(t)$ to maintain the business at moment t. $e_i(\cdot) : [0, T] \to [0, 1]$ of agent i (for $i = 1, 2$) is a measurable function; it is costly and unobservable to the other agent. The efforts of the two agents jointly affect the project's survival. The more efforts they exert, the higher the project's survival rate and the longer its expected lifespan.

A key and novel feature of this model is cost synergies between the agents. We model cost synergies directly in the cost functions; that is, one agent's effort reduces the other's cost of effort. The cost functions of Agents 1 and 2 are defined by $C_1 \equiv k_1(e_1(t) - \varepsilon_{21}e_2(t))^2$ and $C_2 \equiv k_2(e_2(t) - \varepsilon_{12}e_1(t))^2$, respectively.[1] The positive constant k_i (for $i = 1, 2$) represents the unit cost of effort for agent i. The term ε_{21} captures the *influence* of Agent 2's effort on Agent 1's cost of effort; that is, Agent 2's effort reduces Agent 1's cost with an influence factor of ε_{21}. Similarly, ε_{12} is the *influence* of Agent 1's effort on Agent 2's cost. To study the effect of cost saving, we assume that ε_{21}, $\varepsilon_{12} \geq 0$.[2]

For tractability, we assume a simple information structure: the only uncertainty is the timing of project failure. The two agents' efforts jointly determine the project's survival. For convenience, we model the failure rather than the survival of the project. The absolute failure rate (probability density of the failure time) of the project at time $t \in [0, T]$ is given by

$$f(t; e_1(\cdot), e_2(\cdot), m_1, m_2) = m_1 \int_{\tau=0}^{t} (1 - e_1(\tau))\, d\tau + m_2 \int_{\tau=0}^{t} (1 - e_2(\tau)) d\tau,$$

where $m_1(1 - e_1(\tau))$ and $m_2(1 - e_2(\tau))$ are the effects of reduced effort from Agents 1 and 2 on the failure rate, respectively. Such shirking increases the probability of failure by destroying items related to the productivity of the project. Therefore, the probability that the project fails at or before time t, $F(t; e_1(\cdot), e_2(\cdot), m_1, m_2)$ (referred to as $F(t)$ hereafter),

[1] The cost of each agent is convex in his own effort. When the incentive compatibility constraints are satisfied, the quadratic cost functions of Agents 1 and 2 are monotonically increasing in $e_1(\cdot)$ and $e_2(\cdot)$, respectively. Moreover, $e_1(t) - \varepsilon_{21}e_2(t) \geq 0$ and $e_2(t) - \varepsilon_{12}e_1(t) \geq 0$, see (4) and (5). This setting is implicitly adopted in the literature on social and economic networks to capture influences between agents; see, for example, Ballester et al. (2006).

[2] For negative influence parameters, Lazear (1989) models a related concept as "sabotage".

is

$$F(t) = m_1 \int_{s=0}^{t} \int_{\tau=0}^{s} (1 - e_1(\tau)) d\tau ds + m_2 \int_{s=0}^{t} \int_{\tau=0}^{s} (1 - e_2(\tau)) d\tau ds. \tag{1}$$

The expected social surplus, Π, is the expected cash flows net of costs of effort:

$$\Pi = b \int_{t=0}^{T} (1 - F(t)) dt - k_1 \int_{t=0}^{T} (e_1(t) - \varepsilon_{21} e_2(t))^2 dt - k_2 \int_{t=0}^{T} (e_2(t) - \varepsilon_{12} e_1(t))^2 dt. \tag{2}$$

The term $b \int_0^T (1 - F(t)) dt$ represents the expected cash flows of the project over its life, since $1 - F(t)$ is the project's survival probability at time t. The maximization of the expected social surplus satisfies the *incentive compatibility* (IC) constraint of each agent.

Assuming that Agents 1 and 2 have linear utilities above zero and are indifferent to receiving a dollar either today or tomorrow, we express their utilities as the expected payments net of costs of effort:

$$\Pi_1 = \int_{t=0}^{T} c_1(t)(1 - F(t)) dt - k_1 \int_{t=0}^{T} (e_1(t) - \varepsilon_{21} e_2(t))^2 dt,$$
$$\Pi_2 = \int_{t=0}^{T} (b - c_1(t))(1 - F(t)) dt - k_2 \int_{t=0}^{T} (e_2(t) - \varepsilon_{12} e_1(t))^2 dt,$$

where the first term is the expected payments and the second one is the expected costs. The optimal contracting problem aims to maximize the expected social surplus Π subject to IC constraints. We derive the optimal efforts of the two agents using the first-order approach because the agents' optimization problem is convex.

Changing the order of integration, we have the following fact:

$$\int_{t=0}^{T} f(t) \int_{s=0}^{t} \int_{\tau=0}^{s} g(\tau) d\tau ds dt = \int_{\tau=0}^{T} g(\tau) \int_{s=\tau}^{T} \int_{t=s}^{T} f(t) dt ds d\tau. \tag{3}$$

Substituting the probability of failure $F(t)$ in (1) and applying (3) to $\int_{t=0}^{T} c_1(t)(1 - F(t)) dt$, we rewrite Agent 1's utility as

$$\Pi_1 = m_1 \int_{t=0}^{T} e_1(t) dt \int_{s=t}^{T} \int_{\tau=s}^{T} c_1(\tau) d\tau ds - k_1 \int_{t=0}^{T} (e_1(t) - \varepsilon_{21} e_2(t))^2 dt - \pi_1,$$

where π_1 is independent of $e_1(\cdot)$.[3] Maximizing Π_1 over $e_1(\cdot)$ pointwise yields the optimal effort for Agent 1 as

$$e_1(t) = \frac{m_1}{2k_1} \int_{s=t}^{T} \int_{\tau=s}^{T} c_1(\tau) d\tau ds + \varepsilon_{21} e_2(t). \tag{4}$$

[3] $\pi_1 = \int_{t=0}^{T} c_1(t) \Big(1 - m_1 t^2/2 - m_2 \int_{s=0}^{t} \int_{\tau=0}^{s} (1 - e_2(\tau)) d\tau ds\Big) dt.$

Similarly, the optimal effort for Agent 2 is

$$e_2(t) = \frac{m_2}{2k_2} \int_{s=t}^{T} \int_{\tau=s}^{T} (b - c_1(\tau)) d\tau ds + \varepsilon_{12} e_1(t). \tag{5}$$

The optimal contract solves the following reduced-form optimization Problem 1.

Problem 1. *Choose the payment $0 \leq c_1(t) \leq b$ to Agent 1 to maximize the expected social surplus given in (2), subject to IC constraints given in (4) and (5), where the cumulative failure rate at time t, $F(t)$, is given in (1).*

We now explain how cost synergies affect the optimal equilibrium efforts before solving Problem 1. The equilibrium efforts for Agents 1 and 2, derived from the IC constraints in (4) and (5), can be rewritten as follows:

$$e_1(t) = \varepsilon \left(\frac{m_1}{2k_1} \int_{s=t}^{T} \int_{\tau=s}^{T} c_1(\tau) d\tau ds + \varepsilon_{21} \frac{m_2}{2k_2} \int_{s=t}^{T} \int_{\tau=s}^{T} (b - c_1(\tau)) d\tau ds \right),$$
$$e_2(t) = \varepsilon \left(\frac{m_2}{2k_2} \int_{s=t}^{T} \int_{\tau=s}^{T} (b - c_1(\tau)) d\tau ds + \varepsilon_{12} \frac{m_1}{2k_1} \int_{s=t}^{T} \int_{\tau=s}^{T} c_1(\tau) d\tau ds \right),$$

where $\varepsilon \equiv (1 - \varepsilon_{12}\varepsilon_{21})^{-1} \geq 1$ is an amplification factor. In equilibrium, these optimal efforts form a *feedback* or *echo* system, which is similar to Edmans et al. (2013). The optimal efforts are amplified by cost synergies in two ways. First, the terms $c_1(t)$ and $b - c_1(t)$ are nonnegative. The second terms in the parentheses are thus positive. Therefore, an agent's influence directly increases the other agent's optimal effort. Second, the optimal effort can be further increased by the amplification factor $\varepsilon \geq 1$.

For expositional simplicity, we introduce two notations before presenting the optimal contract. Let $s_i = \frac{m_i^2}{4k_i}$ be the severity of agent i's moral hazard, and $\mathcal{R}_{ij} = \frac{m_i/k_i}{m_j/k_j}$ be the relative moral hazard of agent i to agent j, where $i, j = 1, 2$ and $i \neq j$. $\mathcal{R}_{12}$ and $\mathcal{R}_{21}$ are the reciprocals of each other, i.e., $\mathcal{R}_{12}\mathcal{R}_{21} = 1$.

2.2 The Optimal Contract

We will show that in the optimal contract, two agents split the constant cash flow proportionally at all times. The moral hazard and cost synergies jointly determine each agent's share. Specifically, the payment to an agent is increasing in the agent's influence on cost savings. An agent will claim all cash flows if his influence is larger than the moral hazard of the other agent relative to his moral hazard (i.e., $\varepsilon_{12} > \mathcal{R}_{21}$). Moreover, compared to the case without cost synergies, synergies not only reallocate cash flows between the two agents but also improve the expected social surplus.

We summarize technical conditions in Assumption 1 and the optimal contract in Proposition 1:

Assumption 1. *Parameters in* Problem 1 *satisfy the following conditions.*

(i) $\frac{1}{2}(m_1 + m_2)T^2 \leq 1$*;*

(ii) $\frac{m_1}{4k_1} bT^2 (1 + \varepsilon_{21}/\mathcal{R}_{12}) \leq 1 - \varepsilon_{12}\varepsilon_{21}$ *and* $\frac{m_2}{4k_2} bT^2 (1 + \varepsilon_{12}/\mathcal{R}_{21}) \leq 1 - \varepsilon_{12}\varepsilon_{21}$*;*

(iii) $\varepsilon_{12}, \varepsilon_{21} \geq 0$ *and* $\varepsilon_{12}\varepsilon_{21} < 1$.

Assumption 1(i) ensures that the probability of failure $0 \leq F(t) \leq 1$. Assumption 1(ii) implies that $0 \leq e_i \leq 1$, for $i = 1, 2$. Assumption 1(iii) ensures that $\varepsilon_{12}\varepsilon_{21} < 1$, because both m_i and k_i (for $i = 1, 2$) are positive constants. Non-negative influence parameters in Assumption 1(iii) reflect cost synergies between the two agents.

Proposition 1 (Proportional Sharing)**.** *Under* Assumption 1*, two agents share the project's cash flows proportionally. Agent 1 collects* $c_1^*(t)$ *and Agent 2 collects the remainder,* $b - c_1^*(t)$*. The payment to Agent 1,* $c_1^*(t)$ *for* $t \in [0, T]$*, is given by*

$$c_1^*(t) = \begin{cases} b, & if\ \varepsilon_{12} \geq \mathcal{R}_{21} \equiv \frac{m_2/k_2}{m_1/k_1}; \\ 0, & if\ \varepsilon_{21} \geq \mathcal{R}_{12} \equiv \frac{m_1/k_1}{m_2/k_2}; \\ c_1(t), & all\ other\ cases; \end{cases} \tag{6}$$

where $c_1(t)$ *is defined as:*

$$c_1(t) = \frac{s_1}{s_1 + s_2} b\varepsilon \left(1 + \varepsilon_{12}\frac{m_2}{m_1}\right)\left(1 - \frac{\varepsilon_{21}}{\mathcal{R}_{12}}\right). \tag{7}$$

Proof. Let $x(t) \equiv \int_{s=t}^{T} \int_{\tau=s}^{T} c_1(\tau) d\tau ds$ be a state variable. Substituting it into the expected social surplus in (2) and using the integration by parts in formula (3), we have

$$\Pi = \pi_0 + s_1 b\varepsilon \left(1 + \varepsilon_{12}\frac{m_2}{m_1}\right)\left(1 - \frac{\varepsilon_{21}}{\mathcal{R}_{12}}\right) \int_{t=0}^{T} x(t)(T - t)^2 dt - (s_1 + s_2) \int_{t=0}^{T} x^2(t) dt, \tag{8}$$

where π_0 is a constant independent of the state variable $x(\cdot)$.[4] Maximizing Π with respect to $x(\cdot)$ pointwise, we obtain the optimal state as

$$x^*(t) = \int_t^T \int_s^T \frac{s_1}{s_1+s_2} b\varepsilon \left(1+\varepsilon_{12}\frac{m_2}{m_1}\right)\left(1-\frac{\varepsilon_{21}}{\mathcal{R}_{12}}\right) d\tau ds.$$

Given that $x(t) = \int_{s=t}^T \int_{\tau=s}^T c_1(\tau) d\tau ds$, the optimal payment $c_1(\cdot)$ is given by (7). Moreover, we have

$$b - c_1(t) = \frac{s_2}{s_1+s_2} b\varepsilon \left(1+\varepsilon_{21}\frac{m_1}{m_2}\right)\left(1-\frac{\varepsilon_{12}}{\mathcal{R}_{21}}\right). \tag{9}$$

Recall that $0 \leq c_1(t), b - c_1(t) \leq b$. Then, from (7) and (9), we obtain the optimal payment for Agent 1 as given in (6). □

Note in the model without cost synergies (that is, $\varepsilon_{12} = \varepsilon_{21} = 0$), the agent with more severe moral hazard receives a larger fraction of the cash flows ($s_i/(s_1+s_2)$ for $i = 1, 2$). When cost synergies exist, the split is jointly determined by moral hazard and cost synergies.[5]

Corollary 2.1 below summarizes how agents' influences affect the allocation of cash flows. Taking derivatives on both sides of (7), we show that one agent's payment is increasing in his own influence and decreasing in the other agent's influence.

Corollary 2.1. *$c_1(\cdot)$ defined in (7) is monotonically increasing in ε_{12} and decreasing in ε_{21}.*

Substituting the optimal payment for Agent 1 in (6) and (9) into (4) and (5), we have the optimal efforts for Agents 1 and 2 in Corollary 2.2 below.

Corollary 2.2. *Suppose* Assumption 1 *holds. The optimal efforts for Agents 1 and 2 are (i) $e_1^*(t) = \frac{m_1}{4k_1}\varepsilon b(T-t)^2$ and $e_2^*(t) = \varepsilon_{12} e_1^*(t)$ for $\varepsilon_{12} \geq \mathcal{R}_{21}$; (ii) $e_1^*(t) = \varepsilon_{21} e_2^*(t)$ and*

[4] $\pi_0 = bT - (m_1+m_2)T^3/6 - s_2 b^2 T^5/20 + \varepsilon\,(1+\varepsilon_{21}m_1/m_2)\,(s_2 b^2 T^5)/10$.

[5] Agent 1 (Agent 2) collects all the cash flows if $\varepsilon_{12} > \mathcal{R}_{21}$ ($\varepsilon_{21} > \mathcal{R}_{12}$). Moreover, $\varepsilon_{12} > \mathcal{R}_{21}$ implies that $\varepsilon_{21} < \mathcal{R}_{12}$, and vice versa. From Assumption 1(iii) and definitions of $\mathcal{R}_{12}$ and $\mathcal{R}_{21}$, $\varepsilon_{12}\varepsilon_{21} < 1 \equiv \mathcal{R}_{12}\mathcal{R}_{21}$. Dividing by ε_{12} on both sides of the inequality, we can see the result immediately.

$e_2^*(t) = \frac{m_2}{4k_2}\varepsilon b(T-t)^2$ *for* $\varepsilon_{21} \geq \mathcal{R}_{12}$*; and (iii) for all other cases, we have*

$$e_1^*(t) = \frac{m_1(T-t)^2}{4k_1} b\varepsilon \left(\frac{\varepsilon_{21}}{\mathcal{R}_{12}} + \frac{s_1}{s_1+s_2}\varepsilon \left(1+\varepsilon_{12}\frac{m_2}{m_1}\right)\left(1-\frac{\varepsilon_{21}}{\mathcal{R}_{12}}\right)^2 \right),$$

$$e_2^*(t) = \frac{m_2(T-t)^2}{4k_2} b\varepsilon \left(\frac{\varepsilon_{12}}{\mathcal{R}_{21}} + \frac{s_2}{s_1+s_2}\varepsilon \left(1+\varepsilon_{21}\frac{m_1}{m_2}\right)\left(1-\frac{\varepsilon_{12}}{\mathcal{R}_{21}}\right)^2 \right).$$

Proof. The optimal payment for Agent 1 is a constant c_1^*. By (4) and (5), we have

$$e_1^*(t) = \frac{m_1(T-t)^2}{4k_1}\varepsilon\left(c_1^* + (b-c_1^*)\frac{\varepsilon_{21}}{\mathcal{R}_{12}}\right), \; e_2^*(t) = \frac{m_2(T-t)^2}{4k_2}\varepsilon\left((b-c_1^*) + c_1^*\frac{\varepsilon_{12}}{\mathcal{R}_{21}}\right).$$

Thus, cases (i) and (ii) are straightforward by noting the optimal payment for Agent 1 in (6). To derive $e_1^*(t)$ given in (iii), we substitute (7) into the above formula while noting that $c_1^* + (b-c_1^*)\varepsilon_{21}/\mathcal{R}_{12} = c_1^*(1-\varepsilon_{21}/\mathcal{R}_{12}) + b\varepsilon_{21}/\mathcal{R}_{12}$. We can derive $e_2^*(t)$ similarly substituting (9) into the above formula of $e_2^*(t)$. □

Agent 1 does not observe the effort of Agent 2 but correctly expects it in equilibrium. An interesting phenomenon is that if Agent 1 receives no payment, all Agent 1's effort comes from Agent 2, whose effort reduces the cost of Agent 1's effort to zero in equilibrium. Agent 1 serves as an important channel amplifying Agent 2's effort.

Moreover, we show that cost synergies improve the expected social surplus.

Corollary 2.3. *Suppose* Assumption 1 *holds. If one influence factor is strictly positive; that is, if* $\varepsilon_{12} > 0$ *or* $\varepsilon_{21} > 0$*, the expected social surplus is larger than that in the case without cost synergies.*

Proof. Denote $\Pi(\varepsilon_{12}, \varepsilon_{21})$ as the expected social surplus in equilibrium. Completing a square in (8), we have

$$\Pi = \pi_2 + \frac{b^2T^5}{10}s_2\varepsilon\left(1+\varepsilon_{21}\frac{m_1}{m_2}\right) - (s_1+s_2)\int_{t=0}^{T}\left(x(t) - \frac{c_1(t)(T-t)^2}{2}\right)^2 dt + (s_1+s_2)\int_{t=0}^{T} c_1^2(t)\frac{(T-t)^4}{4}dt,$$

where $\pi_2 = bT - (m_1+m_2)T^3/6 - s_2b^2T^5/20$ and $c_1(t)$ is given by (7). Then, the expected social surplus in equilibrium is

$$\Pi(\varepsilon_{12}, \varepsilon_{21}) = \pi_2 + \frac{b^2T^5}{10}\left(s_2\varepsilon\left(1+\varepsilon_{21}\frac{m_1}{m_2}\right) + \frac{s_1^2\varepsilon^2}{2(s_1+s_2)}\left(1+\varepsilon_{12}\frac{m_2}{m_1}\right)^2\left(1-\frac{\varepsilon_{21}}{\mathcal{R}_{12}}\right)^2\right).$$

Taking derivatives on both sides, we have $\frac{\partial \Pi(\varepsilon_{12},\varepsilon_{21})}{\partial \varepsilon_{12}} > 0$ for all $\varepsilon_{12} \geq 0$ and $\varepsilon_{21} \geq 0$.

Similarly, we could rewrite the social surplus as

$$\Pi(\varepsilon_{12},\varepsilon_{21}) = \pi_1 + \frac{b^2T^5}{10}\left(s_1\varepsilon\left(1+\varepsilon_{12}\frac{m_2}{m_1}\right) + \frac{s_2^2\varepsilon^2}{2(s_1+s_2)}\left(1+\varepsilon_{21}\frac{m_1}{m_2}\right)^2\left(1-\frac{\varepsilon_{12}}{\mathcal{R}_{21}}\right)^2\right),$$

where $\pi_1 = bT - (m_1+m_2)T^3/6 - s_1b^2T^5/20$. So we have $\frac{\Pi(\varepsilon_{12},\varepsilon_{21})}{\partial \varepsilon_{21}} > 0$ for all $\varepsilon_{12} \geq 0$ and $\varepsilon_{21} \geq 0$. □

Thus, cost synergies not only adjust cash flow allocations, but also improve efficiency. In the presence of cost synergies, allocating more cash flows to the more influential agent mitigates the efficiency loss due to moral hazard. Figure 1 provides a numerical example that the expected social surplus is monotonically increasing in the influence factor ε_{12}, given that other parameters are constants.

[Figure 1 about here.]

To focus on the effect of cost synergies, we next examine a special case in which the two agents have the same severity of moral hazard; that is, $m_1^2/k_1 = m_2^2/k_2$ ($s_1 = s_2$). Combining this assumption with the payment for Agent 1 in (6), we have the following corollary.

Corollary 2.4. *Suppose* Assumption 1 *holds and* $m_1^2/k_1 = m_2^2/k_2$. *The optimal payment for Agent 1 is (i)* b*, if* $\varepsilon_{12} \geq \mathcal{R}_{21}$*; (ii)* 0*, if* $\varepsilon_{21} \geq \mathcal{R}_{12}$*; (iii)* $\frac{b}{2} + \frac{b}{2}\varepsilon(\frac{\varepsilon_{12}}{\mathcal{R}_{21}} - \frac{\varepsilon_{21}}{\mathcal{R}_{12}})$*, all other cases. Agent 2 collects the remaining cash flows,* $b - c_1^*(t)$ *for* $t \in [0,T]$.

In this symmetric case, two agents still divide the cash flows proportionally. When the agents' moral hazard is equally severe, the agent with a relatively large influence (e.g., $\varepsilon_{12}/\varepsilon_{21} > k_1/k_2$) receives more payments. In the extreme case, when one agent's influence substantially exceeds the other's, the more influential agent collects all cash flows. The less influential agent exerts positive effort but incurs zero cost due to cost synergies.

3 Three-Sided Moral Hazard Model with Cost Synergies

This section presents the basic analysis of a continuous-time model with three-sided moral hazard and cost synergies. We introduce a new agent, Agent 0, who exerts effort at the outset to establish the project. Agents 1 and 2 exert ongoing efforts to maintain the project and generate cash flows. Different timing of Agent 0's effort enriches the dynamic model and expands its application to a broader set of real-world problems.

The model parameters are set as follows. Three agents engage in a project as a team. As in the two-sided moral hazard model, the project spans the period from time 0 to time T. The project generates cash flow at a fixed rate b during its lifespan. At the outset, three agents sign an incentive contract dividing the project's entire cash flows. The payment for agent i (for $i = 1, 2$) is a measurable function $c_i(\cdot) : [0, T] \to [0, b]$. Agent 0 receives the residual $(b - c_1(t) - c_2(t))$ at each time t. Payments must be nonnegative and the budget must be balanced, i.e., $c_1(t) \geq 0$, $c_2(t) \geq 0$, and $c_1(t) + c_2(t) \leq b$.

Agent 0's effort is $e_0 \in [0, \phi]$, where $\phi > 0$ is the maximum level of effort. The effort for agent i (for $i = 1, 2$) is a measurable function $e_i : [0, T] \to [0, 1]$. The effort of each agent is costly and unobservable to other agents. Agent 0's cost is defined as

$$C_0 \equiv \gamma e_0^2,$$

where γ (a positive constant) is the unit cost of effort.

The central feature of this model is that the cost of effort for Agent 1 depends not only on his own effort but also on the effort of Agent 2, and vice versa. The cost functions for Agents 1 and 2 are

$$C_1 \equiv k_1(e_1(t) - \varepsilon_{21} e_2(t))^2, \quad C_2 \equiv k_2(e_2(t) - \varepsilon_{12} e_1(t))^2,$$

respectively, where influence factors $\varepsilon_{12} \geq 0$ and $\varepsilon_{21} \geq 0$ reflect cost synergies.

The absolute failure rate of the project at time $t \in [0, T]$ is

$$f(t; e_0, e_1(\cdot), e_2(\cdot); \phi, m_1, m_2) = (\phi - e_0) + m_1 \int_{\tau=0}^{t} (1 - e_1(\tau)) d\tau + m_2 \int_{\tau=0}^{t} (1 - e_2(\tau)) d\tau,$$

where m_1 and m_2 are positive constants representing each agent's moral hazard. Three sources of moral hazard are additive. Agent 0's effort has a constant impact $\phi - e_0$ on the failure rate throughout the life of the project. The impact of agent i (for $i = 1, 2$) on the failure rate accumulates over time, i.e., $\int_0^t m_i(1 - e_i(\tau)) d\tau$. Therefore, $F(t)$, the probability of failure before or at time t, (the dependence on e_0, $e_1(\cdot)$, $e_2(\cdot)$, ϕ, m_1, m_2 is omitted for simplicity), is given by

$$F(t) = (\phi - e_0)t + m_1 \int_{s=0}^{t} \int_{\tau=0}^{s} (1 - e_1(\tau))d\tau ds + m_2 \int_{s=0}^{t} \int_{\tau=0}^{s} (1 - e_2(\tau))d\tau ds, \tag{10}$$

where $\phi T + \frac{1}{2}(m_1 + m_2)T^2 \leq 1$ to ensure $F(t) \leq 1$ for all effort levels and each $t \in [0, T]$.

The three agents form a three-sided moral hazard problem – maximizing the expected social surplus of the project, which is the expected cash flows net of the total costs of effort:

$$\Pi = \int_{t=0}^{T} b(1 - F(t))dt - \gamma e_0^2 - k_1 \int_{t=0}^{T} (e_1(t) - \varepsilon_{21} e_2(t))^2 dt \; - k_2 \int_{t=0}^{T} (e_2(t) - \varepsilon_{12} e_1(t))^2 dt. \tag{11}$$

The maximization of the expected social surplus is subject to each agent's IC constraint; that is, each agent chooses effort to maximize his own utility, the expected payment net of the cost of effort. The utilities of Agent 0, Agent 1, and Agent 2 are:

$$\begin{aligned}
\Pi_0 &= \int_{t=0}^{T} (b - c_1(t) - c_2(t))(1 - F(t))dt - \gamma e_0^2, \\
\Pi_1 &= \int_{t=0}^{T} c_1(t)(1 - F(t))dt - k_1 \int_{t=0}^{T} (e_1(t) - \varepsilon_{21} e_2(t))^2 dt, \\
\Pi_2 &= \int_{t=0}^{T} c_2(t)(1 - F(t))dt - k_2 \int_{t=0}^{T} (e_2(t) - \varepsilon_{12} e_1(t))^2 dt.
\end{aligned}$$

By changing the order of integration, we replace agents' IC constraints with the first-order conditions in the following lemma.

Lemma 3.1 (IC Constraints)**.** *The optimal efforts of Agents 0, 1, and 2 satisfy the following IC constraints:*

$$e_0 = \frac{1}{2\gamma} \int_{t=0}^{T} \int_{\tau=t}^{T} (b - c_1(\tau) - c_2(\tau))d\tau dt, \tag{12}$$

$$e_1(t) = \frac{m_1}{2k_1} \int_{s=t}^{T} \int_{\tau=s}^{T} c_1(\tau)d\tau ds + \varepsilon_{21} e_2(t), \tag{13}$$

$$e_2(t) = \frac{m_2}{2k_2} \int_{s=t}^{T} \int_{\tau=s}^{T} c_2(\tau)d\tau ds + \varepsilon_{12} e_1(t). \tag{14}$$

Proof of Lemma 3.1. Changing the order of integration, we obtain

$$\int_{t=0}^{T} (b - c_1(t) - c_2(t)) \int_{s=0}^{t} ds dt = \int_{t=0}^{T} \int_{\tau=t}^{T} (b - c_1(\tau) - c_2(\tau))d\tau dt.$$

Therefore, Agent 0's utility is reduced into

$$\Pi_0 = e_0 \int_{t=0}^{T} \int_{\tau=t}^{T} (b - c_1(\tau) - c_2(\tau)) d\tau dt - \gamma e_0^2 + \pi_0,$$

where π_0 is independent of e_0.[6] Maximizing Π_0 over e_0 pointwise, we obtain the optimal effort for Agent 0 in (12).

Substituting (10) into $\int_{t=0}^{T} c_1(t)(1 - F(t))dt$ and changing the order of integration using (3), we rewrite Agent 1's utility as

$$\Pi_1 = m_1 \int_{t=0}^{T} e_1(t) \int_{s=t}^{T} \int_{\tau=s}^{T} c_1(\tau) d\tau ds dt - k_1 \int_{t=0}^{T} (e_1(t) - \varepsilon_{21} e_2(t))^2 dt + \pi_1,$$

where π_1 is independent of $e_1(\cdot)$.[7] Maximizing Π_1 over $e_1(\cdot)$ pointwise yields Agent 1's optimal effort in (13). Similarly, Agent 2's optimal effort is given in (14). □

The dynamic contracting problem is now reduced into an optimization problem, which is summarized below in Problem 2.

Problem 2. *Choose payments $c_1(t)$ and $c_2(t)$ (satisfying $0 \le c_1(t) + c_2(t) \le b$ and $c_i(t) \ge 0$, $i = 1, 2$) to maximize the expected social surplus given in (11), subject to the IC constraints given in (12), (13), and (14), where the cumulative failure rate at time t, $F(t)$, is given in (10).*

Similar to the two-sided model, we first present intuitions on how cost synergies work. Rewriting the equilibrium efforts of Agents 1 and 2 in (13) and (14) leads to the following new conditions:

$$e_1(t) = \frac{m_1}{2k_1} \varepsilon \left(\int_{s=t}^{T} \int_{\tau=s}^{T} c_1(\tau) d\tau ds + \varepsilon_{21} \mathcal{R}_{21} \int_{s=t}^{T} \int_{\tau=s}^{T} c_2(\tau) d\tau ds \right), \tag{15}$$

$$e_2(t) = \frac{m_2}{2k_2} \varepsilon \left(\int_{s=t}^{T} \int_{\tau=s}^{T} c_2(\tau) d\tau ds + \varepsilon_{12} R_{12} \int_{s=t}^{T} \int_{\tau=s}^{T} c_1(\tau) d\tau ds \right). \tag{16}$$

This is a *feedback* or *echo* system. Given that Agents 1 and 2 at most split the cash flow (i.e., $0 \le c_1(t) + c_2(t) \le b$), cost synergies increase the agents' optimal efforts by the non-negative second term in the parentheses in (15) and (16), and by the amplification effect of the feedback system via $\varepsilon = (1 - \varepsilon_{12}\varepsilon_{21})^{-1} \ge 1$.

We summarize technical conditions on model parameters in Assumption 2 below.

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[6] $\pi_0 \equiv \int_{t=0}^{T} (b - c_1(t) - c_2(t)) \left(1 - \phi t - \int_{s=0}^{t} \int_{\tau=0}^{s} (m_1 - m_1 e_1(\tau) + m_2 - m_2 e_2(\tau)) d\tau ds\right) dt.$

[7] $\pi_1 \equiv \int_{t=0}^{T} c_1(t) \left(1 - (\phi - e_0)t - m_1 t^2/2 - m_2 \int_{s=0}^{t} \int_{\tau=0}^{s} (1 - e_2(\tau)) d\tau ds\right) dt.$

Assumption 2. *Parameters in* Problem 2 *satisfy the following conditions:*

(i) $\phi T + \frac{1}{2}(m_1 + m_2)T^2 \leq 1$*;*

(ii) $\frac{bT^2}{4\gamma} < \phi$*;*

(iii) $\frac{m_1}{4k_1} bT^2 (1 + \varepsilon_{21}/\mathcal{R}_{12}) \leq 1 - \varepsilon_{12}\varepsilon_{21}$ and $\frac{m_2}{4k_2} bT^2 (1 + \varepsilon_{12}/\mathcal{R}_{21}) \leq 1 - \varepsilon_{12}\varepsilon_{21}$*;*

(iv) $\varepsilon_{12}, \varepsilon_{21} \geq 0$ and $\varepsilon_{12}\varepsilon_{21} < 1$*.*

Assumption 2(i) guarantees that the probability of failure $F(t) \leq 1$. Assumptions 2(ii) and (iii) ensure that $e_0 \leq \phi$ and $e_i \leq 1$ for $i = 1, 2$. Because m_i and k_i (for $i = 1, 2$) are positive constants, Assumption 2(iii) implies that the amplification factor is greater than one, i.e., $\varepsilon = (1 - \varepsilon_{12}\varepsilon_{21})^{-1} > 1$. For clarity, we make this assumption explicitly in Assumption 2(iv). Non-negative influence parameters in Assumption 2(iv) reflect cost synergies between Agents 1 and 2.

In the next two sections, we discuss the solutions to Problem 2, i.e., the optimal payment schemes under different scenarios.

4 The Optimal Contract for the Symmetric Case

This section provides an optimal contract for Problem 2 when Agents 1 and 2 are symmetric: the ratio of Agent 1's influence to Agent 2's influence equals the ratio of Agent 1's unit cost of effort to Agent 2's unit cost of effort, and both agents have the same severity of moral hazard, i.e., $\varepsilon_{12}/\varepsilon_{21} = m_1^2/m_2^2 = k_1/k_2$. Under the optimal contract, Agent 0, who exerts upfront effort, claims all cash flows until a predetermined cutoff date, after which Agents 1 and 2 share the cash flows equally. The larger the cost synergies, the earlier Agents 1 and 2 start receiving the cash flows. Moreover, in the presence of cost synergies, the expected social surplus is larger than that in the model without cost synergies.

Proposition 2. *Suppose* Assumption 2 *holds and* $\varepsilon_{12}/\varepsilon_{21} = m_1^2/m_2^2 = k_1/k_2$.[8] *In the*

[8]We use the condition $\varepsilon_{12}/\varepsilon_{21} = m_1^2/m_2^2 = k_1/k_2$ for expositional simplicity. In Appendix A, we analyze Problem 2 and prove Proposition 2 under a weaker condition $\varepsilon_{12}m_2/m_1 = \varepsilon_{21}m_1/m_2$ (including the case $\varepsilon_{12} = \varepsilon_{21} = 0$) and $s_1 = s_2$. Note that $m_1^2/m_2^2 = k_1/k_2$ implies $s_1 = s_2$.

optimal contract, there exists a cutoff date t_c *in* $(0, T)$ *determined by*

$$\frac{m_1 + \varepsilon_{12} m_2}{m_1 - \varepsilon_{12} m_2} \frac{t_c^3 - 4T t_c^2 + 6T^2 t_c}{6} + \frac{t_c^3}{2} - \frac{T^2 - t_c^2}{s_1 \gamma} = 0, \tag{17}$$

such that (i) Agent 0 claims all cash flows prior to t_c*, namely,* $c_1^*(t) = c_2^*(t) = 0$ *for* $t < t_c$*; and (ii) Agents 1 and 2 share the cash flows equally after* t_c*, namely,* $c_1^*(t) = c_2^*(t) = \frac{b}{2}$ *for* $t \geq t_c$.

Proof. See Appendix A. □

Note that the cutoff date t_c determined by equation (17) is an implicit function of the influence of Agent 2, i.e., ε_{21}.[9] In the absence of cost synergies, the optimal payment scheme is the same as that of Yang (2010). This can be seen by letting $\varepsilon_{12} = \varepsilon_{21} = 0$ in (17).

Taking the total derivative on both sides of (17) with respect to ε_{12}, ε_{21} and $1/(s_1\gamma)$, respectively, we have the comparative statistics below in Corollary 4.1.

Corollary 4.1. *The cutoff date* t_c *is monotonically decreasing in* ε_{12} *and* ε_{21}*, and increasing in* $1/(s_1\gamma)$.

Proof. See Appendix A.3. □

Greater cost synergies between agents 1 and 2 increase the period over which the two agents split the cash flows of the project (a smaller t_c). Specifically, Agents 1 and 2 receive larger payments than in the case without cost synergies, while Agent 0 receives a smaller payment. Recall that $1/\gamma$, s_1, and s_2 represent the moral hazard severity of Agents 0, 1, and 2, respectively. A larger $1/(s_1\gamma)$ indicates that Agent 0's moral hazard is more severe relative to that of Agents 1 and 2. Hence, Agent 0 receives more payments (a larger t_c) to improve incentives.

Substituting the optimal payment scheme given in Proposition 2 into the IC constraints in Lemma 3.1, we have the optimal efforts summarized in the following corollary:

[9]It can be seen from that $(m_1 + \varepsilon_{12} m_2)/(m_1 - \varepsilon_{12} m_2) = (m_2 + \varepsilon_{21} m_1)/(m_2 - \varepsilon_{21} m_1)$, which is implied by $\varepsilon_{12}/\varepsilon_{21} = m_1^2/m_2^2$.

Corollary 4.2. *If the conditions in* Proposition 2 *hold, the optimal efforts are*

$$e_0^* = \frac{bt_c^2}{4\gamma},$$

$$e_1^*(t) = \frac{m_1}{m_1 - \varepsilon_{12} m_2} \frac{bm_1}{8k_1} \left((T-t)^2 - (t_c - t)^2 1_{[0,t_c)}(t) \right), \tag{18}$$

$$e_2^*(t) = \frac{m_2}{m_2 - \varepsilon_{21} m_1} \frac{bm_2}{8k_2} \left((T-t)^2 - (t_c - t)^2 1_{[0,t_c)}(t) \right). \tag{19}$$

Proof. See Appendix A.3. □

Although all three agents exert suboptimal effort, Agents 1 and 2 exert greater efforts (cf. (18) and (19)) than in the case without cost synergies, due to the amplification factors $m_1/(m_1 - \varepsilon_{12} m_2) > 1$, $m_2/(m_2 - \varepsilon_{21} m_1) > 1$ and a smaller cutoff date t_c (cf. Corollary 4.1).[10] In contrast, Agent 0 exerts less effort than in the case without synergies, because he claims all cash flows for a shorter period of time (a smaller t_c).

The corollary below compares the expected social surplus and the payment scheme with those in the model without cost synergies.

Corollary 4.3. *Suppose the conditions in* Proposition 2 *hold. If one influence factor is positive, then the expected social surplus is larger than that in the case without cost synergies.*

Proof. See Appendix A.3. □

Without cost synergies, the efficiency loss is increasing in the severity of the agents' moral hazard (p.1582, Yang, 2010). Cost synergies mitigates the efficiency loss due to moral hazard and improve the expected social surplus. Figure 2 provides an example that the expected social surplus is monotonically increasing in ε_{12} (equivalently, ε_{21}) in this symmetric case.

[Figure 2 about here.]

5 Optimal Contracts for General Cases

This section provides optimal contracts for Problem 2 for general cases when Agents 1 and 2 are not symmetric. In the optimal contract, Agent 0 receives all cash flows until a known

[10] By $\varepsilon_{12}/\varepsilon_{21} = m_1^2/m_2^2$, we have $1 - \varepsilon_{12} m_2/m_1 = 1 - \sqrt{\varepsilon_{12}\varepsilon_{21}}$. Thus, $(1 - \varepsilon_{12} m_2/m_1)^{-1} > 1$ because of $0 \le \varepsilon_{12}\varepsilon_{21} < 1$. Similarly, we have $(1 - \varepsilon_{21} m_1/m_2)^{-1} > 1$.

cutoff date. If Agent 1's influence is larger than Agent 2's moral hazard relative to Agent 1's (i.e., $\varepsilon_{12} > \mathcal{R}_{21} \equiv \frac{m_2/k_2}{m_1/k_1}$), then Agent 1 claims all cash flows after Agent 0 is fully compensated. Agent 2 receives no payments at all. The optimal contract is summarized in Proposition 3 below. In a more general setting, Agent 0 claims all cash flows before a cutoff date. Subsequently, Agents 1 and 2 receive cash flows alternately with an increasing frequency of switches. Section 5.1 discusses this case.

Proposition 3. *Suppose that* Assumption 2 *holds and* $\varepsilon_{12} > \mathcal{R}_{21}$. *In the optimal contract, there exists a cutoff date* t_c *satisfying*

$$\varepsilon\varepsilon_{12}\left(\frac{m_2}{m_1}+\varepsilon_{21}\right)\frac{t_c^3-4Tt_c^2+6T^2t_c}{3}+t_c^3-\frac{T^2-t_c^2}{s_1\gamma}=0, \tag{20}$$

such that (i) Agent 0 claims all cash flows prior to t_c*, namely,* $c_1^*(t)=c_2^*(t)=0$ *for* $t<t_c$*; and (ii) Agent 1 claims all cash flows after* t_c*, and Agent 2 receives no payments, namely,* $c_1^*(t)=b$ *and* $c_2^*(t)=0$ *for* $t\geq t_c$.

Proof. See Appendix A.3. □

This type of optimal contract cannot be anticipated by the three-sided model without cost synergies. Agent 2 receives nothing over the project's lifespan, because his effort is completely driven by Agent 1's influence (see the optimal effort in (21) below). Given that $\varepsilon_{12} > \mathcal{R}_{21}$ is equivalent to $\varepsilon_{21} < \mathcal{R}_{12}$, we can derive the optimal contract for $\varepsilon_{21} > \mathcal{R}_{12}$ by swapping the subscripts 1 and 2 and Agents 1 and 2 in Proposition 3.

We next show how the cutoff date depends on cost synergies and the relative severity of moral hazard. By taking total derivatives on both sides of (20) with respect to ε_{12}, ε_{21}, and $1/(s_1\gamma)$, respectively, we obtain comparative statistics for the cutoff date in Corollary 5.1 below.

Corollary 5.1. *Suppose that* Assumption 2 *holds and* $\varepsilon_{12} > \mathcal{R}_{21}$. *The cutoff date* t_c *is monotonically decreasing in* ε_{12} *and* ε_{21}*, and increasing in* $1/(s_1\gamma)$.

Proof. See Appendix A.3. □

The cutoff date t_c is jointly determined by the agents' moral hazard and influence parameters. If Agent 0's moral hazard is more severe relative to Agent 1's (a larger $1/(s_1\gamma)$), then

Agent 0 receives greater payments (i.e., a larger t_c). On the other hand, if cost synergies are larger, Agent 1 receives greater payments (i.e., a smaller t_c). Given $\varepsilon_{12} > \mathcal{R}_{21}$, Agent 1 benefits from exerting a larger influence by collecting all cash flows after the cutoff date.

Similar to Corollary 4.2, we calculate the optimal effort using the allocation rule in Proposition 3 and the IC constraints in Lemma 3.1. The results are summarized below in Corollary 5.2.

Corollary 5.2. *Suppose that* Assumption 2 *holds and* $\varepsilon_{12} > \mathcal{R}_{21}$*. The optimal efforts* e_0^**,* e_1^**, and* e_2^* *are*

$$e_0^* = \frac{bt_c^2}{4\gamma},\ e_1^*(t) = \frac{m_1}{4k_1} b\varepsilon \left((T-t)^2 - (t_c - t)^2 1_{[0,t_c)}(t)\right),\ e_2^*(t) = \varepsilon_{12} e_1(t)^*. \tag{21}$$

Cost synergies improve Agent 1's effort by an amplifier $\varepsilon > 1$ and via a smaller t_c. Larger cost synergies allow Agent 1 to collect the cash flows for a longer time period (a smaller t_c, see Corollary 5.1) and motivate Agent 1 to exert a greater effort. An interesting observation is that even though Agent 2's effort stems solely from the Agent 1's influence, his amplification role is important. More specifically, Agent 2's effort reduces the cost of Agent 1, which in turn increases Agent 1's optimal effort. In equilibrium, the effort of Agent 1 reduces Agent 2's cost of effort to zero.

5.1 Discussion

In the three-sided moral hazard model with cost synergies, Agent 0 receives all cash flows until a predetermined cutoff date, while Agents 1 and 2 divide all subsequent cash flows. If Agents 1 and 2 are symmetric, they split the cash flows equally at all times. Cost synergies improve these latter agents' optimal efforts while reducing the effort level of Agent 0. Moreover, the expected social surplus increases as cost synergies increase.

If Agent 1's influence is larger than the ratio of Agent 2's moral hazard to Agent 1's moral hazard ($\varepsilon_{12} > \mathcal{R}_{21}$), then Agent 1 collects all cash flows after Agent 0 is fully compensated. Relative to the case without cost synergies, Agent 1 receives cash flows for a longer period. The presence of Agent 2 improves Agent 1's incentives, even though Agent 2's effort is completely driven by Agent 1's influence.

Other types of optimal contracts exist under different combinations of parameters. If $s_1 > s_2$ and $\varepsilon_{12}/\varepsilon_{21} = m_1^2/m_2^2$, Agent 0 claims all cash flows before the first cutoff date t_c, Agent 1 claims all cash flows from t_c to a second cutoff date t_e, and Agent 2 collects all cash flows after t_e. Appendix A.4 provides an analysis of this case. In a more general case, after Agent 0 is fully paid, Agents 1 and 2 alternate in receiving all cash flows, doing so with an increasing frequency. In a finite time, the switches become infinitely frequent, and the system reaches an accumulation point, at which the two agents start splitting the cash flows in a fixed proportion. The proportion is determined by both the agents' severities of moral hazard and influenced on cost saving. This case corresponds to a chattering control (see, for example, Fuller (1963) and Section 2.1 of Yang (2010)).

Modeling cost synergies between Agents 1 and 2 is economically intuitive and analytically tractable. If we model cost synergies between Agent 0 and Agent 1, then a coupling term, $e_0e_1(\cdot)$, appears in the social surplus in Problem 2. We cannot solve such a model analytically. Indeed, the optimal control problem with such coupling terms is time-inconsistent, and its tractability problem is widely recognized. Marin-Solano and Navas (2009) suggest that these kinds of control problems in continuous time receive less attention because of their complexity. One example of time-inconsistent control in economics is the so-called (quasi) hyperbolic discount functions (see, for example, Phelps and Pollak (1968) and Pollak (1968)).

6 Conclusion

This paper examines a dynamic contracting problem with three-sided moral hazard and cost synergies. One agent exerts upfront effort to set up a project; two others exert ongoing effort to manage it. The agents exerting ongoing effort have cost synergies; that is, one's effort reduces the other's cost of effort. The agents' efforts jointly determine the probability of the project's survival and its expected cash flows.

In the optimal contract, payment timing reflects the timing of agents' efforts as well as the effect of cost synergies. The agent exerting upfront effort is penalized for an early failure, while the two agents with ongoing efforts are penalized for any failure occurring before the project reaches the end of its lifespan. Project cash flows are divided as follows: the agent exerting upfront effort claims all cash flows prior to a cutoff date, and the two agents exerting ongoing effort divide all subsequent cash flows. The allocation of cash flows between the latter two agents shows several patterns. When they are symmetric, they split the cash flows equally. In an extreme case, the agent with substantially larger influence claims all cash flows. In a more typical setting, the two agents alternate in receiving all cash flows with an increasing frequency of payment switches. Compared with a baseline model without cost synergies, the cutoff date is earlier, giving the two agents exerting ongoing effort a larger share of the cash flows. The model provides a dynamic framework for analyzing business contracting problems, such as compensation plans in startups and profit sharing among business partners.

A Solving Problem 2

This section provides proofs for the main results. In Section A.1, we use the maximum principle to determine the optimal strategy for Problem 2. In Section A.2, we further analyze switching functions. All the proofs for the main results are presented in Section A.3. We discuss the optimal contracting problem for general cases in Section A.4.

A.1 Analysis of Problem 2

Define new state variables $x_1(\cdot)$, $x_2(\cdot)$, $y_1(\cdot)$, and $y_2(\cdot)$ as follows:

$$x_1(t) = \int_{s=t}^{T}\int_{\tau=s}^{T} c_1(\tau)d\tau ds,\ x_2(t) = \dot{x}_1(t);\ y_1(t) = \int_{s=t}^{T}\int_{\tau=s}^{T} c_2(\tau)d\tau ds,\ y_2(t) = \dot{y}_1(t). \tag{22}$$

Recall Assumption 2(iv) that $0 \le \varepsilon_{12}\varepsilon_{21} < 1$ and the definition of the amplification factor $\varepsilon \equiv (1-\varepsilon_{12}\varepsilon_{21})^{-1}$. Using (22) to further simplify Equations (12), (15), and (16), we have

$$e_0 = \frac{bT^2}{4\gamma} - \frac{x_1(0)+y_1(0)}{2\gamma},\ e_1(t) = \frac{\varepsilon m_1}{2k_1}x_1(t) + \frac{\varepsilon\varepsilon_{21}m_2}{2k_2}y_1(t),\ e_2(t) = \frac{\varepsilon m_2}{2k_2}y_1(t) + \frac{\varepsilon\varepsilon_{12}m_1}{2k_1}x_1(t). \tag{23}$$

Then Problem 2 is equivalent to a new problem (**P**) given in Lemma A.1 below.

Lemma A.1. *The reduced-form optimization* Problem 2 *is equivalent to the following optimal control problem* (**P**).

$$\begin{cases} \max\limits_{c_1(t),c_2(t)} & \hat{\Pi} \equiv \int_{t=0}^{T} f(t, x_1(t), y_1(t))dt + h(x_1(0), y_1(0)) \\ s.t. & c_1(t) + c_2(t) \le b; \quad c_i(t) \ge 0,\ i = 1, 2; \\ & \dot{x}_1(t) = x_2(t),\ x_1(T) = 0;\ \dot{x}_2(t) = c_1(t),\ x_2(T) = 0; \\ & \dot{y}_1(t) = y_2(t),\ y_1(T) = 0;\ \dot{y}_2(t) = c_2(t),\ y_2(T) = 0; \end{cases} \qquad (\mathbf{P})$$

where $h(x_1, y_1) = -\frac{1}{4\gamma}(x_1 + y_1)^2$ *and*

$$f(t, x_1, y_1) = b\varepsilon(T - t)^2 \left(\left(1 + \varepsilon_{12}\frac{m_2}{m_1}\right) s_1 x_1 + \left(1 + \varepsilon_{21}\frac{m_1}{m_2}\right) s_2 y_1 \right) - s_1 x_1^2 - s_2 y_1^2.$$

Proof of Lemma A.1. Let $\Pi = b(\Pi_0' + \frac{1}{2}T^2 e_0 + \frac{C_0}{b}) + \Pi' - \gamma e_0^2$, where $C_0 = b(T - \frac{\phi T^2}{2} - (m_1 + m_2)\frac{T^3}{6})$ and

$$\Pi_0' = \int_{t=0}^{T} (1 - F(t))dt - \frac{T^2}{2}e_0 - \frac{C_0}{b},$$

$$\Pi' = -k_1 \int_{t=0}^{T} (e_1(t) - \varepsilon_{21}e_2(t))^2 dt - k_2 \int_{t=0}^{T} (e_2(t) - \varepsilon_{12}e_1(t))^2 dt.$$

By the definition of $F(\cdot)$ in (10), we have

$$\Pi_0' = \int_{t=0}^{T} \int_{s=0}^{t} \int_{\tau=0}^{s} m_1 e_1(\tau)d\tau ds dt + \int_{t=0}^{T} \int_{s=0}^{t} \int_{\tau=0}^{s} m_2 e_2(\tau)d\tau ds dt.$$

Given the optimal effort in (23), applying integration by part formula (3), we have

$$\Pi_0' = \varepsilon \left(1 + \varepsilon_{12}\frac{m_2}{m_1}\right) \frac{m_1^2}{4k_1} \int_{t=0}^{T} x_1(t)(T - t)^2 dt + \varepsilon \left(1 + \varepsilon_{21}\frac{m_1}{m_2}\right) \frac{m_2^2}{4k_2} \int_{t=0}^{T} y_1(t)(T - t)^2 dt. \tag{24}$$

Moreover, using the optimal effort in (23), we can derive that

$$\Pi' = -\frac{m_1^2}{4k_1} \int_{t=0}^{T} x_1^2(t)dt - \frac{m_2^2}{4k_2} \int_{t=0}^{T} y_1^2(t)dt. \tag{25}$$

Recall that $\Pi = b(\Pi_0' + \frac{1}{2}T^2 e_0 + \frac{C_0}{b}) + \Pi' - \gamma e_0^2$ and $s_1 = \frac{m_1^2}{4k_1}$, $s_2 = \frac{m_2^2}{4k_2}$. By (24) and (25), we rewrite the expected social surplus as

$$\begin{aligned} \Pi =& \varepsilon b \left(1 + \varepsilon_{12}\frac{m_2}{m_1}\right) s_1 \int_{t=0}^{T} x_1(t)(T - t)^2 dt + \varepsilon b \left(1 + \varepsilon_{21}\frac{m_1}{m_2}\right) s_2 \int_{t=0}^{T} y_1(t)(T - t)^2 dt \\ &+ \frac{bT^2}{2}e_0 + C_0 - \gamma e_0^2 - s_1 \int_{t=0}^{T} x_1^2(t)dt - s_2 \int_{t=0}^{T} y_1^2(t)dt \\ =& \int_{t=0}^{T} f(t, x_1(t), y_1(t))dt + h(x_1(0), y_1(0)) + C_0 + \frac{b^2 T^4}{16\gamma}. \end{aligned}$$

We have the desired conclusion, omitting the constant $C_0 + \frac{b^2T^4}{16\gamma}$. $\square$

Applying the maximum principle for deterministic optimal control problem (**P**), we obtain the Hamiltonian

$$\begin{cases} H(t, x_i, y_i, \lambda_i, \mu_i) = f(t, x_1(t), y_1(t)) + \lambda_1(t)x_2(t) + \lambda_2(t)c_1(t) + \mu_1(t)y_2(t) + \mu_2(t)c_2(t), \\ H(0, x_i(0), y_i(0), \lambda_i(0), \mu_i(0)) = h(x_1(0), y_1(0)), \end{cases}$$

where for $i = 1, 2$, $x_i(\cdot)$ and $y_i(\cdot)$ are state variables, $\lambda_i(\cdot)$ and $\mu_i(\cdot)$ are costate variables ($\lambda_2(\cdot)$ and $\mu_2(\cdot)$ are also called *switching functions*), and $c_i(\cdot)$ is the control variable to be determined. The costates satisfy the following system of differential equations.

$$\begin{cases} \dot{\lambda}_1(t) = 2s_1x_1(t) - b(1 + \varepsilon_{12}\frac{m_2}{m_1})\varepsilon s_1(T - t)^2, & \lambda_1(0) = \frac{x_1(0)+y_1(0)}{2\gamma}; \\ \dot{\lambda}_2(t) = -\lambda_1(t), & \lambda_2(0) = 0; \\ \dot{\mu}_1(t) = 2s_2y_1(t) - b(1 + \varepsilon_{21}\frac{m_1}{m_2})\varepsilon s_2(T - t)^2, & \mu_1(0) = \frac{x_1(0)+y_1(0)}{2\gamma}; \\ \dot{\mu}_2(t) = -\mu_1(t), & \mu_2(0) = 0. \end{cases} \tag{26}$$

The Hamiltonian is linear in control variables $c_1(\cdot)$ and $c_2(\cdot)$. Therefore, the optimal control is a *bang-bang control* or a *singular control.* Specifically, maximizing the Hamiltonian with respect to control variables, we can characterize the optimal policy in the lemma below.

Lemma A.2. *The optimal control for problem* (**P**) *is determined as follows:*

1. *if* $\lambda_2(t) < 0$ *and* $\mu_2(t) < 0$*, then* $c_1(t) = 0$ *and* $c_2(t) = 0$*;*
2. *if* $\lambda_2(t) \geq 0$ *and* $\lambda_2(t) > \mu_2(t)$*, then* $c_1(t) = b$ *and* $c_2(t) = 0$*;*
3. *if* $\mu_2(t) \geq 0$ *and* $\mu_2(t) > \lambda_2(t)$*, then* $c_1(t) = 0$ *and* $c_2(t) = b$*;*
4. *if* $\lambda_2(t) = \mu_2(t) \geq 0$ *and* t *is a singleton, then* $c_1(t)$ *and* $c_2(t)$ *are undetermined;*
5. *if* $\lambda_2(t) = \mu_2(t) \geq 0$ *for all* t *in an open interval, then* $c_1(t) = \frac{s_2+\varepsilon\left(s_1\left(1+\varepsilon_{12}\frac{m_2}{m_1}\right)-s_2\left(1+\varepsilon_{21}\frac{m_1}{m_2}\right)\right)}{s_1+s_2}b$*,* $c_2(t) = \frac{s_1-\varepsilon\left(s_1\left(1+\varepsilon_{12}\frac{m_2}{m_1}\right)-s_2\left(1+\varepsilon_{21}\frac{m_1}{m_2}\right)\right)}{s_1+s_2}b$*.*

Proof of Lemma A.2. The first four cases are obviously true. We only need to verify the last case. If $\lambda_2(t) = \mu_2(t) \geq 0$, then $c_1(t) + c_2(t) = b$ and $\lambda_2(t) - \mu_2(t) = 0$. Equations for the costates $\lambda_2(t)$ and $\mu_2(t)$ in (26) lead to the following:

$$\begin{cases} \lambda_2^{(4)}(t) = -2s_1\ddot{x}_1(t) + 2b(1+\varepsilon_{12}\frac{m_2}{m_1})\varepsilon s_1 = -2s_1c_1(t) + 2b(1+\varepsilon_{12}\frac{m_2}{m_1})\varepsilon s_1, \\ \mu_2^{(4)}(t) = -2s_2\ddot{y}_1(t) + 2b(1+\varepsilon_{21}\frac{m_1}{m_2})\varepsilon s_2 = -2s_2c_2(t) + 2b(1+\varepsilon_{21}\frac{m_1}{m_2})\varepsilon s_2. \end{cases}$$

Combining with the facts $c_1(t)+c_2(t) = b$ and $\lambda_2^{(4)}(t) = \mu_2^{(4)}(t)$, we can obtain the payment scheme (i.e., $c_1(t)$ and $c_2(t)$) for the last case. □

A.2 Analysis of Switching Functions

In this subsection, to make the optimal policy in Lemma A.2 more explicit, we further analyze the switching functions $\lambda_2(\cdot)$, $\mu_2(\cdot)$ and their relationship. Lemmas A.3 and A.4 together indicate that $\lambda_2(\cdot)$ or $\mu_2(\cdot)$ starts from zero, decreases first, then increases after it reaches the minimum point. Furthermore, this behavior of $\lambda_2(\cdot)$ or $\mu_2(\cdot)$ holds for all level of synergy and moral hazard. In addition, Lemma A.5 indicates that $\lambda_2(t) > \mu_2(t)$ for $t \in (0, T]$ if $\varepsilon_{12} > \mathcal{R}_{21}$. In this case, $\lambda_2(\cdot)$ will arrive at 0 first.

Lemma A.3. $\lambda_2(0) = \mu_2(0) = 0$, $\dot{\lambda}_2(0) = \dot{\mu}_2(0) < 0$.

Proof of Lemma A.3. By (26), we have $\lambda_2(0) = \mu_2(0) = 0$ and $\dot{\lambda}_2(0) = \dot{\mu}_2(0)$. Next we prove that $\dot{\lambda}_2(0) < 0$. The payments of Agents 1 and 2 satisfy the following constraints: $c_1(t) + c_2(t) \le b$, $c_1(t) \ge 0$, and $c_2(t) \ge 0$ for all $t \in [0, T]$. According to definitions of state variables $x_1(t)$ and $y_1(t)$ in (22), we have $x_1(0) + y_1(0) \le bT^2/2$, $x_1(0) \ge 0$, and $y_1(0) \ge 0$. Then, from (26), we know that

$$\dot{\lambda}_2(0) = -\lambda_1(0) = -\frac{x_1(0) + y_1(0)}{2\gamma} \le 0. \tag{27}$$

If $\dot{\lambda}_2(0) = 0$, then $x_1(0) = y_1(0) = 0$, which implies that $c_1(t) = c_2(t) = 0$ for $t \in (0, T]$. Thus, $\ddot{\lambda}_2(t) = b(1+\varepsilon_{12}\frac{m_2}{m_1})\varepsilon s_1(T-t)^2 > 0$ and $\ddot{\mu}_2(t) = b(1+\varepsilon_{21}\frac{m_1}{m_2})\varepsilon s_2(T-t)^2 > 0$. Note that $\lambda_2(0) = \dot{\lambda}_2(0) = 0$, we conclude that $\lambda_2(t) > 0$ for $t \in (0, T]$. However, $c_1(t)$ and $c_2(t)$ cannot be zero simultaneously if $\lambda_2(t) > 0$ for $t \in (0, T]$ (cf. Lemma A.2). Therefore, $\dot{\lambda}_2(0) \ne 0$. Combining this fact with (27), we have $\dot{\lambda}_2(0) < 0$. Similarly, we can prove that $\dot{\mu}_2(0) < 0$. □

Lemma A.4. *For any $t \in [0, T)$, (i) $\ddot{\lambda}_2(t) > 0$ and $\ddot{\mu}_2(t) > 0$ if $\varepsilon_{12} = \varepsilon_{21} = 0$; (ii) $\ddot{\lambda}_2(t) > 0$ if $\varepsilon_{12} > 0$; (iii) $\ddot{\mu}_2(t) > 0$ if $\varepsilon_{21} > 0$.*

Proof of Lemma A.4. The first conclusion is the same as that in Lemma 7 of Yang (2010). Then we consider the second case. By (26), we have

$$\ddot{\lambda}_2(t) = -\dot{\lambda}_1(t) = b\left(1 + \varepsilon_{12}\frac{m_2}{m_1}\right)\varepsilon s_1(T-t)^2 - 2s_1x_1(t).$$

Recall that $\varepsilon = (1-\varepsilon_{12}\varepsilon_{21})^{-1} \geq 1$, then we have

$$\ddot{\lambda}_2(t) \geq b\varepsilon_{12}\frac{m_2}{m_1}\varepsilon s_1(T-t)^2 + s_1(b(T-t)^2 - 2x_1(t)). \tag{28}$$

From definitions of the state variables in (22), we have $0 \leq x_1(t) + y_1(t) \leq b(T-t)^2/2$. Combining with the fact $\varepsilon_{12} > 0$, we can see that the first term on the right hand side of (28) is positive and the second term is nonnegative. Therefore, $\ddot{\lambda}_2(t) > 0$. Similarly, we can prove that $\ddot{\mu}_2(t) > 0$ if $\varepsilon_{21} > 0$. □

Lemma A.5. *If $\varepsilon_{12} > \mathcal{R}_{21} \equiv \frac{m_2/k_2}{m_1/k_1}$, then $\ddot{\lambda}_2(t) > \ddot{\mu}_2(t)$ for all $t \in [0, T)$.*

Proof of Lemma A.5. By (26), we have

$$\ddot{\lambda}_2(t) - \ddot{\mu}_2(t) = 2s_2y_1(t) - 2s_1x_1(t) + b\varepsilon\left(\left(1 + \varepsilon_{12}\frac{m_2}{m_1}\right)s_1 - \left(1 + \varepsilon_{21}\frac{m_1}{m_2}\right)s_2\right)(T-t)^2.$$

Adding and subtracting the same term (i.e., $s_1b(T-t)^2$), we have

$$\ddot{\lambda}_2(t) - \ddot{\mu}_2(t) = 2s_2y_1(t) + s_1(b(T-t)^2 - 2x_1(t)) + b(T-t)^2 \cdot \Gamma, \tag{29}$$

where

$$\Gamma = \varepsilon\left(\left(1 + \varepsilon_{12}\frac{m_2}{m_1}\right)s_1 - \left(1 + \varepsilon_{21}\frac{m_1}{m_2}\right)s_2\right) - s_1.$$

By (22), we have $0 \leq x_1(t), y_1(t) \leq b(T-t)^2/2$. The first two terms on the right hand side of (29) are nonnegative, thus we only need to show that $\Gamma > 0$ to ensure $\ddot{\lambda}_2(t) > \ddot{\mu}_2(t)$. Recall that $s_1 = m_1^2/(4k_1)$ and $s_2 = m_2^2/(4k_2)$, we have

$$\varepsilon^{-1} \cdot \Gamma = \varepsilon_{12}\varepsilon_{21} \cdot s_1 - s_2 + \varepsilon_{12}\frac{m_2}{m_1}s_1 - \varepsilon_{21}\frac{m_1}{m_2}s_2 = \varepsilon_{12}\varepsilon_{21}\frac{m_1^2}{4k_1} - \frac{m_2^2}{4k_2} + \varepsilon_{12}\frac{m_1m_2}{4k_1} - \varepsilon_{21}\frac{m_1m_2}{4k_2}.$$

A further identical transformation leads to

$$\frac{\varepsilon^{-1} \cdot \Gamma}{m_1^2/4} = \frac{\varepsilon_{12}}{k_1}\varepsilon_{21} - \frac{1}{k_2}\left(\frac{m_2}{m_1}\right)^2 + \frac{\varepsilon_{12}}{k_1}\frac{m_2}{m_1} - \frac{\varepsilon_{21}}{k_2}\frac{m_2}{m_1} = \left(\frac{\varepsilon_{12}}{k_1} - \frac{1}{k_2}\frac{m_2}{m_1}\right)\left(\varepsilon_{21} + \frac{m_2}{m_1}\right).$$

So $\Gamma > 0$ is equivalent to $\varepsilon_{12} > \frac{m_2/k_2}{m_1/k_1}$. □

According to aforementioned lemmas, the following conclusions hold for switching functions $\lambda_2(\cdot)$ and $\mu_2(\cdot)$: (i) There exists a unique $t_c \in (0,T)$ such that $\lambda_2(\cdot)$ crosses zero from below at t_c and stays positive afterward; that is, $\lambda_2(t_c) = 0$ and $\lambda_2(t) > 0$ for $t > t_c$. Similarly, there exists a unique $t_c \in (0,T)$ such that $\mu_2(t_e) = 0$ and $\mu_2(t) > 0$ for $t > t_e$. (ii) If $\varepsilon_{12} > \frac{m_2/k_2}{m_1/k_1}$, then $\lambda_2(t) > \mu_2(t)$ for all $t \in (0,T)$, which means $t_c < t_e$.

A.3 Proofs of the Main Results

Based on the analyses in Sections A.1-A.2, we can prove the main results.

Proof of Proposition 2. The condition $\varepsilon_{12}/\varepsilon_{21} = m_1^2/m_2^2 = k_1/k_2$ implies that $s_1 = s_2$ and $1 + \varepsilon_{12}m_2/m_1 = 1 + \varepsilon_{21}m_1/m_2$. By (26), $x_1(\cdot)$ and $y_1(\cdot)$ are symmetric, thus $\lambda_2(\cdot) = \mu_2(\cdot)$ are symmetric. Applying Lemmas A.3 and A.4, we conclude $\lambda_2(t) = \mu_2(t)$ for all $t \in [0,T]$. Moreover, there exists a unique $t_c > 0$ such that $\lambda_2(t_c) = \mu_2(t_c) = 0$. According to Lemma A.2 and the analysis of the switching functions ($\lambda_2(t) = \mu_2(t) < 0$ for $t \in (0,t_c)$, $\lambda_2(t) = \mu_2(t) > 0$ for $t \in (t_c,T]$), in the optimal contract, Agent 0 claims all cash flows prior to t_c; afterwards, Agents 1 and 2 share all cash flows equally because of $s_1 = s_2$ and $1 + \varepsilon_{12}m_2/m_1 = 1 + \varepsilon_{21}m_1/m_2$.

Next, we determine the value of t_c. The optimal states $x_1(t)$ and $y_1(t)$ are

$$x_1(t) = y_1(t) = \frac{b}{4}\Big((T-t)^2 - (t_c - t)^2 1_{[0,t_c)}(t)\Big). \tag{30}$$

Hence $x_1(0) = y_1(0) = b(T^2 - t_c^2)/4$. Substituting (30) into (26), we have

$$\begin{aligned}\lambda_2(t) =& bs_1\left(\left(1+\varepsilon_{12}\frac{m_2}{m_1}\right)\varepsilon - \frac{1}{2}\right)\int_{\tau=0}^{t}\int_{s=0}^{\tau}(T-s)^2 ds d\tau \\ &+ \frac{bs_1}{2}\int_{\tau=0}^{t}\int_{s=0}^{\tau}(t_c-s)^2 1_{[0,t_c)}(s) ds d\tau - \frac{x_1(0)+y_1(0)}{2\gamma}t. \end{aligned}\tag{31}$$

Note that

$$\int_{\tau=0}^{t_c}\int_{s=0}^{\tau}(T-s)^2 ds d\tau = \frac{t_c^4 - 4Tt_c^3 + 6T^2t_c^2}{12}, \quad \int_{\tau=0}^{t_c}\int_{s=0}^{\tau}(t_c-s)^2 1_{[0,t_c)}(s) ds d\tau = \frac{t_c^4}{4}. \tag{32}$$

The point t_c is determined by the equation $\lambda_2(t_c) = 0$. By (31) and (32), t_c satisfies the following equation

$$\left(\left(1+\varepsilon_{12}\frac{m_2}{m_1}\right)\varepsilon - \frac{1}{2}\right)\frac{t_c^4 - 4Tt_c^3 + 6T^2t_c^2}{12} + \frac{t_c^4}{8} - \frac{T^2t_c - t_c^3}{4s_1\gamma} = 0.$$

Note that $\varepsilon_{12}/\varepsilon_{21} = m_1^2/m_2^2$ implies $\varepsilon_{21} = \varepsilon_{12}m_2^2/m_1^2$. We have

$$\varepsilon = \frac{1}{1-\varepsilon_{12}\varepsilon_{21}} = \frac{1}{1-\varepsilon_{12}\cdot\varepsilon_{12}m_2^2/m_1^2}$$

Thus,

$$2\left(\left(1+\varepsilon_{12}\frac{m_2}{m_1}\right)\varepsilon - \frac{1}{2}\right) = \frac{m_1+\varepsilon_{12}m_2}{m_1-\varepsilon_{12}m_2}.$$

The cutoff date, $t_c > 0$, satisfies the following condition:

$$\frac{m_1+\varepsilon_{12}m_2}{m_1-\varepsilon_{12}m_2}\frac{t_c^3-4Tt_c^2+6T^2t_c}{24} + \frac{t_c^3}{8} - \frac{T^2-t_c^2}{4s_1\gamma} = 0,$$

which is equivalent to (17). □

Proof of Corollary 4.1. By $\varepsilon_{12}/\varepsilon_{21} = m_1^2/m_2^2$ and $0 \le \varepsilon_{12}\varepsilon_{21} < 1$ in Assumption 2(iv), we have $m_1 - \varepsilon_{12}m_2 > 0$ and $m_2 - \varepsilon_{21}m_1 > 0$. Taking the total derivative on both sides of (17) with respect to ε_{12} and $1/(s_1\gamma)$, respectively, we have

$$\frac{\partial t_c}{\partial \varepsilon_{12}} = -\frac{\frac{2m_1m_2t_c}{(m_1-\varepsilon_{12}m_2)^2}\left((t_c-2T)^2+2T^2\right)}{\frac{m_1+\varepsilon_{12}m_2}{m_1-\varepsilon_{12}m_2}\left(3(t_c-\frac{4}{3}T)^2+\frac{2}{3}T^2\right)+9t_c^2+\frac{12t_c}{s_1\gamma}} < 0,$$
$$\frac{\partial t_c}{\partial(1/(s_1\gamma))} = \frac{6(T^2-t_c^2)}{\frac{m_1+\varepsilon_{12}m_2}{m_1-\varepsilon_{12}m_2}\left(3(t_c-\frac{4}{3}T)^2+\frac{2}{3}T^2\right)+9t_c^2+\frac{12t_c}{s_1\gamma}} > 0.$$

Similarly, we can show that $\partial t_c/\partial\varepsilon_{21} < 0$. □

Proof of Corollary 4.2. By the allocation rule in Proposition 2, for $i = 1, 2$, we have

$$\int_{s=t}^{T}\int_{\tau=s}^{T} c_i(\tau)d\tau ds = \frac{b}{2}\int_{s=t}^{T}\int_{\tau=s}^{T} 1_{[t_c,T]}(\tau)d\tau ds = \frac{b}{4}\Big((T-t)^2-(t_c-t)^2 1_{[0,t_c)}(t)\Big). \quad (33)$$

From the condition $\varepsilon_{12}/\varepsilon_{21} = m_1^2/m_2^2 = k_1/k_2$, we have

$$\varepsilon\left(\frac{m_1}{2k_1}+\varepsilon_{21}\frac{m_2}{2k_2}\right) = \frac{m_1}{2k_1}\frac{m_1}{m_1-\varepsilon_{12}m_2},\ \varepsilon\left(\frac{m_2}{2k_2}+\varepsilon_{12}\frac{m_1}{2k_1}\right) = \frac{m_2}{2k_2}\frac{m_2}{m_2-\varepsilon_{21}m_1}. \quad (34)$$

Substituting (33) and (34) into (15) and (16), we have the optimal efforts for Agents 1 and 2 in (18) and (19), respectively. The optimal effort for Agent 0 is obvious. □

Proof of Corollary 4.3. Without loss of generality, we focus on ε_{12}. We use $\Pi(\varepsilon_{12})$ and $t_c(\varepsilon_{12})$ to indicate the dependence of the influence parameter. The second conclusion is directly implied by Corollary 4.1. We prove the first conclusion by showing that $\frac{\partial \Pi(\varepsilon_{12})}{\partial \varepsilon_{12}}$ is positive. For simplicity, let

$$\alpha = \varepsilon_{12}\frac{m_2}{m_1}, \qquad \pi_0 = b\left(T - \frac{\phi T^2}{2} - \frac{(m_1+m_2)T^3}{6}\right) + \frac{b^2T^4}{16\gamma}.$$

By Lemma A.1, the expected social surplus is

$$\Pi(\varepsilon_{12}) = \pi_0 - \frac{1}{\gamma}x_1^2(0) - 2s_1\int_{t=0}^{T} x_1^2(t)dt + 2s_1\int_{t=0}^{T}\frac{b(T-t)^2}{1-\alpha}x_1(t)dt. \tag{35}$$

Note that

$$\int_0^T (T-t)^2(t_c-t)^2 1_{[0,t_c)}(t)dt = \frac{T^2t_c^3}{3} - \frac{Tt_c^4}{6} + \frac{t_c^5}{30}. \tag{36}$$

Using (36) and the optimal state given in (30), we have the following formulas:

$$x_1(0) = \frac{b(T^2-t_c^2)}{4}, \tag{37}$$

$$\int_0^T x_1^2(t)dt = \frac{b^2}{16}\left(\frac{2t_c^5}{15} + \frac{Tt_c^4}{3} - \frac{2T^2t_c^3}{3} + \frac{T^5}{5}\right), \tag{38}$$

$$\int_0^T b(T-t)^2x_1(t)dt = \frac{b^2}{4}\left(\frac{T^5}{5} - \left(\frac{t_c^5}{30} - \frac{Tt_c^4}{6} + \frac{T^2t_c^3}{3}\right)\right). \tag{39}$$

Substituting (37)-(39) into (35), we have

$$\begin{aligned}\Pi(\varepsilon_{12}) =& \pi_0 - \frac{b^2}{\gamma}\left(\frac{T^2-t_c^2}{4}\right)^2 - 2s_1\frac{b^2}{16}\left(\frac{2t_c^5}{15} + \frac{Tt_c^4}{3} - \frac{2T^2t_c^3}{3} + \frac{T^5}{5}\right)\\ &+ \frac{2s_1}{1-\alpha}\frac{b^2}{4}\left(\frac{T^5}{5} - \left(\frac{t_c^5}{30} - \frac{Tt_c^4}{6} + \frac{T^2t_c^3}{3}\right)\right).\end{aligned}$$

Let $\widetilde{\Pi}(\varepsilon_{12}) = [\Pi(\varepsilon_{12}) - (\pi_0 - s_1b^2T^5/40)]/(s_1b^2)$. Then we have

$$\widetilde{\Pi}(\varepsilon_{12}) = -\frac{(T^2-t_c^2)^2}{16s_1\gamma} - \left(\frac{t_c^5}{60} + \frac{Tt_c^4}{24} - \frac{T^2t_c^3}{12}\right) + \frac{1}{1-\alpha}\left(\frac{T^5}{10} - \frac{t_c^5}{60} + \frac{Tt_c^4}{12} - \frac{T^2t_c^3}{6}\right). \tag{40}$$

Recall that the critical date $t_c(\varepsilon_{12})$ satisfies (17), that is,

$$\frac{T^2-t_c^2}{s_1\gamma} = \frac{1+\alpha}{1-\alpha}\frac{t_c^3 - 4Tt_c^2 + 6T^2t_c}{6} + \frac{t_c^3}{2}.$$

Substituting the above equation into (40), we have

$$\begin{aligned}\widetilde{\Pi}(\varepsilon_{12}) =& -\frac{(T^2-t_c^2)}{16}\left(\frac{1+\alpha}{1-\alpha}\frac{t_c^3-4Tt_c^2+6T^2t_c}{6} + \frac{t_c^3}{2}\right)\\ &-\left(\frac{t_c^5}{60} + \frac{Tt_c^4}{24} - \frac{T^2t_c^3}{12}\right) + \frac{1}{1-\alpha}\left(\frac{T^5}{10} - \frac{t_c^5}{60} + \frac{Tt_c^4}{12} - \frac{T^2t_c^3}{6}\right)\\ =& \frac{(\alpha-2)t_c^5 + 15T^2t_c^3 - 10(\alpha+1)T^3t_c^2 + 15(\alpha+1)T^4t_c - 24T^5}{240(\alpha-1)}.\end{aligned} \tag{41}$$

By (41) and the relationship between $\Pi(\cdot)$ and $\widetilde{\Pi}(\cdot)$, we have

$$\begin{aligned}\frac{\partial \Pi(\varepsilon_{12})}{\partial \varepsilon_{12}} \equiv& \frac{\partial \widetilde{\Pi}(\varepsilon_{12})}{\partial \varepsilon_{12}} = \frac{I_1}{48(1-\alpha)} \cdot \left(-\frac{\partial t_c}{\partial \varepsilon_{12}}\right) + \frac{I_2}{240(1-\alpha)^2} \cdot \frac{\partial \alpha}{\partial \varepsilon_{12}}, \qquad (42)\\ I_1 =& 3(\alpha+1)T^2(T-t_c)^2 + (2-\alpha)(3T^2 - t_c^2)t_c^2 + 2(\alpha+1)T^3 t_c,\\ I_2 =& (T-t_c)\Big(16T^4 + 6T^3(T-t_c) + 14T^2 t_c^2 + (T^4 - Tt_c^3) + (T^4 - t_c^4)\Big).\end{aligned}$$

It is obvious that $\frac{\partial \alpha}{\partial \varepsilon_{12}} = \frac{m_2}{m_1} > 0$ and $I_2 > 0$, and $-\frac{\partial t_c}{\partial \varepsilon_{12}}$ is positive by Corollary 4.1. Moreover, substituting the symmetric condition $\varepsilon_{21} = \varepsilon_{12} m_2^2/m_1^2$ into $0 \le \varepsilon_{12}\varepsilon_{21} < 1$ in Assumption 2(iv), we have $\alpha \equiv \varepsilon_{12} m_2/m_1 \in [0, 1)$. It implies $2 - \alpha > 0$, and thus I_1 is positive. Therefore, $\frac{\partial \Pi(\varepsilon_{12})}{\partial \varepsilon_{12}}$ is positive because all components in (42) are positive. □

Proof of Proposition 3. If $\varepsilon_{12} > \mathcal{R}_{21}$, according to Lemmas A.3-A.5, we conclude that $\lambda_2(t) > \mu_2(t)$ for all $t \in (0, T)$. Moreover, there exists a critical date $t_c \in (0, T)$ such that $\lambda_2(t_c) = 0$, $\lambda_2(t) < 0$ for $t \in (0, t_c)$ and $\lambda_2(t) > 0$ for $t \in (t_c, T)$. The optimal policy now follows from Lemma A.2. Next, we will determine the critical date t_c. The optimal state variables $x_1(t)$ and $y_1(t)$ are

$$x_1(t) = \frac{b}{2}\Big((T-t)^2 - (t_c - t)^2 1_{[0,t_c)}(t)\Big), \; y_1(t) = 0. \qquad (43)$$

Hence $x_1(0) = b(T^2 - t_c^2)/2$ and $y_1(0) = 0$. Substituting (43) into (26), we have

$$\begin{aligned}\lambda_2(t) =& bs_1\Big(\Big(1 + \varepsilon_{12}\frac{m_2}{m_1}\Big)\varepsilon - 1\Big) \int_{\tau=0}^{t}\int_{s=0}^{\tau} (T-s)^2 ds d\tau + bs_1 \int_{\tau=0}^{t}\int_{s=0}^{\tau} (t_c - s)^2 1_{[0,t_c)}(s) ds d\tau\\ &- \frac{x_1(0) + y_1(0)}{2\gamma} t. \qquad (44)\end{aligned}$$

The critical point t_c is the solution to $\lambda_2(t_c) = 0$. By (32) and (44), t_c satisfies the following equation

$$bs_1\Big(\Big(1 + \varepsilon_{12}\frac{m_2}{m_1}\Big)\varepsilon - 1\Big)\frac{t_c^4 - 4Tt_c^3 + 6T^2 t_c^2}{12} + bs_1\frac{t_c^4}{4} - \frac{b(T^2 - t_c^2)}{4\gamma} t_c = 0,$$

which is equivalent to (20). □

Proof of Corollary 5.1. By taking total derivatives on both sides of (20) with respect to

ε_{12}, ε_{21}, and $1/(s_1\gamma)$, respectively, we have

$$\frac{\partial t_c}{\partial \varepsilon_{12}} = -\frac{\varepsilon^2(\frac{m_2}{m_1}+\varepsilon_{21})((t_c-2T)^2+2T^2)t_c}{\varepsilon\varepsilon_{12}(\frac{m_2}{m_1}+\varepsilon_{21})(3(t_c-\frac{4}{3}T)^2+\frac{2}{3}T^2)+9t_c^2+\frac{6t_c}{s_1\gamma}} < 0,$$
$$\frac{\partial t_c}{\partial \varepsilon_{21}} = -\frac{\varepsilon^2\varepsilon_{12}(1+\varepsilon_{12}\frac{m_2}{m_1})((t_c-2T)^2+2T^2)t_c}{\varepsilon\varepsilon_{12}(\frac{m_2}{m_1}+\varepsilon_{21})(3(t_c-\frac{4}{3}T)^2+\frac{2}{3}T^2)+9t_c^2+\frac{6t_c}{s_1\gamma}} < 0,$$
$$\frac{\partial t_c}{\partial(1/(s_1\gamma))} = \frac{T^2-t_c^2}{\varepsilon\varepsilon_{12}(\frac{m_2}{m_1}+\varepsilon_{21})((t_c-\frac{4}{3}T)^2+\frac{2}{9}T^2)+3t_c^2+\frac{2t_c}{s_1\gamma}} > 0.$$

□

A.4 Optimal Contracts for General Cases in Section 5.1

Recall Problem 2 and associated analysis in Sections A.1 and A.2. To derive the optimal contract, we define the costates with assuming that $\varepsilon_{12}/\varepsilon_{21} = m_1^2/m_2^2$. Under this assumption, the following equality holds:

$$b\left(1+\varepsilon_{12}\frac{m_2}{m_1}\right)\varepsilon = b\left(1+\varepsilon_{21}\frac{m_1}{m_2}\right)\varepsilon := b_0.$$

Then, from (26), the costates satisfy the following equation system:

$$\begin{cases} \dot{\lambda}_1(t) = 2s_1x_1(t) - b_0s_1(T-t)^2, & \lambda_1(0) = \frac{x_1(0)+y_1(0)}{2\gamma}; \\ \dot{\lambda}_2(t) = -\lambda_1(t), & \lambda_2(0) = 0; \\ \dot{\mu}_1(t) = 2s_2y_1(t) - b_0s_2(T-t)^2, & \mu_1(0) = \frac{x_1(0)+y_1(0)}{2\gamma}; \\ \dot{\mu}_2(t) = -\mu_1(t), & \mu_2(0) = 0. \end{cases}$$

The above system of equations, which determines the costates, is the same as that (i.e., equations (2) and (3)) presented in Yang (2010). In addition, the Hamiltonian given in Section A.1 does not depend on the influence parameters ε_{12} and ε_{21}. Therefore, the optimal contract is the same as that given in Yang (2010).

According to the electronic companion of Yang (2010), we have another type of optimal contract. Specifically, if $\varepsilon_{12}/\varepsilon_{21} = m_1^2/m_2^2$ and $s_1 > s_2$, then, (i) $\lambda_2(\cdot)$ crosses zero from below first at $t_c \in (0, T)$, (ii) $\mu_2(\cdot)$ exceeds $\lambda_2(\cdot)$ at a date $t_e \in (t_c, T)$. Therefore, in the optimal contract, (i) Agent 0 collects all the cash flows (i.e., b) before the cutoff date t_c; (ii) Agent 1 collects all the cash flows during (t_c, t_e); (iii) Agent 2 receives all the cash flows from t_e until the end of the project. A more general case is discussed in Section 2.1 of Yang (2010), that is, Agents 1 and 2 alternate in receiving all cash flows with an increasing frequency after Agent 0 is fully compensated.

References

Ballester, C., A. Calvo-Armengol, Y. Zenou. 2006. Who's who in networks. Wanted: The key player. *Econometrica* 74(5), 1403-1417.

Dybvig, P., N. Lutz. 1993. Warranties, durability, and maintenance: two-sided moral hazard in a continuous-time model. *The Review of Economic Studies*, 60(3), 575-597.

Edmans, A., I. Goldstein, J.Y., Zhu. 2013. Contracting with synergies. *National Bureau of Economic Research*.

Falk, A., A. Ichino. 2006. Clean evidence on peer effects. *Journal of Labor Economics* 24, 39-57.

Fuller, A. 1963. Study of an optimum non-linear system. *Journal of Electronic Control* 15, 63-71.

Holmström, B. 1982. Moral hazard in teams. *Bell Journal of Economics* 13, 324-340.

Holmström, B., P., Milgrom. 1987. Aggregation and linearity in the provision of intertemporal incentives. *Econometrica* 55, 303-328.

Georgiadis, G. 2015. Projects and team dynmaics. *Review of Economic Studies* 82, 187–218.

Lazear, P. 1989. Pay equality and industrial politics. *Journal of Political Economy* 97, 561-580.

Kandel, E., P. Lazear. 1992. Peer pressure and partnerships. *Journal of Political Economy* 100, 801-817.

Marin-Solano, J., J. Navas. 2009. Non-constant discounting in finite horizon: The free terminal time case. *Journal of Economic Dynmaics & Control* 33, 666-675.

Mas, A., E. Moretti. 2009. Peers at work. *American Economic Review* 99, 112-145.

Phelps, E.S., R.A. Pollak. 1968. On second-best national saving and game-equilibrium growth. *Review of Economic Studies* 35, 185–199.

Pollak, R.A.,1968. Consistent planning. *Review of Economic Studies* 35, 201-208.

Schättler, H., J. Sung. 1993. The first-order approach to the continuous-time principal-agent problem with exponential utility. *Journal of Economic Theory* 61, 331-371.

Sung, P. 1995. Linearity with project selection and controllable diffusion rate in continuous-time principal-agent problems. *Rand Journal of Economics* 74, 297-332.

Winter, E. 2010. Transparency and incentives among peers. *RAND Journal of Economics* 41, 504-523.

Yang, J. 2010. Timing of effort and reward: three-sided moral hazard in a continuous-time model. *Management Science*, 56(9), 1568-1583.

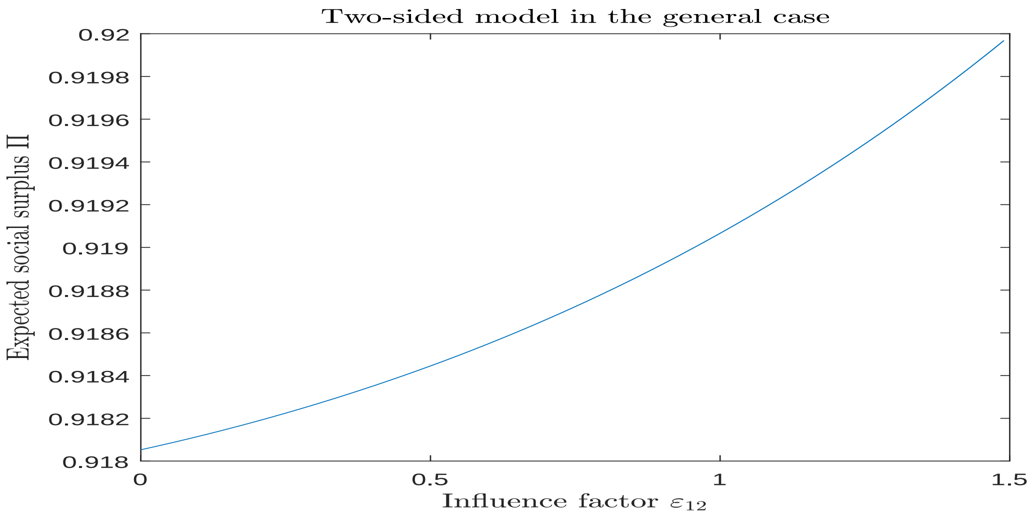

Figure 1: This picture plots the expected social surplus versus the influence factor ε_{12} in the two-sided model. Other parameters are given as $\varepsilon_{21} = 0.1$, $m_1 = 0.2$, $m_2 = 0.3$, $k_1 = k_2 = 1$, $T = 1$, $b = 1$.

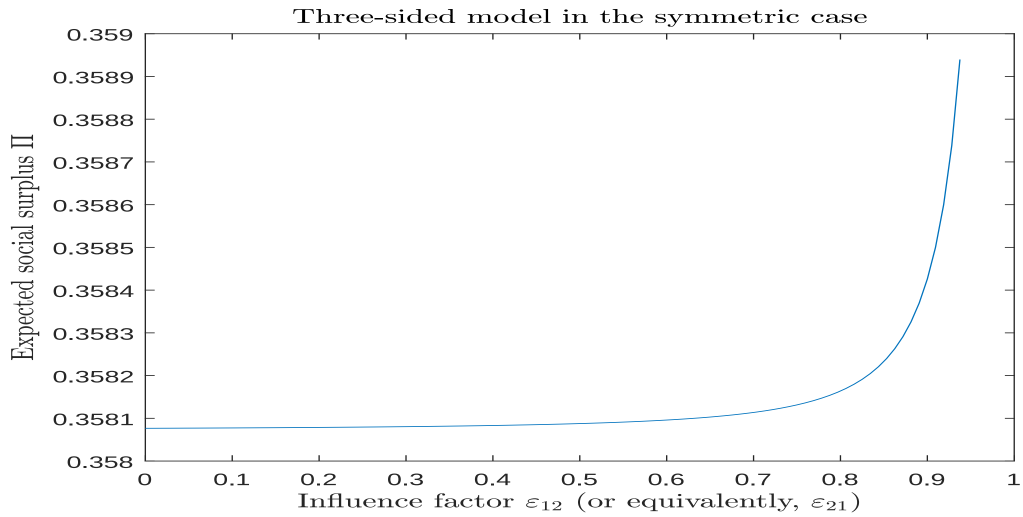

Figure 2: This picture plots the expected social surplus versus the influence factor ε_{12} in the symmetric case by setting $\varepsilon_{12} = \varepsilon_{21}$. Other parameters are given as $m_1 = m_2 = 0.5$, $k_1 = k_2 = 1$, $T = 1$, $b = 1$, $\phi = 1$, $\gamma = 1$.

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