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Abstract

Theoretical differences between different voting rules have been well-studied, and several paradoxical situations are known. For instance, the use of different voting rules not only can lead to different winners for the same preference profile, but also might the winner under one voting rule be the loser under another voting rule. Also, cyclic collective preferences – as in Condorcet cycles – can make it impossible to determine the winner of an election under a specific voting rule in use. Based on data collected in an online-survey in connection with the 2015 parliament election in the Austrian federal state of Styria, we provide an empirical analysis of whether different voting rules yield different outcomes in real-world elections, and whether paradoxical situations show up in real-world data. For our findings, we generate statistical confidence levels by the use of a nonparametric bootstrap.

Keywords: election, voting rules, empirical study, Condorcet.

1 Introduction

Social choice theory studies the aggregation of individual preferences into a social preference. Many real-world situations are eligible for an analysis using tools from that theory, such as elections, choosing from job candidates, making investment decisions, searching the internet, and so on. Since the beginning of modern social choice theory in the 1950s, the focus of traditional social choice theory has mainly been put on theoretical aspects. Normative approaches, as in impossibility theorems (see, e.g., Arrow (1963), Sen (1979) or Gibbard (1973) and Satterthwaite (1975)), the study of theoretical properties of voting rules (see, e.g., Brams and Fishburn (2002), Laruelle and Valenciano (2011), and Nehring et al. (2016)), the analysis of paradoxical situations (see, e.g., Nurmi (1999), Saari (2000a,b) and Gehrlein (2006)), and comparisons of outcomes, i.e., either winners or whole rankings, of different social choice rules (see, e.g., Saari (1994), Ratliff (2001) or Klamler (2005)) were some of the main topics discussed in the literature. This led to an overwhelming number of negative results, which rather was in contrast to the experience coming from actual elections and decision processes.

In the early 2000s, first attempts started to analyze the consequences of those theoretical results for the practical use of social choice rules based on empirical data. In particular, Regenwetter et al. (2006) opened the new research-field of behavioral social choice, where, essentially, data from actual elections were tested against the negative predictions stemming from the theoretical literature. This resulted in the surprising conclusion that many of the theoretical problems could not be found in real-world decision processes.

The main focus in the early literature was put on the famous Condorcet paradox, i.e., the existence of majority cycles in preference profiles. As shown by Regenwetter et al. (2006), in most of the elections for which preference data is available, the Condorcet paradox almost never occurs although theoretical results predict otherwise (Gehrlein (2006)). Second, some papers looked at the actual consensus among social choice rules in real-world elections (e.g., Regenwetter et al. (2007) or Popov et al. (2014)) indicating a surprising overlap of outcomes among various social choice rules in such situations. This is in contrast to the theoretical investigations of differences among social choice rules in the sense that for various preference profiles winners and losers of social choice rules could fundamentally contradict each other (Saari (2000a,b)).[1]

Although certain steps forward have been made in the behavioral approach to social choice theory, a massive expansion of that literature has been prevented by the unavailability of real-world election data. The collection of such data with sufficiently rich preference information is both difficult and costly. Some exceptions can be found in the papers by Baujard et al. (2014), Roescu (2014), Alòs-Ferrer and Granič (2012) and Darmann et al. (2017) who provide interesting insights into the performance of selected social choice rules for various real-world elections. We add to that literature by providing an analysis of data collected in an online-survey in connection with the 2015 parliament elections in the Austrian federal state of Styria.

Complementary to the above papers, we rely on an approach used, e.g., by Regenwetter et al. (2007), namely the generation of statistical confidence levels through a nonparametric bootstrap (see Efron and Tibshirani (1993)). The underlying idea is to investigate certain outcomes in case there are small changes in the ballot frequencies. Hence, based on our survey data, we created numerous fictitious ballots which we analyzed with respect to various different voting rules. Regenwetter et al. (2007) and Popov et al. (2014) applied bootstrap methods essentially to a huge dataset from the American Psychological Association (APA) presidential elections and focused on issues like the comparison of voting outcomes under different voting rules and Condorcet cycles. In our paper we want to extend their analysis to a different set of data from the 2015 elections to the Styrian parliament including a larger set of voting rules such as, e.g., evaluative voting. This became possible because we collected very rich preference data.

In particular, our online-survey data consisted of various data about voters' preferences. Those were obtained by different query-modes such as ranking all candidates, disapproving candidates and individually evaluating the candidates. Based on this preference data, which had been collected via a representative online survey in the two weeks before the actual elections, we are able to apply different voting rules and compare the respective outcomes. Because, obviously, there was

[1] For a more detailed discussion of the theoretical results in social choice theory we refer to Popov et al. (2014).

no real election held on the basis of the collected data, we expect voters to have behaved sincerely.[2]

This paper shows, by generating statistical confidence levels through a non-parametric bootstrap as in Regenwetter et al. (2007), that certain paradoxical situations from the theoretical literature do not occur in this real-world dataset. In particular, we found that there is large agreement in the outcomes of the voting rules based on the different preference information and Condorcet cycles are virtually inexistent. This supports the findings in the behavioral social choice literature.

The rest of the paper is structured as follows: Section 2 introduces the set of voting rules used in the paper. Section 3 provides a detailed theoretical example leading to a plethora of paradoxical results. This is followed, in Section 4, by an overview over the experimental design of our online survey and the used method. Our empirical results are presented in Section 5 before Section 6 concludes the paper.

2 Voting Rules

In collecting the data in the online-survey a main goal was to use a wide variety of preference query modes[3], which allows for a comparison of different voting rules based on the preference data given. In particular, we aimed at getting data about the voters' rankings of the candidates, their possibility of dividing candidates into different classes with qualitative meaning and about cardinal valuations giving the option of expressing intensities in the preferences over candidates. In that respect, the applied voting rules are (informally) defined and grouped as follows:

1. *Ranking Information*
   (a) **Plurality Rule**
   Plurality Rule is the most commonly used voting rule in real-world elections. It is based on voters casting one vote for exactly one candidate. The ranking of the candidates is determined by ordering them according to the total number of votes they receive (plurality scores).[4]
   (b) **Plurality Runoff**
   Plurality Runoff is a two-round system used, e.g., in presidential elections in France and Austria. It works as follows: we start, in the first round, with the Plurality Rule and, in case there is no absolute majority for one of the candidates, in the second round we have a runoff, i.e., a majority vote between the two candidates with the highest plurality scores.
   (c) **Hare System**
   The Hare System is a voting rule based on the sequential elimination of candidates with the lowest plurality scores until there is one candidate

[2] We also asked voters to communicate their actual vote in the election. It turned out that a certain percentage of voters did vote strategically in the real election. However, in the online-survey there were no incentives for strategic behavior.

[3] See also Grundner (2017) for a detailed analysis of the behavior of the voters under the different preference query modes.

[4] In principle no complete ranking information is needed for Plurality Rule. However, we determined the Plurality Rule outcome from the ranking information collected in the online-survey.

with an absolute majority. This means that, in the first round, the plurality scores are determined and the candidate with the lowest plurality score is eliminated. All voters that voted for that candidate now vote for their second ranked candidate. Updating the plurality scores of the candidates we eliminate again the candidate with the lowest (updated) plurality score. This process is repeated until there is a candidate with an absolute majority. In the real-world this voting rule is used in elections in several countries and for determining the winner of the Oscar for the best movie.[5]

(d) **Borda Rule**
The Borda Rule is a scoring rule which uses pre-defined points assigned to the ranks in the individual preferences. In particular, the Borda Rule, for each voter, assigns zero points to the bottom ranked candidate, one point to the candidate ranked next to bottom, up to $n-1$ points to the top ranked candidate (out of n candidates). The Borda score of a candidate is given by adding up these points for each voter. Finally, the Borda ranking is determined by ranking the candidates according to their Borda scores (the higher the score, the better the rank).

(e) **Condorcet Method**
The Condorcet Method (also called simple majority rule) uses Condorcet's concept of pairwise majorities. For each pair of candidates, a and b, we consider a *socially preferred* to b whenever the number of voters preferring a over b is larger than the number of voters preferring b over a. The top-ranked candidate in the Condorcet ranking is the candidate that wins the pairwise contest against every other candidate, the second-ranked candidate the one that wins against every other but the top-ranked candidate and so forth. However, such a Condorcet ranking will not exist for all preference profiles because there might be social preference cycles, i.e., Condorcet cycles (see Gehrlein (2006) for a discussion of Condorcet cycles).

2. *Qualitative Information*

Disapproval Voting
Disapproval Voting (Brams and Fishburn (1983)) allows each voter to divide candidates into those she approves and those she disapproves of, i.e., the candidates are divided into two classes with qualitative meaning. The disapproval ranking is determined by summing the disapprovals for each candidate over all voters. Disapproval Voting can be seen as the underlying concept in web-applications such as doodle.

3. *Cardinal Information*

(a) **100 Points Rule**
In the 100 Points Rule voters can freely assign 100 points among the set of candidates. This offers the possibility of indicating certain intensities in the preferences among candidates. The candidates are ranked according to their average number of points they receive (the higher that number, the better the rank).

(b) **$\pm$ 20 Points Rule**
In the $\pm$ 20 Points Rule voters are asked to place each candidate on the

[5] In case of single-winner elections the Hare system is also called single transferable vote.

the interval $[-20, +20]$. Again, this gives the chance to indicate intensities. Also, candidates are ranked according to their average number of points they receive.

3 A Theoretical Example

As discussed in the introduction, theoretical results usually aim at normative results or finding paradoxical situations. In particular, many papers in the literature state specific preference profiles in which different voting rules lead to extreme differences in the outcome (see, e.g., Saari (1994)). In this section we want to add to this plethora of paradoxes by providing another example in which the application of the voting rules presented in Section 2 leads to very different results. In contrast to other examples we tried to extend the number of paradoxical results by including rules based on qualitative and cardinal information.

Consider the following preference profile in Table 1 with 15 voters and five candidates. The table states that five voters prefer a over b over c over d over e, four voters prefer e over b over c over d over a, and so on.

5	4	3	2	1
a	e	d	c	b
b	b	e	d	c
c	c	b	e	d
d	d	c	b	e
e	a	a	a	a

Table 1: Ordinal preference profile with 15 voters.

Let us assume that voters vote sincerely and apply certain voting rules to this preference profile.

Voting rules based on ranking information. Obviously, under Plurality Rule candidate a turns out to be the winner with five votes. If we consider Plurality Runoff, then candidates a and e will proceed to the second round because they have the highest plurality score (but still not an absolute majority). In the second round e wins against a by 10 to 5 votes. Another option is to apply the Borda Rule. Under the Borda Rule, we assign 4 points to the top ranked candidates of the voters, 3 points to the second ranked candidates, and so on. In the above preference profile this leads to candidate b winning with 39 points. Thus, all of these rules identify a different winner.[6] Next, let us consider the outcome according to the Condorcet Method. Because candidate a is ranked at the bottom by 10 voters, every other candidate will be socially preferred to a. Hence, candidate a is the Condorcet loser, the worst ranked candidate according to the Condorcet Method. Considering the other four candidates, we see that b is socially preferred to c, c to d, d to e and e to b. Therewith, we have a Condorcet cycle of the form $b \succ c \succ d \succ e \succ b$, making it impossible to use the Condorcet Method to determine a winner (or a ranking).

As far as the Hare system is concerned, in the first round we eliminate the candidate

[6] Actually, the plurality winner a is considered to be the worst alternative under the Borda rule.

with the lowest plurality score, i.e., candidate b. In the second round, the voter who voted for b in the first round now votes for c. Thus, in the second round, c and d are the candidates with the lowest plurality scores (each of them receives 3 votes); we need a pre-defined tie-breaking rule to determine the candidate to be eliminated (we refer to O'Neill (2004) for some tie-breaking rules in connection with the Hare system). For instance, applying forwards tie-breaking the candidate with the higher plurality score in the first round proceeds to the next round, which is candidate d; this results in candidate e being eliminated in round 3 and finally candidate d winning the election. On the other hand, applying Borda tie-breaking we eliminate d instead of c because c has the higher Borda score; this leads to an elimination of candidate c in the next round and candidate e finally winning the election.

Voting rules based on qualitative or cardinal information. The previous voting rules were based on ordinal preference information. However, we could also use different information as used in Disapproval Voting and various types of evaluative voting such as the 100 Points Rule or the ± 20 Points Rule. The following Table 2 presents hypothetical values for those rules consistent with the individual rankings in Table 1.

$A(B)$	5	$A(B)$	4	$A(B)$	3	$A(B)$	2	$A(B)$	1
$30(20)$	a	$30(20)$	e	$70(10)$	d	$45(10)$	c	$35(10)$	b
$25(2)$	b	$25(5)$	b	$15(-1)$	e	$40(-1)$	d	$30(5)$	c
$20(1)$	c	$20(3)$	c	$10(-2)$	b	$10(-2)$	e	$25(-5)$	d
$15(-5)$	d	$15(-3)$	d	$5(-5)$	c	$5(-3)$	b	$10(-7)$	e
$10(-6)$	e	$10(-5)$	a	$0(-7)$	a	$0(-4)$	a	$0(-10)$	a

Table 2: Preference profile of 15 voters with additional preference information.

Table 2 indicates for each alternative in each ranking two numbers, namely the value assigned to the candidate under the 100 Points Rule (A) and (in brackets) the value assigned to the same candidate under the ± 20 Points Rule (B). All the numbers are consistent with the ranking of the candidates.[7] The horizontal lines in the rankings indicate the threshold between candidates which are approved (above the line) and candidates disapproved (below the line). Those lines are made consistent with the values of the ± 20 Points Rule in the sense that only candidates that receive positive values under that rule are approved.[8] Let us start with Disapproval Voting. Assigning one point to every candidate above the lines in the rankings, candidate c wins with 12 points. Applying the 100 Points Rule, we see that candidate d wins with 450 points (or an average of 30 points). Finally, under the ± 20 Points Rule candidate a wins with 41 points (or an average of 2.73 points). Interestingly, the social rankings of the 100 Points Rule and the ± 20 Points Rule are exactly the opposite. Whereas the 100 Points Rule ranks the candidates as $d \succ c \succ b \succ e \succ a$, the ± 20 Points Rule leads to the social ranking $a \succ e \succ b \succ c \succ d$.

[7] Beware that individual answers to different query modes need not be consistent. See Grundner (2017) for a study of different query modes using real-world data.

[8] To make the table more readable, we assumed that every group of voters with the same preference ranking also has the same valuation and approval set. Of course this does not hold in general.

Summarizing the application of the different voting rules on the above preferences, there is a plethora of possible outcomes. Actually each alternative is the winner under one of the discussed voting rules and the Condorcet Method leads to a social preference cycle. The results are collected in Table 3.

Plur	Runoff	Hare	Borda	Cond	Appr	100 points	± 20 points
a	e	d/e	b	$\emptyset$	c	d	a

Table 3: Winning candidates under different voting rules.

4 Experimental Design and Methodology

The data used in this study was collected in the two weeks prior to the recent parliamentary elections in the Austrian federal state of Styria, which took place on 31 May 2015. In cooperation with the *Institut fuer Strategieanalysen (ISA)* we developed an online questionnaire which was sent to a representative sample of which 916 voters responded. This sample consisted of experienced respondents, who had already participated in other online-surveys. Two weeks before the actual election date the participants received a link to a corresponding web-page on which they could access the questionnaire. The questionnaire included various different questions concerning personal views about the candidates, general attitudes towards the political system, preferences about coalitions in the regional government and, in particular, questions about their preferences over the candidates running for the election. In that respect, we asked for three different types of preference information. First, respondents were required to provide a strict ordinal ranking of the candidates. Second, we asked the candidates for qualitative statements about the candidates. In particular, they had to state the candidates which they disapprove of.[9] Third, they were allowed to provide cardinal preference information in two different ways. One question asked them to freely assign 100 points among the set of candidates, the second question allowed them to independently assign each candidate points from the interval $[-20, 20]$.

All the respondents had to answer the questions in the same order, backward steps during the survey were not possible. Hence, certain inconsistencies in the answers about preferences are to be expected.[10] Concerning the respondents' statement of their ordinal preferences, the questionnaire "forced" them to provide complete rankings as the questionnaire could not be continued otherwise. On the one hand, this guaranteed a sufficiently large set of rather unique ranking data, on the other hand, some respondents may have had difficulties in providing consistent answers. A major advantage of obtaining full linear orders from the respondents over partial orders is that assumptions on the underlying complete orders over the candidates need not be made, thus avoiding certain models of the latter as suggested in Regenwetter et al. (2007).

In our analysis we generated statistical confidence levels through a nonparametric bootstrap as applied to the American Psychological Association presiden-

[9] As the respondents stated the candidates they would not want to be seen in the parliament, we assume, in this binary setting, that all the other candidates are seen as acceptable. See Brams and Fishburn (1983) for a discussion on approval voting.

[10] See Grundner (2017) for a study about consistency under different query modes.

tial elections by Regenwetter et al. (2007). This approach allows us to evaluate whether the voting outcomes are unaffected by small perturbations of the ballot frequencies, respectively the statistical confidence level of whether the outcomes are unaffected by small perturbations. In the bootstrap, from the original dataset we created 1000 samples; each sample was made up of 916 pseudoprofiles (this corresponds to the number of voters in the original dataset) by sampling, with replacement, ballots from the original empirical data. Then, for each sample we determined and compared the outcomes under the different voting rules.

5 Empirical Results

Let us start with the voting outcomes under different voting rules for the originally collected dataset. The applied voting rules are those presented in Section 2 and are based on different types of preference information stated by the voters. The "point estimate" results for the received data in the online survey under the various voting rules is presented in Table 4.[11] For instance, applying Plurality Rule yields a collective ranking with a top-ranked, c second-ranked and so on, and f bottom-ranked.

VOTING RULE	RANKING
Plurality Rule	*acbdeghf*
Plurality Runoff	*acbdeghf*
Hare System	*acbdeghf*
Condorcet Method	*abdcegfh*
Borda Rule	*abdecgfh*
$\pm$20 Points Rule	*abdcegfh*
100 Points Rule	*abcdegfh*
Disapproval Voting	*abedgchf*

Table 4: Outcomes under different voting rules in the original dataset.

Interestingly, the outcomes seem to depend on the preference information used by the voting rule. In particular, all voting rules based on Plurality Rule (and therefore in at least one step only using information about the top ranked candidates), rank candidate c second and candidate b third. Whenever a voting rule uses

[11] For the sake of readability we used letters instead of party names in the text and tables. The following list states the parties behind those letters:

a ... SPÖ - Sozialdemokratische Partei Österreichs (Social Democratic Party of Austria)
b ... ÖVP - Österreichische Volkspartei (Austrian People's Party)
c ... FPÖ - Freiheitliche Partei Österreichs (Freedom Party of Austria)
d ... Die Grünen - Die Grüne Alternative (The Greens - The Green Alternative)
e ... KPÖ - Kommunistische Partei Österreichs (Communist Party of Austria)
f ... TS - Team Stronach
g ... NEOS - Das Neue Österreich und Liberales Forum (The New Austria and Liberal Forum)
h ... Piraten - Piratenpartei Österreichs (Pirate Party of Austria)

On a left-right ideological scale, the parties Greens, KPÖ, SPÖ and Piraten are considered rather left, NEOS central, ÖVP and TS central to right, and FPÖ rightwing. This common perception of the parties is confirmed by the result of a question contained in the survey and available from the authors upon request.

more preference information, whether complete rankings, qualitative evaluations or cardinal information, this changes and candidates b and c switch their relative positions in the social ranking. Moreover, Plurality Rule, Plurality Runoff and the Hare System lead to exactly the same outcome. All voting rules using more preference information agree on the order of all candidates but candidate c, except for Disapproval Voting, which deviates substantially. The winner, however, is the same (candidate a) under all voting rules. Finally, a Condorcet cycle does not exist, actually the outcome under the Condorcet method is almost identical to the Borda ranking, which is somewhat surprising given the ongoing debate about the (theoretical) differences between the Borda Rule and the Condorcet Method (see, e.g., Klamler (2005)).

To be able to evaluate the sensitivity of the findings concerning small perturbations within the ballot counts, we bootstrapped the sample. In particular, we generated 1000 nonparametrically bootstrapped pseudo-profiles by drawing as many voters for the pseudo-profile as there were voters in the original dataset (916 voters). This allowed us to analyse the differences in the outcomes of the various voting rules with a statistical confidence level. Table 5 provides all the outcomes for each of the voting rules with their bootstrapped statistical confidence (BC).

In particular, in Table 5, for the sake of readability for each voting rule we report all outcomes with a confidence level of above 1% in decreasing order. It turned out that the Borda Rule led to the lowest number of different rankings (three), whereas Disapproval Voting resulted in 20 different outcomes, thus reaching the highest number of different outcomes among the considered voting rules.[12]
Table 5 shows that, for example, the most likely outcome under the Borda Rule is the ranking *abdecgfh* with a bootstrapped confidence level of 87.1%. When comparing each rule's ranking with the highest bootstrapped confidence level to its point estimate (the results for the original dataset displayed in Table 4), it turns out that these perfectly match. In addition, we can also confirm the Condorcet winner and loser with 100% bootstrapped confidence.

Condorcet cycles. Concerning the Condorcet Method, we also checked for the occurrence of Condorcet cycles. While the original dataset did not reveal any such cycle, in 13 out of our 1000 samples we found a Condorcet cycle among some candidates. None of these, however, included the Condorcet winner or loser; more precisely, 11 samples contained a cycle between candidates c, e and g ($c \succ e$, $e \succ g$ and $g \succ c$), and 2 cycles concerning c, d and e ($c \succ d$, $d \succ e$ and $e \succ c$). I.e., in only 1.3% of our samples a Condorcet cycle occurred, which is a reasonably low number and does in fact confirm the findings of Popov et al. (2014) who also detected virtually no incidence of Condorcet cycles in their analysis of the APA presidential elections. Hence, the Condorcet Method may indeed perform well in real-life situations.

Note that the reason for the absence of cycles could, in theory, lie in the single-peakedness of the preferences (see, e.g., Black (1958)). In political elections, if we consider voters to evaluate political parties along a "left-right line" with respect to their political spectrum, this might in fact be a plausible reason. However, the em-

[12] Actually, Plurality Rule led to a total of 9 different rankings, Plurality Runoff to 7, the Hare System to 13, the Condorcet Method to 4, the Borda Rule to 3, Disapproval Voting to 20, and, finally, both the ± 20 Points Rule and the 100 Points Rule to 5 different outcomes each. These numbers are rather low compared with the theoretical possibility of 545835 different outcomes for 8 candidates (with possible ties).

Voting rule	Outcome	BC (%)	Voting rule	Outcome	BC (%)
Plurality Rule	$acbdeghf$	64.7	Borda Rule	$abdecgfh$	87.1
	$acbdegfh$	24.1		$abdegcfh$	7.1
	$abcdeghf$	6.3		$abdcegfh$	5.8
	$abcdegfh$	2.5	$\pm$ 20 Points	$abdcegfh$	53.6
	$acbdgehf$	1.5		$abdecgfh$	22.6
	...			$abdceghf$	12.4
Plurality Runoff	$acbdeghf$	65		$abdecghf$	11.2
	$acbdegfh$	24.2	100 Points	$abcdegfh$	53.2
	$abcdeghf$	6.3		$acbdegfh$	39
	$abcdegfh$	2.5		$abcdeghf$	4.3
	$acbdgehf$	1.5		$acbdeghf$	3.4
	...		Disappr. Vote	$abedgchf$	40.8
Hare System	$acbdeghf$	43.2		$baedgchf$	40.1
	$acdbeghf$	20.4		$aebdgchf$	2.8
	$acbdegfh$	16.6		$beadgchf$	2.7
	$abcdeghf$	6.9		$abedghcf$	2.6
	$acdbegfh$	6.2		$baedghcf$	2.4
	$abcdegfh$	3.2		$ebadgchf$	1.8
	$acbdgehf$	1.5		$baedgcfh$	1.2
	...			$abdegchf$	1.4
Condorcet Method	$abdcegfh$	45.3		$eabdgchf$	1.3
	$abdecgfh$	33.1		$badegchf$	1
	$abdegcfh$	14.8		...	
	$abcdegfh$	5.5			
	cycles	1.3			

Table 5: Outcomes under different voting rules in the bootstrapped samples with statistical confidence levels (BC).

pirical absence of Condorcet cycles in our samples is not due to single-peakedness, since none of our bootstrapped samples contained a single-peaked preference profile.

Finally, we investigate whether or not the theoretical differences between outcomes of different voting rules (see the example in Section 3) do actually show up when using preference data from real-world elections. In this respect, we compare the outcomes of the voting rules in each bootstrapped sample. Let us start with a comparison of the outcomes under the Condorcet Method and the Borda Rule.

Borda Rule and Condorcet Method. Although the outcomes with the highest bootstrapped confidence are different (there is a swap between candidates c and e concerning ranks 4 and 5), 42% of all bootstrapped samples deliver exactly the same ranking outcome for both voting rules. When comparing the winners only, we actually have a 100% agreement, i.e., candidate a wins in each sample irrespective of whether Borda Rule or Condorcet Method is applies. The same holds for the losers in both voting rules (the unique loser, in fact, is always h). The

consensus between the both voting rules is emphasized by the fact that we have a 100% agreement on the candidates ranked first and second, and an agreement of 94.3% over all samples on the candidates ranked first, second, and third. Hence, although historically known for their differences in outcomes, our results reveal a surprising agreement between the Condorcet Method and the Borda Rule.[13]

Plurality Rule, Plurality Runoff and Hare System. Next, we compare the three voting rules based on plurality counts, i.e., those that, at least in the first step of the voting rule, only use the top ranked candidates from the individual ranking information: Plurality Rule, Plurality Runoff Rule and the Hare System. As displayed in Table 5, the outcomes with the highest confidence level under Plurality Rule, the Plurality Runoff Rule and the Hare System coincide. However, the bootstrapped confidence level of the outcome with highest confidence level under the Hare System is about 20 percentage points lower than the ones under the two other rules. In particular, only two thirds of the bootstrapped samples between the Hare System and Plurality Rule respectively Plurality Runoff Rule deliver the identical outcome, whereas almost 100% of the samples lead to the same outcome under Plurality Rule and Plurality Runoff Rule.

Disapproval Voting, ± 20 Points Rule, and 100 Points Rule. Finally, we compare the three voting rules which are not based on pure ranking information but on valuations or qualitative statements as in the ± 20 Points Rule, the 100 Points Rule and Disapproval Voting. Because we have already observed that Disapproval Voting is the only voting rule that deviates substantially from the outcomes of all other voting rules, we expect there to be also a substantial difference when comparing it to the evaluative voting methods ± 20 Points Rule and 100 Points Rule, which might be related to the different informational structure underlying Disapproval Voting. Indeed, although the winner of the ± 20 Points Rule and the 100 Points Rule is the same in all but one of the bootstrapped samples, only in 488 of those samples, which is less than half of all cases, is the Disapproval Voting winner identical to that candidate. In addition, the Disapproval Voting loser, i.e., the candidate ranked last according to Disapproval Voting, deviates from the ± 20 Points Rule loser in about 75% of the cases and from the 100 Points Rule loser in about 90% of the samples. To put this into perspective, there is also no complete agreement among the ± 20 Points loser and the 100 Points loser, but differences only occur in about 20% of all cases (recall the theoretical possibility of the two rules determining completely contrary rankings as in the example provided in Section 3). However, as an interesting fact, limiting the consensus between these two methods to a certain degree, note that the ranking of the top three candidates for those two voting rules does not agree in any of the 1000 bootstrapped samples. Hence, one can conclude that the different options how to assign points to candidates may in fact influence the outcome.

[13] If we compare this with the virtual absence of Condorcet cycles though, then Saari (1999) showed already that in the absence of such cycles the Condorcet Method and the Borda Rule do indeed provide the same outcome in a lot of situations.

6 Conclusion

This paper analyzed data from an online-survey in connection with the 2015 election to the parliament in the Austrian federal state of Styria. Using a bootstrap approach to take into account small perturbations in the ballot frequencies, we checked for the occurrence of paradoxical outcomes in the form of Condorcet cycles and for possible differences in outcomes among various voting rules. Because in the theoretical literature the existence of cycles and differences in voting outcomes seems to be of rather great importance (see, e.g., Saari (2000a,b) and Gehrlein (2006)), we tested whether such results also show up in real-world preference data.

In particular, our analysis showed the following. First, Condorcet cycles did not show up in the survey data and were virtually absent in the 1000 bootstrapped samples. This confirms the empirical findings of Regenwetter et al. (2006) and stands in contrast to theoretical results based on certain specific assumptions about the underlying distribution of preferences which predict a significant number of such cycles. In addition, the Condorcet winners and losers did not change throughout our samples. Second, we analysed how different the outcomes of the investigated voting rules will be. In principle we found a high degree of similarity in the rankings derived from the different voting rules both in the original data and the bootstrapped samples. Comparing the Borda winner with the Condorcet winner, we found a 100% agreement over all bootstrapped samples; in $42,4\%$ of the samples the whole collective ranking of the candidates determined by the two rules was identical. This is somewhat surprising given the theoretical debate about differences between the Borda Rule and the Condorcet Method (see, e.g., Klamler (2005)).
Another strong agreement of almost 100% can be seen between the Plurality Rule and the Plurality Runoff Rule. More significant deviations can only be found when comparing Disapproval Voting with the other voting rules. Compared to evaluative voting rules, Disapproval Voting leads to the identical winner only in less than half of the bootstrapped samples. Among the evaluative voting rules considered, i.e., the ± 20 Points Rule and the 100 Points Rule, the winner is always the same in our bootstrapped samples. However, significant differences in the rankings of the top three candidates show up: the collective rankings determined by the both rules do not coincide in any of the 1000 samples even if restricted to the top three ranks.

Certainly, this study has left out various other interesting questions. In particular, it would be interesting to analyze the actual occurrence of strategic behavior of voters under different voting rules, i.e., how voters would act strategically in different voting situations. However, this would require a different experimental approach and is left for future research.

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