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Abstract

We consider a general framework for multitask moral hazard problems with observable and hidden actions. Ideally, the principal in our framework can design optimal contracts that depend on both observable (and verifiable) actions and realized outcomes. Given a mild assumption on the existence of a punishment scheme, we identify a general equivalence result, dubbed the "forcing principle," which states that every optimal contract in our framework is *strategically equivalent* to a simple forcing contract, which only specifies an outcome-contingent reward scheme and an action profile, and the agent receives the outcome-contingent reward only if he follows the recommended observable actions (and is otherwise punished severely). The forcing principle has useful implications: It confers analytic advantage for the existence and computation of optimal contracts in our setting. It also highlights and makes explicit the importance of the existence of the punishment scheme in characterizing first-best benchmarks in moral hazard problems.

Keywords: Forcing Contract, Forcing Principle, Moral Hazard, Observable Actions, First-best Benchmark

JEL Classification: C61, C62, D82, D86

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1 Introduction

In many real-world contracting problems, an agent's decision will involve various dimensions, some of which may be more easily observed than others. Holmström and Milgrom (1991)[10] provide the first formal analysis of principal-agent problems with multidimensional tasks. An important insight of this study (and subsequent literature on multitasking) is that the optimal design of incentive contracts depends crucially on the precision with which actions can be measured. While this literature has produced many useful insights, much of the work is somewhat limited in scope, and the literature has tended to focus on the particular case of linear contracts, an agent with exponential utility, and signals that are jointly normally distributed.[1]

In this paper, we study an optimal contracting model with moral hazard and multidimensional tasks, but diverge from previous multitasking studies in two dimensions.[2] First, we focus on an environment in which the agent takes both observable and unobservable actions. Such scenarios are not difficult to imagine. For example, although the time a worker spends at work may be observable, the mental or physical effort he devotes to his job may not be easily measured. In a relationship between a venture capitalist and an entrepreneur, while the entrepreneur's investment in capital is easily verifiable, the entrepreneur's daily effort to run the business may be too costly to monitor. Finally, in the context of insurance provision, the insurer may be able to observe the number of doctor's visits per year, but the daily preventive care taken by the insured party is difficult to observe. Although other studies have allowed for both observable and unobservable actions, we explicitly take both into account, and explore—in a general framework—how the agent's observable actions enter optimally into contracts in a somewhat simple manner.

Second, we impose minimal structure: We only require the outcome, action, and reward spaces to be metric spaces, and each of these sets can be multi-dimensional, potentially non-compact, finite or infinite, countable or uncountable. Moreover, contracts in our framework are only assumed to be Borel measurable, although we allow for the possibility of other contracting constraints. In addition, we allow for arbitrary interdependencies between the agent's actions, arbitrary risk attitudes, and non-separable and discontinuous utility functions.

Specifically, we consider an abstract moral hazard setting in which a principal hires an agent for a project with multiple tasks. The agent chooses both observable and hidden

[1] This is the so-called "linear-exponential-normal (LEN)" model in the literature.

[2] We equate multitasks with multidimensional actions in our analysis, although there could be literal differences in different contexts.

actions, resulting in stochastic outcomes in which both observable actions and realized outcomes are verifiable. With so little structure in our baseline model, one might hypothesize that an optimal contract (should one exist) would employ complex reward schemes that link the agent's payment non-trivially to both observable actions and outcomes.[3]

To the contrary, we establish a general *forcing principle*, which demonstrates the relatively simple manner by which observable actions should optimally be taken into account by the principal. Under a relatively mild assumption which ensures the existence of a severe punishment available to the principal, we first show that there is no loss of generality for the principal to restrict attention to a particular set of contracts (i.e., punishing contracts) by which the agent receives outcome-contingent payment *only if* he has chosen the observable actions recommended by the principal, and is severely punished (i.e., obtaining a payoff no more than the agent's outside option) otherwise.[4] Under such punishing contracts, both the principal and the agent obtain the same expected utilities as in any other optimal contract in the setting. Using this result as a building block, we then establish the forcing principle: The solution to the original contracting problem can be obtained by solving an equivalent, but simplified forcing contracting problem in which the principal "chooses" the observable action directly, and can restrict attention to outcome-contingent reward schemes in which reward variations are independent of the observable actions. We also highlight the relevance of the punishment condition by demonstrating that the equivalence may not be preserved if the condition is violated (see Example 3).

The simple contracts (in which the optimal contracts depend on observable actions in a simple fashion) are closely related to the concept of *forcing contracts* in many applications of contract theory. The use of forcing contracts dates to the early moral hazard literature, and this structure is frequently mentioned in the contracting literature, especially in the characterization of first-best benchmarks (see Section 4.1).[5] However, as far as we are aware, the usefulness and robustness of such contracts has *not* been explored (at least not in a general framework). Our forcing principle provides a single sufficient assumption on the existence of a suitable punishment scheme available to the principal, under which any optimal contract can be equivalently written as a simple forcing contract. Crucially, we do not impose the usual technical assumptions (e.g., continuity, compactness, etc.) to establish the principle. Our result thus demonstrates remarkable robustness of our forcing

[3]For example, the famous Gantt's task and bonus plan in industrial management specifies a worker's reward scheme that varies in both the worker's output and working hours in a complex way, which are called *hybrid* reward schemes hereafter. See Example 1.

[4]A punishing contract is defined as $\tilde{k}$ in Proposition 1 in Section 3.

[5]See Mirlees (1974)[18] and Grossman and Hart (1983)[8], among many others.

principle (and hence the optimality of simple forcing contracts) in contracting problems with partially observable actions, in the sense that the forcing principle is not sensitive to the details of the model. Indeed, the power of our result comes from the general conditions under which it holds, implying its applicability to a wide range of settings.

Our forcing principle has useful practical implications in general settings of applications with both observable and unobservable actions. In particular, it implies that there is no need for the principal to design and implement complex hybrid contracts with rewards that depend intricately on both observable actions and outcomes. In fact, as long as the principal can determine whether the *correct* action has been taken, she need not possess the ability to precisely measure every observable action of the agent.

The forcing principle also has useful theoretical applications. The first application of the forcing principle has to do with the characterization of first-best benchmarks in moral hazard problems with only observable and verifiable actions, which is a special case of our framework (without hidden actions). The standard approach in the literature is to characterize the first-best benchmark using a forcing contract (similar to our simple forcing contracts) in which the principal forces an action on the agent without any incentive compatibility constraint. The forcing principle, when applied to such a setting, implies that the standard approach of considering a forcing contract for the first-best benchmark is valid *if* a suitable and severe punishment scheme is available to the principal. When this assumption is violated, however, we demonstrate via an example that the forcing contract employed in the standard approach may fail to identify the first-best benchmark. Our result hence highlights the importance of the assumption on the existence of a severe punishment scheme in the characterization of first-best benchmarks, which has not been emphasized explicitly in the literature.[6]

Another useful application of the forcing principle is that it greatly simplifies the contracting problem and enables us to establish the existence of optimal *deterministic* contracts in our setting under mild topological conditions. Our approach here is most closely related to Page (1987)[20], who establishes the existence of optimal deterministic contracts in a pure moral hazard setting. Page's approach is fairly general, allowing in particular for an infinite dimensional contract space and for the principal and the agent to hold different beliefs about the random outcomes generated by the agent's actions. Nevertheless, his approach cannot be directly applied to our setting with both observable and hidden actions.[7] We establish that under the forcing principle, one avoids the

[6] This is perhaps a minor issue for the moral hazard literature, as we discuss in detail at the end of Section 4.1. Nevertheless, we think that it is still sensible and useful to raise the issue explicitly.

[7] Technically, it is difficult to establish sequential continuity of the two parties' expected utility func-

complications of a hybrid reward scheme, and Page's approach can then be adapted to establish existence in our setting.

Finally, our forcing principle can simplify the analytical computation of optimal contracts in our moral hazard setting. In particular, the principal can now search for optimal contracts in a strictly smaller set, and the agent's incentive constraint is simplified to the choice of only hidden actions. Technically, a standard first-order approach (FOA) cannot be directly applied to find the optimal contract in the original general contracting problem; however, given the forcing principle, the FOA approach can be applied in the equivalent but simpler contracting problem. We discuss these simplifications at greater length in Section 4.3 (and Example 4), in which we explicitly compute the optimal contract for a simple moral hazard problem with the help of our forcing principle.

1.1 Related Literature

As stated, our paper is related to the literature on multitask principal-agent problems that originated with Holmström and Milgrom (1991)[10]. One particular result in this literature is that the power of incentives for some tasks depends on the principal's ability to monitor various aspects of the agent's performance. High-powered incentive contracts can reduce performance, because the agent can shift his effort from poorly measured activities to better measured and more highly compensated ones. While our paper also suggests that observed actions be used solely as thresholds rather than active components of the agent's payments, our results are obtained in an abstract setting and are motivated by consideration of *simplicity*.

Our paper is also related to a strand of literature that analyzes optimal contracting issues in some specific multitask moral hazard problems with both hidden and observable actions (e.g., Laux 2001[17]; Zhao 2008[25]; and Chen 2010[3], 2012[4]). These studies consider similar settings, in which a cost-minimizing principal hires risk-neutral agent(s) protected by limited liability to work on multiple tasks and the agents' effort choices in some tasks are observable. However, they analyze different issues such as task clustering and job design (Laux 2001[17]); choosing between input monitoring and output monitoring (Zhao 2008[25] and Chen 2010[3]); and all-or-nothing payment structures (Chen 2012[4]). While these studies are important precursors and have characterized optimal contracts as forcing contracts similar to the ones in this study, we consider a more general setting and a different objective, by which we aim to establish a general forcing principle and put

tions via Delbaen's Lemma, crucial in Page's approach, when the observable action component also enters the reward scheme. We discuss this in detail in Section 4.2.

this principle to work. Nevertheless, in the context of this literature, our paper makes an important contribution by providing a foundation for focusing on forcing contracts rather than on more complicated reward schemes in these settings, which, given our forcing principle, is without loss of generality.

Finally, the existence of optimal contracts is an important issue in moral hazard problems, and the forcing principle also enables us to establish the existence of an optimal contract in a general moral hazard setting with both hidden and observable actions. As noted above, Page (1987)[20] provides the first methodological contribution on the existence of deterministic optimal contracts in a (pure) moral hazard model with a general topological structure. The most general existence result to date is established recently by Kadan, Reny and Swinkels (2017)[13], who provide conditions for the existence of optimal (randomized) mechanisms for general principal-agent problems that cover a wide array of economic settings and encompass pure moral hazard problems, pure adverse selection problems, and problems with both, as special cases. While general, the existence result in Kadan, Reny and Swinkels (2017)[13] is mainly for optimal randomized mechanisms. However, we impose the restriction that the mechanisms in our setting are deterministic, and our existence result is not implied by theirs.[8] Our existence analysis hence is more closely related to that in Page (1987)[20], but our setting differs from his by including observable actions. Specifically, our analysis takes advantage of the forcing principle and follows Page's approach to establish the existence of an optimal deterministic contract in general moral hazard settings with both unobservable and observable actions.

2 The Model

We consider a principal-agent relationship in which a principal (she) hires an agent (he) to make a one-time choice of multi-dimensional actions on a project. The timing of the contracting game is standard: The principal proposes a take-it-or-leave-it contract to the agent. The agent then decides whether to accept the contract. If he accepts the contract, the agent chooses a vector of actions and is then rewarded, according to the contract, upon the outcomes generated by his actions.

The agent chooses two types of actions, observable and unobservable. We assume that the agent's observable actions are contractible and his unobservable actions are not.

[8] Section 12 of Kadan, Reny and Swinkels (2017)[13] contains results on optimality of deterministic contracts in pure moral hazard and other restricted settings, which do not cover our moral hazard setting with both hidden and observable actions.

Let $A = A_0 \times A_1$ be the set of all possible action profiles available to the agent where A_0 is a metric space containing all possible observable actions and A_1 is a metric space containing all possible unobservable actions available to the agent.[9] A typical action profile of the agent is denoted as $(a_0, a_1) \in A$ with an observable component $a_0 \in A_0$ and an unobservable component $a_1 \in A_1$.

After the agent chooses his actions (a_0, a_1), nature moves by stochastically generating an *outcome* ω based on (a_0, a_1), where $\omega \in \Omega$ and Ω is a metric space of all possible outcomes. Here an outcome ω can be multi-dimensional and may include both monetary components (e.g., sales revenues generated by a salesperson) and non-monetary components (e.g., service evaluations of the salesman by clients). Let $B(\Omega)$ denote the Borel σ-field over the outcome space Ω. For each action profile (a_0, a_1), let $P(\cdot; a_0, a_1) : \Omega \to [0, 1]$ be a probability measure defined on $(\Omega, B(\Omega))$. Observe that $P(\cdot; a_0, a_1)$ is (a_0, a_1)-dependent, and it is common knowledge between the principal and the agent that the observable outcomes will be generated according to the (same) probability measure $P(\cdot; a_0, a_1)$.

We next define the principal's strategies. A strategy of the principal is a *reward scheme* contingent on outcome ω and action a_0, which are both observable and contractible. Let D be a metric space of all possible end-of-period *rewards* for the agent, with typical element $d \in D$. Like outcome ω, a reward d can be multi-dimensional and contain both monetary (e.g., salaries) and non-monetary (e.g., promotions) components. Define a function set

$$\mathcal{M} := \{s : \Omega \to D | s \text{ is a Borel Measurable function}\},$$

with element s. $\mathcal{M}$ is hence the set of *pure* outcome-contingent reward schemes.

Let $\mathcal{K} \subseteq \mathcal{M}$ represent all available, pure outcome-contingent reward schemes under certain primitive restrictions on contracting practices.[10] Given that observable actions are contractible, it is natural for the principal to also include observable actions in her contract. We define the set of all possible *hybrid* reward schemes to be

$$\mathcal{R}_{\mathcal{K}} := \{f : \Omega \times A_0 \to D | f \text{ is Borel measurable, and } f(\cdot, a_0) \in \mathcal{K} \ \forall a_0 \in A_0\},$$

with typical element f. By definition, a hybrid reward scheme f typically depends on

[9] Here and in the sequel, the *observability* of a given object implies both verifiability by a third party and measurability of the object with respect to the corresponding distribution function.

[10] To be specific, $\mathcal{K}$ denotes the set of all practically available pure outcome-contingent reward schemes, given current market or technology conditions, legal customs, and the principal's abilities for computation and accounting, etc., as in Page (1987)[20]. For instance, $\mathcal{K}$ can be the set of bounded and monotone contracts or a set of contracts that are restricted to be linear in outcomes.

both outcomes and observable actions.

The principal benefits from ω and pays $f(a_0,\omega)$ to the agent. Denote the principal's vNM utility function as $u:\Omega\times D\times A\to\mathbb{R}$, and the agent's vNM utility function defined over rewards and actions as $v:D\times A\to\mathbb{R}$. Both u and v are Borel measurable functions. The expected utility of each party can be written as:

$$\begin{aligned}\text{Principal:}\qquad U(f,a_0,a_1)&=\int_\Omega u(\omega,f(\omega,a_0),a_0,a_1)P(d\omega;a_0,a_1),\\ \text{Agent:}\qquad V(f,a_0,a_1)&=\int_\Omega v(f(\omega,a_0),a_0,a_1)P(d\omega;a_0,a_1).\end{aligned}$$

Notice that to allow for general preferences, we have implicitly built the reward and the effort cost in the utility functions u and v.

The principal's objective is to design an optimal (possibly) *hybrid* contract (f,a_0,a_1) consisting of a reward scheme f and action recommendations (a_0,a_1). The principal's problem (**P1**) is formulated as follows:[11]

$$(\mathbf{P1})\quad \begin{array}{ll} & \max_{(f,a_0,a_1)\in\mathcal{R}_\mathcal{K}\times A} U(f,a_0,a_1)\\ \text{s.t.} & \text{(IR): } V(f,a_0,a_1)\geq r\\ & \text{(IC): } (a_0,a_1)\in\arg\max_{(a_0',a_1')\in A} V(f,a_0',a_1')\end{array}$$

As usual, the (IR) constraint guarantees the agent's expected utility from the contract to be at least his reservation utility r, and the (IC) constraint requires that the recommended action profile be optimal for the agent. Importantly, notice that in the (IC) constraint, though a_0 is verifiable, there is no *forcing* in (**P1**), since the principal does not directly "choose" the recommended a_0 for the agent.[12]

Definition 1 *A contract (f,a_0,a_1) is **feasible** if $f\in\mathcal{R}_\mathcal{K}$ and f satisfies (IR) and (IC).*

To illustrate the general principal-agent problem, consider a concrete example:

Example 1 *Consider an employer-employee relationship. The employee chooses observable working hours $a_0\in[0,24]$ and hidden effort $a_1\in[0,1]$. An action profile (a_0,a_1) stochastically generates a monetary outcome $z\in Z$ via a distribution function $G(z;a_0,a_1)$*

[11] In (**P1**) if A_0 is a singleton, then the problem reduces to a pure moral hazard model, while if A_1 is a singleton, then (**P1**) corresponds to the first-best contracting problem with only observable (and verifiable) actions. We use the notation "max" rather than "sup" in the principal's and agent's problems in (**P1**). The existence of solutions in (**P1**) will be established with further assumptions in Section 4.

[12] For future comparison, such *forcing* will take place in (**P2**) in Section 3.

with density $g(z; a_0, a_1)$. The employer offers a monetary reward $d \in D$ according to a scheme $f : Z \times A_0 \to D$. The employee's payoff is $v(d, a_0, a_1) = \varphi(d) - C(a_0, a_1)$, with $\varphi(d)$ being the utility from the reward d and $C(a_0, a_1)$ the cost from (a_0, a_1). The principal's utility is $u(z - d)$. The resulting principal-agent problem is

$$\begin{array}{rl} & \max_{(f,a_0,a_1)\in\mathcal{R}_{\mathcal{K}}\times A} \int_Z u(z - f(z, a_0))g(z; a_0, a_1)dz \\ s.t. & \begin{array}{l} \int_Z \varphi(f(z, a_0))g(z; a_0, a_1)dz - C(a_0, a_1) \geq r \\ (a_0, a_1) \in \arg\max_{(a_0',a_1')\in A} \int_Z \varphi(f(z, a_0')g(z; a_0', a_1')dz - C(a_0', a_1') \end{array} \end{array}$$

Overall, we have imposed little structure in our model in order to obtain a general moral hazard framework. In particular, the spaces A_0, A_1, Ω and D can all have multiple dimensions, with some being discrete and some continuous, and the two parties' utility functions are allowed to be non-separable in the corresponding arguments. As we shall see shortly, together with an additional assumption, our forcing principle will hold in such a general framework, so that a simple forcing contract solves the principal's optimization problem (**P1**).

3 The Forcing Principle

The celebrated informativeness principle of Holmström (1979)[9] posits that additional information that is incrementally informative about an agent's action should be included in the optimal contract. It stands to reason that optimal contracts in our setting should involve complex reward schemes that depend non-trivially on both the outcomes and the observable actions as addressed in (**P1**).[13] A case in point here is Gantt's famous task and bonus plan in industrial management (Periasamy 2010[21], p. 390):

Example 2 *Gantt's task and bonus plan involves the careful design of an incentive system using time rate, differential piece rate and bonus. To illustrate, the principal observes both the time worked a_0 and output ω (but not how hard the agent works a_1). Gantt's plan specifies a wage scheme $f(\omega, a_0)$ with parameters α, β, and $\overline{\omega}$:*

$$f(\omega, a_0) = \begin{cases} \alpha a_0 & \textit{if } \omega < \overline{\omega} \\ (\alpha + 20\%)a_0 & \textit{if } \omega = \overline{\omega} \\ \beta\omega & \textit{if } \omega > \overline{\omega} \end{cases}$$

[13]It is known, however, that the informativeness principle may not hold in a multitask principal-agent models (see, e.g., Holmström and Milgrom 1987, 1991).

i.e., a time rate α is applied when output ω is below a standard $\overline{\omega}$, a bonus time rate $(\alpha + 20\%)$ is employed when ω hits the standard, and a higher piece rate β is employed otherwise. This task and bonus plan was identified as quite successful by Gantt (1919)[7].[14]

Given our general framework and the complexity of reward schemes, however, the characterization of optimal contracts in (**P1**) is rather unmanageable. An alternative and simpler contracting approach for the principal is to downplay (but not neglect) the role of observable actions and focus instead on the main *performance indicator* of the agent, i.e., on the outcome ω.[15] Specifically, the principal now imposes an explicit choice of a_0 and searches for a pure *outcome-contingent* reward scheme, rather than a complex hybrid reward scheme. Such a contracting procedure can be formally described as

$$
(\mathbf{P2}) \quad \begin{array}{ll} & \max_{(s,a_0,a_1)\in\mathcal{K}\times A} \int_\Omega u(\omega, s(\omega), a_0, a_1) P(d\omega; a_0, a_1) \\ \text{s.t.} & \text{(IR): } \int_\Omega v(s(\omega), a_0, a_1) P(d\omega; a_0, a_1) \geq r \\ & \text{(IC): } a_1 \in \arg\max_{\hat{a}_1\in A_1} \int_\Omega v(s(\omega), a_0, \hat{a}_1) P(d\omega; a_0, \hat{a}_1) \end{array}
$$

An optimal contract solving (**P2**) is called a *forcing* contract. Such a formulation of forcing contracts is analogous to the standard treatment of observable and verifiable actions in the literature: Since observable actions are verifiable, the choice of a_0 can be forced upon the agent, and it is as if the principal now directly chooses a_0 for the agent. As a result, the choice of a_0 does not enter the (IC) constraint.

The contracting problem (**P2**) is simpler than (**P1**): The (IC) constraint in (**P2**) only involves a_1 and is hence simpler for both the principal and the agent. In addition, the principal chooses a pure outcome-contingent reward scheme in $\mathcal{K}$ (rather than a more complex hybrid reward scheme in $\mathcal{R}_\mathcal{K}$). The simpler formulation in (**P2**) then motivates the natural question of whether optimal solutions for (**P2**) are also optimal contracts for (**P1**), and vice versa. We demonstrate in the rest of this section that this is indeed the case for the contracting problems (**P1**) and (**P2**) as long as a punishment scheme is available for the principal (Assumption 1). As a result, simple forcing contracts are optimal for the original contracting problem (**P1**).

The forcing contracts solving (**P2**) (and (**P1**) under Assumption 1) are *simple* in the sense that compared to optimal hybrid contracts, such forcing contracts are easier to

[14] "For Bethlehem Steel, the average monthly output of the shop from March 1, 1900, to March 1, 1901, was 1,173,000 pounds, and from March 1, 1901, to August 1, 1901 (having implemented the task and bonus system) was 2,069,000 pounds. The shop had 700 men in it and we were paying on the bonus plan only about 80 workmen out of the entire 700." (Chapter VII, Gantt 1919).

[15] This intuitive approach is perhaps consistent with many business practices and academic research emphasizing the importance of **key performance indicators** for successes in various organizations.

characterize in (**P2**) and the reward schemes for these contracts only need to be outcome-contingent.[16] In addition, again compared to hybrid contracts, these forcing contracts simplify the decision making for both the principal and the agent in the setting.

Importantly, notice that optimal forcing contracts from (**P2**) are *not* necessarily always (weakly) worse than optimal contracts from (**P1**): Although the principal can choose a more general hybrid reward scheme in (**P1**), the two contracting problems are not *nested*: While the principal restricts attention to outcome-contingent reward schemes in (**P2**), the (IC) constraint in (**P2**) is less stringent in that it does not require the agent to optimally choose a_0. As a result, the set of *feasible contracts* (cf. Definition 1) in (**P2**) is not necessarily strictly smaller than that in (**P1**). Indeed, we show later in Example 3 (Section 4.1) that a forcing contract for (**P2**) can actually lead to a strictly better outcome for the principal compared to an optimal hybrid contract for (**P1**).

To formally investigate the relationship between the contracting problems (**P1**) and (**P2**), we start with the following assumption:

Assumption 1 (Punishment) *There exists a punishment scheme $t \in \mathcal{K}$ such that for all $(a_0, a_1) \in A$, $\int_\Omega v(t(\omega), a_0, a_1) P(d\omega; a_0, a_1) \leq r$.*

Assumption 1 says that the principal has the option of using a (possibly outcome-contingent) punishment scheme that offers the agent no more than his reservation utility, no matter what actions he takes. Thus, facing the reward scheme t, the agent will have no incentive to participate given his outside option, so the punishment scheme t is *severe*. The existence of such a punishing reward scheme is a relatively mild assumption in contracting scenarios. For example, as long as the principal has the option of offering a sufficiently low reward scheme, i.e., if $f(\omega, a_0) = c$ for all (ω, a_0) is an element in $\mathcal{R}_\mathcal{K}$ such that the agent's utility $v(c, a_0, a_1) \leq r$ for all (a_0, a_1), then Assumption 1 will be satisfied.[17]

We first present an intermediate step toward the equivalence between (**P1**) and (**P2**). Specifically, we show that every reward scheme in a feasible hybrid contract can be equivalently transformed into a simple reward scheme with an outcome-contingent component and a punishment component, which leads to the same expected utility to both the principal and the agent (Proposition 1).[18]

[16] The outcome-contingent portion of the reward scheme however is similar to an optimal contract solving a canonical principal-agent model in Holmström (1979), which may not take a simple form.

[17] If on the other hand the agent is protected by limited liability or minimum-wage legislation from receiving harsh putative wages, then Assumption 1 will be violated. See Section 4.1.

[18] All proofs in the paper are relegated to an Appendix.

Proposition 1 *Suppose Assumption 1 holds. For any feasible contract* $k^* = (f^*, a_0^*, a_1^*)$*, consider another contract* $\tilde{k} = (\tilde{f}, a_0^*, a_1^*)$ *such that* $\tilde{f}(\omega, a_0) = \begin{cases} s(\omega), & \text{if } a_0 = a_0^* \\ t(\omega), & \text{otherwise} \end{cases}$*, where* $s(\omega) \equiv f^*(\omega, a_0^*)$*. Then the contract* $\tilde{k}$ *is also feasible, and the principal and the agent receive the same expected utilities under* k^* *and* $\widetilde{k}$*.*

One perhaps useful angle from which to view Proposition 1 is that it has a similar flavor as the *revelation principle* in the context of mechanism design: Proposition 1 implies that for problem **(P1)**, it is without loss of generality to consider a proper subset of hybrid contracts with typical element $\tilde{k}$ rather than a general hybrid contract in $\mathcal{R}_{\mathcal{K}}$.[19] The scheme $\tilde{f}$ in the contract $\tilde{k}$ employs a simple punishment to induce the agent to choose the principal's recommended observable actions. The observable actions merely serve the role of a "threshold" in the reward scheme (for punishment), and the main variations in the reward scheme are then contingent only on the outcome ω to incentivize the agent to choose the recommended unobservable actions. The contract $\tilde{k}$ in Proposition 1 can be regarded as a forcing contract with punishment, or a punishing contract. It is intuitive and perhaps also consistent with some contracts in practice. For example, to enforce an eight-hour workday requirement (the recommended observable action a_0^*) for a worker, a manager can lower his wage or fire him, if the worker fails to meet the requirement.

Given Proposition 1, we can restrict the principal's choice of reward schemes to the class of pure outcome-contingent reward schemes, augmented by the punishment scheme $t(\omega)$ in Assumption 1. Using this as a crucial building block, Theorem 1 below formally presents our *forcing principle*, which states that the original contracting problem (**P1**) can be equivalently transformed into the simple forcing contracting problem (**P2**) under Assumption 1.

Theorem 1 (Forcing Principle) *Under Assumption 1, **(P1)** is strategically equivalent to **(P2)**, i.e.,*

1. *Given an optimal solution* (f^*, a_0^*, a_1^*) *to **(P1)**, there is an optimal solution* $(s^\dagger, a_0^\dagger, a_1^\dagger)$ *to **(P2)**, where* $(a_0^\dagger, a_1^\dagger) = (a_0^*, a_1^*)$*,* $s^\dagger(\omega) = f^*(\omega, a_0^*)$ *for all* $\omega \in \Omega$*, and the agent receives reward scheme* $t(\omega)$ *in Assumption 1 whenever* $a_0 \neq a_0^\dagger$*.*

[19] To be specific, $\tilde{k}$ in Proposition 1 is a feasible contract for the contracting problem ($\tilde{\mathbf{P}}\mathbf{1}$) in the proof of Theorem 1 below, which *only* differs from (**P1**) in that the reward schemes in ($\tilde{\mathbf{P}}\mathbf{1}$) are restricted to be outcome-contingent (i.e., in $\mathcal{K}$ rather than in $\mathcal{R}_{\mathcal{K}}$).

2. *Given an optimal solution* $(s^\dagger, a_0^\dagger, a_1^\dagger)$ *to* ***(P2)****, there is an optimal solution* (f^*, a_0^*, a_1^*) *to* ***(P1)****, where* $(a_0^*, a_1^*) = (a_0^\dagger, a_1^\dagger)$ *and for all* $\omega \in \Omega$*,*

$$f^*(\omega, a_0) = \begin{cases} s^\dagger(\omega), & \textit{if } a_0 = a_0^* \\ t(\omega), & \textit{otherwise} \end{cases} \tag{1}$$

3. *Both the principal and the agent attain the same expected utility in* ***(P1)*** *and* ***(P2)****.*

The forcing principle summarized in Theorem 1 implies that for a general moral hazard problem with observable and hidden actions, an optimal contract can be found by solving a simpler problem in which the principal chooses a_0 directly, so long as the agent can be sufficiently punished when deviating from the recommended observable action.[20] Moreover, both the principal and the agent will obtain the same expected utilities under (**P1**) and (**P2**), and hence neither one has any incentive to deviate from the simpler contracting procedure (**P2**).

The optimality of the simple forcing contract in (**P2**) is of practical significance. In implementing the simple forcing contract, the principal only needs to detect whether the agent has deviated from the recommended observable action. In particular, there is no need for the principal to specify different punishments for different deviations in the observable actions—in practice, measuring such deviations precisely can be quite costly. Thus, the informational demands of these simple contracts can be significantly less than what is required for more general contracts.

Finally, Assumption 1 is crucial for Theorem 1. As we show in the proof of Theorem 1, the equivalence (**P1**) and (**P2**) is obtained by establishing a successive sequence of equivalent maximization problems. Assumption 1 plays two important roles in driving the equivalence among these maximization problems: First, Assumption 1 enables us to consider an intuitive and restricted set of feasible contracts, i.e., punishing contracts in Proposition 1. Second, Assumption 1 also allows for a reduction of the (IC) constraint in (**P1**) to one that only involves the choice of unobservable actions. Absent Assumption 1, the equivalence of these maximization problems breaks down. As illustrated previously, without Assumption 1, the sets of feasible contracts in (**P1**) and (**P2**) are not *nested*, which obscures a general comparison between optimal contracts in (**P1**) and (**P2**). Indeed, absent of Assumption 1, it is difficult to characterize the general relationship between

[20] In such simple contracts, the principle can simply state some necessary or minimal requirements on a_0 for the agent to get any compensation (for example, minimal working hours, project completion deadlines, exclusive sales territories, etc).

the two contracting problems (**P1**) and (**P2**) without imposing further structure in our general moral hazard framework.

4 Applications of the Forcing Principle

The forcing principle is powerful. In particular, it does not require stringent conditions or mathematical structures on the primitives other than Assumption 1 on the existence of a suitable punishment, which renders it applicable in many applications. We present three specific applications in this section to demonstrate the theoretical advantages conferred by the forcing principle.

4.1 First-Best Benchmark in Pure Moral Hazard

Our first application of the forcing principle is that it can shed light on the characterization of the first-best benchmark in pure moral hazard problems, which arises frequently in the literature. Our general framework in Section 2 reduces to a perfect information setting *without* moral hazard when the set of unobservable actions A_1 is empty. Such a setting (with only observable and verifiable actions) is typically employed to characterize the first-best benchmark for the underlying principal-agent problem.

A conventional treatment in the literature when characterizing first-best benchmarks is to take the actions out of the hands of the agent and let the principal dictate what action the agent should choose.[21] The optimal contract in a first-best benchmark typically takes the form of a forcing contract and is derived from the following optimization problem, denoted again here as (**P2**), in which we impose $A_1 = \varnothing$ for (**P2**) in Section 3:

$$(\mathbf{P2}) \quad \begin{array}{c} \max_{(s,a_0)\in\mathcal{K}\times A_0} \int_\Omega u(\omega, s(\omega), a_0)P(d\omega; a_0) \\ \text{s.t. (IR): } \int_\Omega v(s(\omega), a_0)P(d\omega; a_0) \geq r \end{array}$$

Applying Theorem 1 to this setting directly leads to the following corollary:

Corollary 1 *Suppose Assumption 1 holds and that A_1 is empty (or a singleton). Then problem* (**P1**) *can be equivalently reduced to* (**P2**)*. The first-best benchmark takes the form of a simple forcing contract where the principal directly chooses an action for the agent.*

[21]See, for instance, the characterization of first-best benchmarks in standard textbooks (Laffont and Martimort (2009)[16] and Bolton and Dewatripont (2005)[2]).

Although it is directly implied by Theorem 1, Corollary 1 validates the standard approach to characterizing first-best benchmarks in the moral hazard literature. Importantly, though, the equivalence result in Corollary 1 holds only under the existence of a severe punishment scheme $t(\omega)$ (Assumption 1), and such an assumption to our knowledge has not been explicitly pointed out in the literature. We next present a simple example to show that Corollary 1 (and also Theorem 1) depends critically on the availability of a suitable punishment scheme.

Example 3 *Consider a principal-agent problem with $\Omega = \{0,1\}$, $A_0 = \{1,2\}$, $A_1 = \{a_1\}$, and $D \subset \mathbb{R}$, i.e., there are two outcomes, "success" ($\omega = 1$) and "failure" ($\omega = 0$), and two observable actions, with an outside option of zero. Moral hazard is degenerated, given that A_1 is a singleton. Let $p(a_0)$ be the probability of success with $p(2) > p(1) > 0$. Let $f_\omega(a_0)$ be the transfer to the agent after $\omega \in \{0,1\}$ and action a_0 from the agent. Given a utility function $v(d, a_0) = d - a_0$, the agent's expected utility is*

$$V(f_\omega, a_0) = p(a_0) f_1(a_0) + (1 - p(a_0)) f_0(a_0) - a_0.$$

Given a utility function $u(\omega, d) = \alpha\omega - d$ with $\alpha > 0$, the principal's expected utility is

$$U(f_\omega, a_0) = p(a_0)(\alpha - f_1(a_0)) - (1 - p(a_0)) f_0(a_0).$$

The principal's revenue is α from a successful project (and 0 from a failed project).

Let the agent's outside option be 0. Suppose the principal faces a minimum-wage constraint requiring $f_\omega(a_0) \geq 2$ for each $\omega \in \{0,1\}$ and each $a_0 \in \{1,2\}$, i.e., the agent's minimal wage is 2 as long as he participates. Hence, Assumption 1 is violated here.

*First consider problem (**P2**), in which the principal chooses an outcome-contingent reward scheme $\{s_\omega\} = \{s_0, s_1\}$:*

$$\begin{aligned} &\max_{(s_1, s_2, a_0)} \{p(a_0)(\alpha - s_1) - (1 - p(a_0)) s_0\} \\ s.t.\ &\text{(IR): } p(a_0) s_1 + (1 - p(a_0)) s_0 - a_0 \geq 0 \\ &\text{(MW): } s_\omega \geq 2 \text{ for } \omega \in \{0,1\} \end{aligned}$$

*Since the agent is "forced" to choose a specific a_0, the (IC) constraint can be ignored. Given the minimum-wage constraint (MW), one can verify that the optimal contract for (**P2**) is $(a_0^* = 2, s_0^* = s_1^* = 2)$ whenever $\alpha > 0$ and $p(2) > p(1)$, and the principal's expected utility is $U(s_\omega^*, a_0^*) = \alpha p(2) - 2$.*[22]

[22] Throughout Example 3, we assume that the principal is always willing to hire the agent given the

Next consider the principal's problem (**P1**) *formulated as follows:*

$$\begin{array}{ll} & \max_{(f_1,f_2,a_0)}\{p(a_0)(\alpha - f_1(a_0)) - (1-p(a_0))f_0(a_0)\} \\ & \text{(IR): } p(a_0)f_1(a_0) + (1-p(a_0))f_0(a_0) - a_0 \geq 0 \\ s.t. & \text{(IC): } a_0 \in \arg\max_{a_0' \in \{1,2\}} p(a_0')f_1(a_0') + (1-p(a_0'))f_0(a_0') - a_0' \\ & \text{(MW): } f_\omega(a_0) \geq 2 \text{ for } \omega \in \{0,1\},\ a_0 \in \{1,2\} \end{array}$$

Notice that the (MW) *constraint implies that the agent can ensure himself an expected utility of at least 1 by choosing $a_0 = 1$, i.e.,* (IR) *always holds. We discuss two cases:*

Case 1 : $\alpha \geq \frac{1}{p(2)-p(1)}$.

Given the (MW) *constraint, the optimal payment scheme can be derived as*

$$f^*_\omega(a_0) = \begin{cases} 2 + \frac{1}{p(2)}, & \text{if } \omega = 1 \text{ and } a_0 = 2, \\ 2, & \text{otherwise.} \end{cases}$$

And under $f^*_\omega(a_0)$, it is optimal for the principal to induce $a^*_0 = 2$ given that $\alpha \geq \frac{1}{p(2)-p(1)}$ and the principal's expected utility from the optimal contract $(a^*_0 = 2, f^*_\omega(a_0))$ is $U(f^*_\omega, a^*_0) = \alpha p(2) - 3$.

Case 2 : $0 < \alpha < \frac{1}{p(2)-p(1)}$.

The optimal payment scheme is $f^{**}_\omega(a_0) = 2$ for all $\omega \in \{0,1\}$ and $a_0 \in \{1,2\}$. Under $f^{**}_\omega(a_0)$, it is optimal for the principal to induce $a^*_0 = 1$ given that $\alpha \in \left(0, \frac{1}{p(2)-p(1)}\right)$ and the principal's expected utility from the optimal contract $(a^{**}_0 = 1, f^{**}_\omega(a_0))$ is $U(f^{**}_\omega, a^{**}_0) = \alpha p(1) - 2$.

In both cases, problem (**P1**) *is* ***not*** *equivalent to problem* (**P2**)*. Moreover, one can verify that from a social welfare point of view it is efficient to induce $a_0 = 1$ if $\alpha \in \left(0, \frac{1}{p(2)-p(1)}\right)$, while inducing $a_0 = 2$ is efficient when $\alpha \geq \frac{1}{p(2)-p(1)}$.*[23] *Hence, the optimal contract for* (**P1**) *always leads to the "first-best" outcome (maximized total surplus), which is* ***not*** *true for the optimal contract solving* (**P2**)*. On the other hand, the optimal contract for* (**P2**) *is strictly better for the principal, i.e., $U(s^*_\omega, a^*_0) > \max\{U(f^*_\omega, a^*_0), U(f^{**}_\omega, a^{**}_0)\}$. Hence, regardless of whether the first-best benchmark is defined from the principal's point of view or from the angle of social surplus, the two problems* (**P1**) *and* (**P2**) *lead to different outcomes.*

minimum wage constraint—this can be guaranteed by adjusting the level of the minimum wage without qualitatively affecting the structure of the optimal contracts.

[23] The socially optimal action in the setting is identified by comparing the net surpluses generated by $a_0 = 2$ and $a_0 = 1$, which are $(\alpha p(2) - 2)$ and $(\alpha p(1) - 1)$, respectively.

Example 3 shows that Assumption 1 is an indispensable condition for the problems (**P1**) and (**P2**) to be strategically equivalent. The key problem here is that the minimum-wage constraint prevents the existence of a suitable punishment scheme and an equivalent forcing contract to implement the solution to (**P1**).

Example 3 also highlights the important role of Assumption 1 in characterizing first-best benchmarks for moral hazard problems. In particular, the standard approach of using a forcing contract to represent the first-best contract when the agent's actions are observable and verifiable may not be valid without Assumption 1.

While Assumption 1 is a mild assumption, a principal in a real-life contracting problem may be restricted from using arbitrarily large sticks in punishing poor performance from an agent—for example, limited liability and minimum-wage legislation can both prevent the agent from being punished too harshly in practice.[24] Example 3 shows that for such settings, it can be problematic to use a forcing contract solving (**P2**) to represent the corresponding first-best benchmark in that such a forcing contract may not characterize the *actual* first-best benchmark: In Example 3, it is optimal from a social welfare point of view to implement a less costly action $a_0 = 1$ when the project value α is small, but a forcing contract from (**P2**) ignores the relevant (IC) constraint and implements $a_0 = 2$. Hence, for general moral hazard problems where Assumption 1 is violated, using a forcing contract from (**P2**) may set up a *false* first-best benchmark, leading to erroneous efficiency evaluations of an underlying second-best contract.

Granted that the first-best benchmark in the literature is sometimes only a hypothetical situation and in others the existence of such a punishment is (implicitly or explicitly) assumed and satisfied, the problematic issue in Example 3 does not pose much of a problem to the moral hazard literature in general. Nevertheless, we believe that it is worthwhile to point out the importance of Assumption 1 explicitly.

4.2 Existence of Optimal Contracts

Our second application of the forcing principle concerns the existence of an optimal solution to the original contracting problem (**P1**). To be specific, the forcing principle enables us to restrict attention to a simple class of forcing contracts and therefore to establish the existence of optimal contracts under mild topological conditions in our setting.

[24] A number of studies in the previous literature have studied moral hazard problems where limited liability or a lower/minimum bound on the agent's payment is imposed (so that Assumption 1 is possibly violated). See, for example, Sappington (1983)[24]; Innes (1990)[11]; Kim (1997)[11]; Dewatripont, Legros and Matthews (2003)[6]; and recent contributions Jewitt, Kadan and Swinkels (2008)[12] and Kadan and Swinkels (2013)[14] and references therein.

Our approach to existence is similar to Page's (1987)[20] general topological approach, which establishes the existence of optimal contracts in a pure moral hazard setting under certain topological assumptions, provided that the set of admissible reward schemes—called contracts in Page's setting—satisfies a sequential compactness property. The main idea of Page's approach is to identify conditions that guarantee certain sequential compactness and non-emptiness of the set of feasible contracts—recall Definition 1—and (sequential) continuity of the principal's expected utility, so that a solution to the principal's problem always exists, akin to the Weierstrass extreme value theorem.[25]

We first establish sequential continuity of the two parties' expected utility functions (by Delbaen's Lemma; Delbaen 1974[5]) and sequential closedness of the set of feasible contracts.[26] These results then imply that the expected utility of the principal admits a finite supremum over the set of feasible contracts. Finally, we show that there is a sequence of feasible contracts converging to this finite supremum with its limit being an optimal solution to **(P2)**.

Notice that Page's (1987) approach *cannot* be applied directly under a hybrid reward scheme. The difficulty lies in establishing the sequential continuity of the two parties' expected utility functions using the Delbaen's Lemma, given that observable actions also enter a hybrid reward scheme. To be specific, consider the sequential continuity of the agent's expected utility. An application of Delbaen's Lemma requires the condition that for a sequence of reward schemes and actions $\{(f_n, a_n)\}_n$ in $\mathcal{R}_{\mathcal{K}} \times A$ with f_n converging to $f^0 \in \mathcal{K}$ pointwise on Ω and $(a_{0,n}, a_{1,n})$ converging to $(a_0^0, a_1^0) \in A$ under the metric on A, $v(f_n(a_{0,n}, \omega), a_{0,n}, a_{1,n})$ converges to $v(f^0(a_0^0, \omega), a_0^0, a_1^0)$ for each $\omega \in \Omega$. However, given that a_0 is also an argument of f, the continuity of the agent's utility function v is insufficient to guarantee such convergence. Delbaen's Lemma hence cannot be directly applied to establish the sequential continuity of the expected utility functions. Consequently, it is also difficult to establish sequential closedness of the set of feasible contracts.

Our forcing principle helps avoid such complications created by hybrid reward schemes. In particular, the structure of reward schemes in **(P2)** resembles that in Page's (1987) moral hazard model, which renders Delbaen's Lemma applicable, and an approach similar to Page's can then be employed to establish existence for **(P2)**. By Theorem 1, the existence of a solution to **(P2)** then implies the existence of a solution to **(P1)**.

[25]Notice that we will consider sequential topological properties for the reward-scheme sets. Since such reward-scheme sets are sets of functions, they may not always be metrizable from the outset. Consequently, sequential topological properties will be more meaningful for the proof of existence (for instance, sequential compactness is not equivalent to compactness for sets of functions).

[26]See, for example, a statement of Delbaen's Lemma in Section 3 of Page (1987) or Delbaen (1974).

As in Page (1987), we start with a few assumptions:

Assumption 2 (Action) *The action spaces A_0 and A_1 are compact metric spaces.*

Assumption 3 (Outcome) *The outcome space Ω is a compact subset of a Euclidean space.*

Assumption 4 (Sequential Continuity of P) *For each closed $E \subseteq \Omega$ and each sequence $\{(a_{0,n}, a_{1,n})\} \to (a_0, a_1)$ in A, $P(E; a_{0,n}, a_{1,n}) \to P(E; a_0, a_1)$.*

Assumption 5 (Reward) *The reward space D is a closed interval in $\mathbb{R}$.*

Assumption 6 (Continuity of Utility) *u is continuous on $\Omega \times D \times A$ and v is continuous on $D \times A$.*

Assumption 7 (Sequential Compactness of $\mathcal{K}$) *The outcome-contingent reward scheme constraint set $\mathcal{K}$ is a sequentially compact subset of $\mathcal{M}$ under the topology of pointwise convergence.*

Assumptions 2, 3, and 5 are standard and impose compactness on, respectively, the action sets, the outcome set, and the reward set. Assumption 4 represents the sequential continuity of $P(E;\cdot,\cdot)$ for each measurable and closed $E \subset \Omega$, which, recall, is the probability of outcomes from actions.[27] Assumption 6 imposes continuity of the two parties' utility functions. Notice that under the above assumptions (2, 3, 5, 6), the utility functions u and v are also bounded.[28] Finally, Assumption 7, as in Page (1987), imposes restrictions over available reward schemes in terms of a sequentially compact set.

We now provide a more compact representation of the principal's optimization problem in **(P2)** in order to establish our existence result. First consider the agent's incentive issue in **(P2)**. Define a set-valued mapping $A^* : \mathcal{K} \times A_0 \twoheadrightarrow A_1$ as

$$A^*(s, a_0) = \arg\max_{a_1' \in A_1} V(s, a_0, a_1'),$$

[27] This continuity assumption, as in Page (1987), is weaker than the differentability assumption of $F(x, a)$, which is the probability distribution induced on the monetary outcome x given action a in Holmström (1979).

[28] This is different from a recent study on the issue of the existence of optimal contracts in a setting with unbounded utility (Moroni and Swinkels 2014). Our setting here also differs from that in Kadan, Reny and Swinkels (2017), in which both parties' utility functions are only assumed to be bounded below.

i.e., $A^*(s, a_0)$ denotes the set of optimal unobservable actions a_1 for the agent given (a_0, s) specified by the principal. In addition, denote the agent's optimal level of expected utility under the contract (s, a_0) as

$$V^*(s, a_0) = \max_{a_1' \in A_1} V(s, a_0, a_1').$$

Next consider the agent's participation in **(P2)**. Define the *participation-guarantee set* $\mathcal{L}(r)$ as the set of incentive-compatible contracts that guarantee the agent's participation:

$$\mathcal{L}(r) := \{(s, a_0) \in \mathcal{K} \times A_0 : V^*(s, a_0) \geq r\}.$$

We assume that the reservation utility r is sufficiently low so that $\mathcal{L}(r)$ is non-empty:

Assumption 8 (Reservation Utility) *The reservation utility r is such that $\mathcal{L}(r) \neq \emptyset$.*

Finally, the constraint set of **(P2)** is then the graph of A^* restricted to $\mathcal{L}(r)$:

$$Gr(A^*) := \{(s, a_0, a_1) \in \mathcal{K} \times A : (s, a_0) \in \mathcal{L}(r) \text{ and } a_1 \in A^*(s, a_0)\}.$$

Given the above notation, we can compactly rewrite **(P2)** as follows:

$$\begin{gathered} \max_{(s,a_0,a_1) \in Gr(A^*)} U(s, a_0, a_1) \\ \text{where } Gr(A^*) = \{(s, a_0, a_1) \in \mathcal{K} \times A : (s, a_0) \in \mathcal{L}(r) \text{ and } a_1 \in A^*(s, a_0)\}. \end{gathered}$$

The existence of optimal solutions to the constrained optimization problem **(P2)** and also **(P1)** can then be established as follows:

Proposition 2 (Existence of Optimal Contract) *Under Assumptions 1-8, there exists an optimal contract (s^*, a_0^*, a_1^*) that solves the principal-agent problem (**P2**). Moreover, there also exists an optimal solution to the original principal-agent problem (**P1**).*

4.3 Computation of Optimal Contracts

The forcing principle can also confer advantages when analytically deriving the optimal contracts for our moral hazard contracting problems, since the optimal contracts now take a simpler functional form. This is particularly relevant when applying the commonly employed first-order approach (FOA): When using the FOA for moral hazard with observable actions, one must assume, somewhat ad hoc, differentiability of f (and further calculations may still be difficult given that f is a function of both ω and a_0). Instead, by solving the simplified problem (**P2**) for the reward scheme $s(\omega)$, one can avoid such problems when applying the FOA. Example 4 provides an illustration:

Example 4 *A principal hires an agent for a project with binary outcomes, success or failure. The agent chooses $(a_0, a_1) \in A = [0,1]^2$ where a_0 is the observable time input and a_1 the unobservable effort intensity. The principal obtains π if the project is successful (with probability $P(a_0, a_1)$), and 0 if the project fails. The payment scheme consists of a transfer $r_s : A_0 \to \mathbb{R}$ after a success and a transfer $r_f : A_0 \to \mathbb{R}$ after a failure. The agent's utility is $v(d) - C(a_0, a_1)$ from transfer d where $C(a_0, a_1)$ is the effort cost. Let $v(0) = 0$ and $r = 0$, and hence Assumption 1 holds. The principal's problem is:*

$$\max_{(r_s, r_f; a_0, a_1) \in \mathcal{M} \times A} (\pi - r_s(a_0))P(a_0, a_1) - r_f(a_0)(1 - P(a_0, a_1))$$

$$\text{s.t.} \quad \begin{array}{l} \text{(IR): } v(r_s(a_0))P(a_0, a_1) + v(r_f(a_0))(1 - P(a_0, a_1)) - C(a_0, a_1) \geq 0 \\ \text{(IC): } (a_0, a_1) \in \arg\max_{(a_0', a_1') \in A} v(r_s(a_0'))P(a_0', a_1') + v(r_f(a_0'))(1 - P(a_0', a_1')) - C(a_0', a_1') \end{array}$$

The FOA cannot be applied without assuming the differentiability of $r_s(\cdot)$ and $r_f(\cdot)$. Such differentiability, however, is hard to justify (see Example 2) and may not necessarily be true for optimal transfers $r_s^*(a_0)$ and $r_f^*(a_0)$. Nevertheless, our forcing principle implies that an optimal contract can be solved from a simplified problem:

$$\max_{(r_s, r_f; a_0, a_1) \in \mathbb{R} \times \mathbb{R} \times A} (\pi - r_s)P(a_0, a_1) - r_f(1 - P(a_0, a_1))$$

$$\text{s.t.} \quad \begin{array}{l} \text{(IR): } v(r_s)P(a_0, a_1) + v(r_f)(1 - P(a_0, a_1)) - C(a_0, a_1) \geq 0 \\ \text{(IC): } a_1 \in \arg\max_{a_1' \in A_1} v(r_s)P(a_0, a_1') + v(r_f)(1 - P(a_0, a_1')) - C(a_0, a_1') \end{array}$$

Here, the principal only solves for two scalars r_s, r_f and the FOA can be readily applied. An optimal contract that solves the simplified problem $(r_s^\dagger, r_f^\dagger; a_0^\dagger, a_1^\dagger)$ specifies a recommended effort $(a_0^\dagger, a_1^\dagger)$, a transfer of $r_s^\dagger$ or $r_f^\dagger$ depending on the outcome when $a_0^\dagger$ is observed, and a transfer of 0 regardless of the outcome when $a_0^\dagger$ is not observed. While such a form of optimal contract is obvious here, Theorem 1 establishes the optimality of $(r_s^\dagger, r_f^\dagger; a_0^\dagger, a_1^\dagger)$ for the original problem formally.

We now *explicitly* compute the optimal contract for a moral hazard problem with both observable and hidden actions using the forcing principle. Consider an agency problem in which a risk-neutral funder (principal) delegates an R&D project to a risk-neutral researcher (agent).[29] The project requires two inputs from the researcher: an investment

[29] This example is motivated by Rietzke and Chen (2016)[23], but without adverse selection.

$x \in [0, 1]$ and effort $y \in [0, 1]$. The researcher's cost function $C(x, y)$ takes the form

$$C(x, y) = x^3 + y.$$

The investment x is observable and verifiable, while the effort choice y is unobservable. The outcome of the project, given (x, y), is binary (success or failure) and the probability of success is $p(x, y) = x\sqrt{y}$. The funder receives a revenue of $W > 0$ if the project succeeds and a zero revenue otherwise. For simplicity, we normalize W to be 1. We assume that the researcher is protected by limited liability, with an outside option of zero.

The funder's contract specifies a reward scheme, a recommended investment x, and a recommended effort y. By the forcing principle, we can consider a reward scheme consisting of a transfer $r_f \in [0, W]$ if the project fails and a transfer $r_s \in [0, W]$ if the project succeeds only after the recommended x is observed (and a transfer of 0 otherwise), where the transfers can be understood as research grants. The funder's contracting problem is:

$$\begin{array}{l} \max_{r_s, r_f \in [0,W]; x, y \in [0,1]} x\sqrt{y}(W - r_s) - (1 - x\sqrt{y})r_f \\ \text{s.t.} \begin{array}{l} \text{(IR): } x\sqrt{y}r_s + (1 - x\sqrt{y})r_f - x^3 - y \geq 0, \\ \text{(IC): } y \in \arg\max_{y'} x\sqrt{y'}r_s + (1 - x\sqrt{y'})r_f - x^3 - y'. \end{array} \end{array}$$

Since the two parties are risk neutral, we can specify $r_f = 0$, reducing the contracting problem to

$$\begin{array}{l} \max_{r_s \in [0,W]; x, y \in [0,1]} x\sqrt{y}(W - r_s) \\ \text{s.t.} \begin{array}{l} \text{(IR): } x\sqrt{y}r_s - x^3 - y \geq 0 \\ \text{(IC): } y \in \arg\max_{y'} x\sqrt{y'}r_s - x^3 - y' \end{array} \end{array}$$

We now solve for the optimal contract in two stages. First, using the first-order approach, we derive the *unique-valued* best effort response of the agent, i.e., the optimal effort from (IC), given r_s and x, as[30]

$$y^*(r_s, x) = \left(\frac{x r_s}{2}\right)^2.$$

[30]It can be readily verified that fixing x, the agent's expected utility is strictly concave in effort y, and the classic first-order approach can be applied here. When the first-order approach cannot be applied, one can employ a polynomial approach introduced by Renner and Schmedders (2015) which (approximately) transforms the principal's (bilevel) optimization problem into a simpler nonlinear optimization problem.

Since $y^*(r_s, x)$ is uniquely valued, we replace y with $y^*(r_s, x)$ and the problem becomes

$$\max_{r_s \in [0,W]; x \in [0,1]} \frac{x^2 r_s}{2}(W - r_s) \text{ s.t. } r_s^2 \geq 4x.$$

The (IR) condition is then binding, given that the funder's expected utility strictly increases in x for any r_s. This allows us to solve for r_s explicitly.[31] Finally, the closed-form solution to the optimal contract can be derived as (recall that $W = 1$):

$$r_s^* = \frac{5W}{6}, x^* = \frac{r_s^{*2}}{4}, y^* = \frac{r_s^{*6}}{64}.$$

Appendix: Proofs

Proof of Proposition 1.

Let $k^* = (f^*, a_0^*, a_1^*)$ be a feasible contract. By Assumption 1, there is a Borel measurable schedule, $t : \Omega \to D$ such that $\forall (a_0, a_1) \in A$, $\int_\Omega v(t(\omega), a_0, a_1) P(d\omega; a_0, a_1) \leq r$. Define $s : \Omega \to D$ such that $s(\omega) = f^*(\omega, a_0^*)$ for all ω, and consider the proposed $\tilde{f}$ with the simple punishment $t(\omega)$:

$$\tilde{f}(\omega, a_0) = \begin{cases} s(\omega), & \text{if } a_0 = a_0^* \\ t(\omega), & \text{otherwise} \end{cases}$$

By construction, $s \in \mathcal{K}$, $\tilde{f} \in \mathcal{R}_{\mathcal{K}}$ and s, $\tilde{f}$ are both Borel-measurable. Moreover, $\tilde{f}(\omega, a_0^*) = f^*(\omega, a_0^*)$. We next show that $\tilde{k} = (\tilde{f}, a_0^*, a_1^*)$ satisfies both (IR) and (IC).

Under $\tilde{k}$, if the agent follows the recommendation a_0^*, then for all ω, $\tilde{f}(\omega, a_0^*) = s(\omega) = f^*(\omega, a_0^*)$. Hence, the agent's expected utility under $\tilde{k}$ from choosing a_0^* is

$$\int_\Omega v(\tilde{f}(\omega, a_0^*), a_0^*, a_1^*) P(d\omega; a_0^*, a_1^*) = \int_\Omega v(f^*(\omega, a_0^*), a_0^*, a_1^*) P(d\omega; a_0^*, a_1^*) \geq r,$$

where the inequality follows from feasibility of k^*. Thus, $\tilde{k}$ satisfies (IR).

To show (IC), let (a_0', a_1') be an arbitrary action profile for the agent, and consider the reward schedule $\tilde{f}$. If $a_0' \neq a_0^*$ then $\tilde{f}(\omega, a_0') = t(\omega)$ for all ω. Given Assumption 1, the agent has no incentive not to choose a_0^*. If $a_0' = a_0^*$ then $\tilde{f}(\omega, a_0') = f^*(\omega, a_0^*)$ for all ω.

[31] It can be verified that after substituting the binding (IR) constraint, the objective function $(r_s^5(W - r_s))$ is strictly quasiconcave in the relevant range, and the unique global maximizer can be derived via the first-order condition as $r_s^* = \frac{5W}{6}$.

Now if the agent chooses (a_0^*, a_1'), his expected utility is:

$$\begin{aligned}\int_\Omega v(f^*(\omega, a_0^*), a_0^*, a_1')P(d\omega; a_0^*, a_1') &\leq \int_\Omega v(f^*(\omega, a_0^*), a_0^*, a_1^*)P(d\omega; a_0^*, a_1^*) \\ &= \int_\Omega v(\tilde{f}(\omega, a_0^*), a_0^*, a_1^*)P(d\omega; a_0^*, a_1^*)\end{aligned}$$

The above follows from the optimality of (a_0^*, a_1^*) for the agent under the reward scheme f^* and that $\tilde{f}(\omega, a_0^*) = f^*(\omega, a_0^*)$ for all ω. Hence, the contract $\tilde{k}$ satisfies both (IC) and (IR), and is feasible. Moreover, the fact that $\tilde{f}(\omega, a_0^*) = f^*(\omega, a_0^*)$ for all ω implies that the principal and the agent attain the same expected utility under $\tilde{k}$ and k^*. $\square$

Proof of Theorem 1.

By Proposition 1, for any feasible contract (f, a_0', a_1'), there is $s \in \mathcal{K}$ with $s(\omega) \equiv f(\omega, a_0')$ and a corresponding feasible punishment contract $(\tilde{f}, a_0', a_1')$ with

$$\tilde{f}(\omega, a_0) = \begin{cases} s(\omega), & \text{if } a_0 = a_0' \\ t(\omega), & \text{otherwise} \end{cases}, \tag{2}$$

and the principal obtains the same expected utility under $(\tilde{f}, a_0', a_1')$ and (f, a_0', a_1'). Since we can always choose a feasible contract (f, a_0', a_1') with a punishment contract $(\tilde{f}, a_0', a_1')$, the original problem **(P1)** is equivalent to the following **($\tilde{\mathbf{P}}$1)**:

$$(\tilde{\mathbf{P}}\mathbf{1}) \quad \begin{aligned} &\max_{(s,a_0',a_1')\in\mathcal{K}\times A} \int_\Omega u(\omega, \tilde{f}(\omega, a_0'), a_0', a_1')P(d\omega; a_0', a_1') \\ \text{s.t.} \quad &(\text{IR}'):\ \int_\Omega v(\tilde{f}(\omega, a_0'), a_0', a_1')P(d\omega; a_0', a_1') \geq r \\ &(\text{IC}'):\ (a_0', a_1') \in \arg\max_{(\tilde{a}_0,\tilde{a}_1)\in A} \int_\Omega v(\tilde{f}(\omega, \tilde{a}_0), \tilde{a}_0, \tilde{a}_1)P(d\omega; \tilde{a}_0, \tilde{a}_1) \end{aligned}$$

Notice that the reward scheme $\tilde{f}$ in **($\tilde{\mathbf{P}}$1)** takes a specific form that is determined jointly by $s(\omega)$ and a_0' (see (2)).

Consider a new problem ($\hat{\mathbf{P}}\mathbf{1}$) and recall that $t(\omega)$ is defined in Assumption 1:

$$(\hat{\mathbf{P}}\mathbf{1}) \quad \begin{aligned} &\max_{(s,a_0',a_1')\in\mathcal{K}\times A} \int_\Omega u(\omega, s(\omega), a_0', a_1')P(d\omega; a_0', a_1') \\ \text{s.t.} \quad &(\text{IR}):\ \int_\Omega v(s(\omega), a_0', a_1')P(d\omega; a_0', a_1') \geq r \\ &(\text{IC}_0):\ \int_\Omega v(s(\omega), a_0', a_1')P(d\omega; a_0', a_1') \geq \int_\Omega v(t(\omega), a_0, a_1)P(d\omega; a_0, a_1),\ \forall a_0, a_1 \\ &(\text{IC}_1):\ a_1' \in \arg\max_{a_1\in A_1} \int_\Omega v(s(\omega), a_0', a_1)P(d\omega; a_0', a_1) \end{aligned}$$

We claim that given the definition in (2), the set of constraints (IR′) and (IC′) in ($\tilde{\mathbf{P}}\mathbf{1}$) is equivalent to the set of constraints (IR), (IC$_0$), and (IC$_1$) in ($\hat{\mathbf{P}}\mathbf{1}$), i.e., any $(s, a_0', a_1') \in$

$\mathcal{K} \times A$ satisfying (IR$'$) and (IC$'$) also satisfies (IR), (IC$_0$), and (IC$_1$), and vice versa.

First suppose that (s, a_0', a_1') satisfies (IR$'$) and (IC$'$). Then (IR) holds for (s, a_0', a_1') given the definition of $\tilde{f}(\omega, a_0)$ in (2) and (IR$'$). Next, since (s, a_0', a_1') satisfies (IC$'$), we have that for all $(a_0, a_1) \in A$,

$$\int_\Omega v(\tilde{f}(\omega, a_0'), a_0', a_1')P(d\omega; a_0', a_1') \geq \int_\Omega v(\tilde{f}(\omega, a_0), a_0, a_1)P(d\omega; a_0, a_1). \tag{3}$$

In particular, by the definition of $\tilde{f}(\omega, a_0)$, for $a_0 \neq a_0'$, (3) implies

$$\begin{aligned}\int_\Omega v(\tilde{f}(\omega, a_0'), a_0', a_1')P(d\omega; a_0', a_1') &= \int_\Omega v(s(\omega), a_0', a_1')P(d\omega; a_0', a_1') \\ &\geq \int_\Omega v(t(\omega), a_0, a_1)P(d\omega; a_0, a_1),\end{aligned}$$

which is (IC$_0$) in ($\tilde{\mathbf{P}}\mathbf{1}$). Moreover, again by (3), we have for all $a_1 \in A_1$,

$$\begin{aligned}\int_\Omega v(\tilde{f}(\omega, a_0'), a_0', a_1')P(d\omega; a_0', a_1') &\geq \int_\Omega v(\tilde{f}(\omega, a_0'), a_0', a_1)P(d\omega; a_0', a_1) \\ &= \int_\Omega v(s(\omega), a_0', a_1)P(d\omega; a_0', a_1),\end{aligned}$$

which is (IC$_1$) in ($\tilde{\mathbf{P}}\mathbf{1}$). Hence, (s, a_0', a_1') satisfies (IR), (IC$_0$), and (IC$_1$).

Now suppose that (s, a_0', a_1') satisfies (IR), (IC$_0$), and (IC$_1$). Then similarly (IR$'$) holds for (s, a_0', a_1') given the definition of $\tilde{f}(\omega, a_0)$ in (2) and (IR). We now show that (s, a_0', a_1') also satisfies (IC$'$). Suppose not, i.e., given s, there is $(a_0'', a_1'') \neq (a_0', a_1')$ in A such that (s, a_0'', a_1'') satisfies (IC$_0$) and (IC$_1$) but

$$\int_\Omega v(\tilde{f}(\omega, a_0''), a_0'', a_1'')P(d\omega; a_0'', a_1'') > \int_\Omega v(\tilde{f}(\omega, a_0'), a_0', a_1')P(d\omega; a_0', a_1'). \tag{4}$$

Suppose $a_0'' \neq a_0'$. Then by definition of $\tilde{f}(\omega, a_0)$ and $t(\omega)$, we have

$$\int_\Omega v(\tilde{f}(\omega, a_0''), a_0'', a_1'')P(d\omega; a_0'', a_1'') = \int_\Omega v(t(\omega), a_0'', a_1'')P(d\omega; a_0'', a_1'') \leq r,$$

contradicting (IR) and (4). Hence, we must have $a_0'' = a_0'$, i.e., (4) becomes

$$\int_\Omega v(\tilde{f}(\omega, a_0'), a_0', a_1'')P(d\omega; a_0', a_1'') > \int_\Omega v(\tilde{f}(\omega, a_0'), a_0', a_1')P(d\omega; a_0', a_1'),$$

which further implies

$$\int_{\Omega} v(s(\omega), a_0', a_1'')P(d\omega; a_0', a_1'') > \int_{\Omega} v(s(\omega), a_0', a_1')P(d\omega; a_0', a_1'),$$

which, however, contradicts (IC_1). Hence, (s, a_0', a_1') satisfies (IR') and (IC').

Given the above claim and the definition of $\tilde{f}(\omega, a_0)$ in (2), the maximization problems ($\hat{\mathbf{P}}\mathbf{1}$) and ($\tilde{\mathbf{P}}\mathbf{1}$) are hence equivalent.[32] Now since in ($\hat{\mathbf{P}}\mathbf{1}$), for any recommended a_0', (IC_0) holds automatically given (IR) and Assumption 1, we further rewrite **($\hat{\mathbf{P}}\mathbf{1}$)** as

$$\begin{array}{ll} & \max_{(s,a_0',a_1')\in\mathcal{K}\times A} \int_{\Omega} u(\omega, s(\omega), a_0', a_1')P(d\omega; a_0', a_1') \\ \text{s.t.} & \begin{array}{l} \text{(IR): } \int_{\Omega} v(s(\omega), a_0', a_1')P(d\omega; a_0', a_1') \geq r \\ (\mathrm{IC}_1)\text{: } a_1' \in \arg\max_{a_1\in A_1} \int_{\Omega} v(s(\omega), a_0', a_1)P(d\omega; a_0', a_1) \end{array} \end{array}$$

which is exactly (**P2**). Since (**P1**) and (**P2**) are equivalent optimization problems under Proposition 1, it follows that bullet points 1, 2, and 3 hold as desired. $\square$

Proof of Proposition 2

This proof follows the proof of existence of optimal contracts in Page (1987)[20] by replacing sequences of f_n's by sequences of $(s_n, a_{0,n})$'s. Delbaen's Lemma (Delbaen 1974[5]) will imply that U and V are sequentially continuous. Moreover, the constraint set of **(P2)** $Gr(A^*) = \{(s, a_0, a_1) \in \mathcal{K} \times A : (s, a_0) \in \mathcal{L}(r),\ a_1 \in A^*(s, a_0)\}$ is non-empty and sequentially closed, and $U^* := \sup_{(s,a_0,a_1)\in Gr(A^*)} U(s, a_0, a_1)$ is finite. Next, since U^* is a supremum, there is a sequence $\{(s_n, a_{0,n}, a_{1,n})\}_n$ in $Gr\,(A^*)$ such that $U(s_n, a_{0,n}, a_{1,n}) \to U^*$. Given the sequential compactness of $\mathcal{K}$ and the compactness of A, there is a subsequence $\{(s_{n_k}, a_{0,n_k}, a_{1,n_k})\}_k$ in $Gr\,(A^*)$ and a triple (s^*, a_0^*, a_1^*) in $\mathcal{K} \times A$ such that $s_{n_k} \to s^*$ pointwise on Ω and $(a_{0,n_k}, a_{1,n_k}) \to (a_0^*, a_1^*)$ under the metric on A. Since $Gr\,(A^*)$ is sequentially closed, $(s^*, a_0^*, a_1^*) \in Gr\,(A^*)$. Finally, the sequential continuity of U implies that $U(s^*, a_0^*, a_1^*) = U^*$. Therefore, (s^*, a_0^*, a_1^*) is the solution to **(P2)**. $\square$

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