Graz Economics Papers – GEP

GEP 2017 –01

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May 2018

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Abstract

We explore the theoretical justification of adopting *bilateral* mechanism design, which is a simplification of canonical *collective* mechanism design, in general *multi-agency* contracting games under Bayesian Nash equilibrium. We establish *interim payoff equivalence* between collective and bilateral mechanism design in the *quasi-separable environment*, in which interdependent valuations and correlated types are allowed. We employ interim payoff equivalence to further show the equivalence between optimal bilateral and collective mechanism design, when the principal's payoff exhibits certain relations with separate agents' payoffs. Our analysis can also incorporate individual rationality and budget balance constraints and the asymptotic equivalence.

Keywords: Bayesian Nash equilibrium, bilateral mechanism, collective mechanism, interim payoff equivalence, quasi-separable environment

JEL Classification: C72 D82 D86

*This paper is a revision of one part of my Indiana University CAEPR working paper titled "Bilateral Mechanism Design: Practical Contracting in Multi-Agency." For their helpful remarks, suggestions, and comments, thanks are due to Frank Page, Christoph Kuzmics, Johannes Horner, Masaki Aoyagi, Bo Chen, Xianwen Shi, Yongchao Zhang, Rongzhu Ke, Nian Yang, Jie Zheng, Zhiqi Chen, Mingjun Xiao. I am solely responsible for any errors. This work is supported by the National Natural Science Foundation of China (Grant No.71673133).

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1 Introduction

Consider a standard adverse selection model with a *multi-agency* setting in which one principal (female) contracts with multiple agents (male). The principal can write a *collective* (multilateral) mechanism[1] that specifies each agent's contract[2] based on the reports of all agents, or a *bilateral* mechanism[3] with each agent, one by one, specifying each agent's contract based solely on his individual report. When will the bilateral mechanism do as well as the collective mechanism?

Bilateral mechanisms, as a simplified class of collective mechanisms, have attracted much attention in the literature of contract and mechanism design over recent years.[4] Several studies suggest that bilateral mechanism design would be a more practical solution to deal with private, decentralized information due to its simplicity. For instance, McAfee and Schwartz (1994) point out that designing a complete and comprehensive multilateral (collective) contract or mechanism might indeed be practically demanding, and the associated costs of auditing and processing information might significantly rise with the number of parties involved. A recent paper by Dequiedt and Martimort (2015) further argues that in addition to saving on haggling and transaction costs, bilateral contracts are also the only feasible arrangement when antitrust laws preclude multilateral agreements.

Although many authors directly adopt bilateral mechanisms as the analytical object in the contracting contexts, this paper is the first to explore the theoretical justification for adopting bilateral mechanisms by examining the strategic equivalence of bilateral and collective mechanism design. It can therefore be regarded as an important complement or extension of studies in the previous literature of bilateral contracting. We focus on the contracting game with pure strategies, since they are addressed in many applications and practices. Moreover, we focus on *Bayesian mechanism design*.[5]

Our analysis provides economically interesting conditions for the equivalence between (optimal) bilateral and collective Bayesian mechanism designs. In that case, we can use bilateral mechanism design to substitute collective mechanism design without loss of generality. Information structure with respect to Bayesian updated beliefs provides a possibility for such equivalence when the agents have no allocation externalities and the principal can separately draw welfare from contracts taken by different agents.

Specifically, we first find that the collection of bilateral mechanisms *interim-payoff-equivalent*

[1] Some authors may also call it a *grand mechanism*.

[2] It normally consists of allocation and transfer.

[3] Some authors may also call it a *bilateral contract*.

[4] See McAfee and Schwartz (1994), Segal (1999), Han (2006), Hansen and Motta (2012), and Dequiedt and Martimort (2015), among many others.

[5] Bayesian mechanism design is importatnt in this context for three reasons. First, the parties in a real-world contracting game may still have finer information with respect to Bayesian updating. Second, a (nontrivial) Bayesian mechanism is more likely to exist under the general models than an ex post (or dominant-strategy) mechanism. Third, Bayesian mechanism design will indeed provide more leeway for the equivalence we address compared with ex post (or dominant-strategy) mechanism design.

to all collective Bayesian incentive compatible mechanisms is exactly the collection of all bilateral Bayesian incentive-compatible mechanisms in the *quasi-separable environment*, in which each agent can separate his direct utility from his own contract and type and valuation adjustment from all agents' types in a linearly additive form of his payoff. A quasi-separable environment is an extension of the *separable environment* introduced by Chung and Ely (2006) and is useful in many economic applications and previous studies, such as procurement, vertical contracting, nonlinear pricing, resource allocation, etc. Unlike single-agency, multi-agency suggests significant interaction and interdependence between the agents. This is still embodied in a quasi-separable environment, as follows: Contract sets and payoff forms for different agents can be different; the agents' types can be correlated; each agent's (expected) payoff can depend not only on his own type, but also on those of the other agents–that is, *information externalities* (or *interdependent valuations*) are still allowed, which has attracted attention in practice and in a number of recent studies.[6]

Furthermore, comparing collective and bilateral Bayesian incentive-compatible mechanism designs boils down to comparing collective Bayesian incentive-compatible mechanisms and their interim-payoff-equivalent bilateral mechanisms in the quasi-separable environment. The equivalence between optimal bilateral and collective Bayesian mechanism designs can be established, when the principal's payoff component involving the allocation with respect to each agent is a linear transformation of that agent's payoff involving the allocation given his type (in the sense of expectation), or when each of the principal's payoff components is a concave transformation of each agent's payoff (given his own type) under private valuations. These conditions appear in many application examples, including the situation in which the principal's interests are consistent with those of the agents–such as efficient mechanism design–and the situation that the principal has conflicts of interest with the agents, such as procurement, vertical contracting, etc.

Interim payoff equivalence is an important concept and analytical tool in the mechanism design literature. Recently, Manelli and Vincent (2010), Gershkov et al. (2013), and Kushnir (2015) apply it to examine the equivalence of Bayesian and dominant-strategy mechanism design. This paper introduces a new route of the application of interim payoff equivalence. The idea behind our equivalence results is similar to theirs: *A posteriori* simplification of the multi-agency contracting procedure can be offset by a finer *a priori* information (common knowledge) structure in terms of Bayesian updated beliefs and certain specific contracting environments.

We also address several extensions or discussions based on our main results. (1) It is not technically difficult to incorporate the interim individual rationality and ex ante budget balance constraints in our results. (2) We show an asymptotic equivalence result by which optimal bilateral BIC mechanism design approaches optimal collective BIC mechanism design as the degree of strategic interdependence of the agents' contributions to the principal approaches zero, even if the exact equivalence does not exactly hold. (3) We discuss the equivalence results under explicit primitive constraints *across* the contracts for different agents. (4) Some mathematical

[6] See Jehiel et al. (1999), Jehiel and Moldovanu (2001), and Mezzetti (2004), among many others.

generalizations of our model and analysis are presented.

2 Model

2.1 Primitives

We consider a *pure-strategy* multi-agency contracting game with one principal and n agents indexed by $i \in \mathcal{N} = \{1, \cdots, n\}$. Agent i has some private *type* $\theta_i \in \Theta_i$, where Θ_i is a closed subset of a Euclidean space. We write $\theta = (\theta_i)_{i\in\mathcal{N}} \in \Theta = \prod_{i=1}^{n} \Theta_i$ and $\theta_{-i} = (\theta_j)_{j\in\mathcal{N}\setminus\{i\}} \in \Theta_{-i}$ $= \prod_{j\neq i}^{n} \Theta_j$. Let μ_i be a probability measure defined on Θ_i and μ be a probability measure on Θ. μ characterizes the *common prior over the agents' types.*[7] Let $\mu_{-i}(\cdot|\theta_i)$ denote a conditional probability measure on θ_i over Θ_{-i} and represent agent *i's interim (Bayesian updated) belief about the other players' types* after learning his own type θ_i.[8] All the relevant probability measures can be equivalently represented by the corresponding probability distributions.

The principal will specify a *contract*[9], $k_i \in \mathcal{K}_i$, for agent i.[10] It consists of *allocation* $x_i \in X_i$, where X_i is a subset of a Euclidean space, and *transfer* $t_i \in T_i$, where T_i is a subset of $\mathbb{R}$. Thus, $k_i = (x_i, t_i) \in \mathcal{K}_i = X_i \times T_i$. Write $x = (x_i)_{i\in\mathcal{N}}$, $t = (t_i)_{i\in\mathcal{N}}$, $X = \prod_{i=1}^{n} X_i$, and $T = \prod_{i=1}^{n} T_i$. The set of all possible joint contracts is given by the (joint) product contract set $\mathcal{K} = \prod_{i=1}^{n} \mathcal{K}_i$. Its typical element is $k = (k_i)_{i\in\mathcal{N}}$. Write $k_{-i} = (x_{-i}, t_{-i}) = (x_j, t_j)_{j\in\mathcal{N}\setminus\{i\}}$. In this baseline model, we actually assume there are no primitive constraints *across* the contracts for different agents;[11] i.e., any contract set available to any agent is not correlated with the contract set available to another agent. This is frequently observed in many applications, such as procurement, vertical contracting, nonlinear pricing, employee compensation, etc..

Let $V_i : X \times T_i \times \Theta \to \mathbb{R}$ denote *agent i's payoff function.* It takes the quasi-linear form

$$V_i(x, t_i, \theta) = v_i(x, \theta) - t_i,$$

where $v_i : X \times \Theta \to \mathbb{R}$ is jointly (Borel) measurable and also continuous in $x \in X$ for each $\theta \in \Theta$. Agent i's *interim payoff function* is defined by

$$\int_{\Theta_{-i}} V_i(x, t_i, \theta)\mu_{-i}(d\theta_{-i}|\theta_i).$$

[7] Note that different agents' types are allowed to be correlated.

[8] We can allow that $\mu_{-i}(\cdot|\theta_i)$ is not necessarily derived from the prior μ on Θ. Such derivability is only required for the equivalence between optimal bilateral mechanism and optimal collective mechanism in Section 3.2.

[9] Some authors may also call it an *outcome, alternative, decision,* or *allocation.*

[10] $\mathcal{K}_i$ may contain an element k_0 that denotes "no contracting."

[11] We will discuss the case in which such constraints are allowed in Section 4.3.

Let $U : \mathcal{K} \times \Theta \to \mathbb{R}$ denote *the principal's payoff function.* It takes the quasi-linear form

$$U(x, t, \theta) = u(x, \theta) + \sum_{i=1}^{n} t_i,$$

where $u : X \times \Theta \to \mathbb{R}$ is jointly (Borel) measurable and also continuous in $x \in X$ for each $\theta \in \Theta$.

All of the sets in the primitives are assumed to be subsets of Euclidean spaces. They can actually be mathematically generalized to be Polish spaces in our analysis. This will be discussed in Section 4.4.

2.2 Bayesian Mechanism Design

Typically, the principal-agent contracting game over mechanisms unfolds as follows: In stage 1, the principal proposes a commonly observable mechanism to the agents. In stage 2, the agents unilaterally learn their own true type and simultaneously send reports to the principal. In stage 3, through the pre-offered mechanism, the principal assigns contracts to the agents after learning their reports. In stage 4, after the agents' participation,[12] the contracts are simultaneously executed.

Definition 1 *A **collective mechanism** is a list of Borel measurable functions*

$$\mathbf{k} = (\mathbf{x}, \mathbf{t}) = (\mathbf{k}_i : \Theta \to \mathcal{K}_i)_{i \in \mathcal{N}} = ((\mathbf{x}_i : \Theta \to X_i)_{i \in \mathcal{N}}, \mathbf{t}_i : \Theta \to T_i)_{i \in \mathcal{N}})$$

satisfying $(\mathbf{k}_1(\theta), \cdots, \mathbf{k}_n(\theta)) \in \mathcal{K}$ *for each* $\theta \in \Theta$.[13] *A **bilateral mechanism** is a list of Borel measurable functions*

$$\overline{\mathbf{k}} = (\overline{\mathbf{x}}, \overline{\mathbf{t}}) = (\overline{\mathbf{k}}_i : \Theta_i \to \mathcal{K}_i)_{i \in \mathcal{N}} = ((\overline{\mathbf{x}}_i : \Theta_i \to X_i)_{i \in \mathcal{N}}, \overline{\mathbf{t}}_i : \Theta_i \to T_i)_{i \in \mathcal{N}})$$

satisfying $(\overline{\mathbf{k}}_1(\theta_1), \cdots, \overline{\mathbf{k}}_n(\theta_n)) \in \mathcal{K}$ *for each* $\theta \in \Theta$. *Each of its components specifies a contract to agent* i *for each type report profile of single agent* i.

Each component $\mathbf{k}_i$ (respectively, $\overline{\mathbf{k}}_i$) of a collective (respectively, bilateral) mechanism specifies a contract to agent i for each type report profile of all agents (respectively, of single agent i). Let $\mathcal{F}(\Theta, \mathcal{K})$ (respectively, $\overline{\mathcal{F}}(\Theta, \mathcal{K})$) denote the collection of collective (respectively, bilateral) mechanisms.

Remark 1 *The well-known revelation principle allows us to restrict attention to Bayesian incentive-compatible direct mechanisms out of general Bayesian mechanisms.*[14] *Thus, our analysis focuses on* direct *mechanisms.*

[12] We can permit that not all agents eventually participate by including "no contracting," k_0, as an element in some individual contract set(s).

[13] $\mathcal{K}$ here serves as imposing some ex post constraint on the mechanisms.

[14] It is easy to verify that the revelation principle holds for both bilateral and collective mechanism design.

Intuitively, collective mechanisms evaluate all agents' type reports to specify each individual agent's contract, whereas bilateral mechanisms ignore it and merely evaluate every individual agent's type report to specify that individual agent's contract. Bilateral mechanism design simplifies collective mechanism design by ignoring other agents' reports when specifying the contract for any individual agent.

Each mechanism offered by the principal induces a simultaneous-moved subgame for the agents, in which **Bayesian Nash equilibrium (BNE)** is considered to be the solution concept in our analysis.

Definition 2 *A collective mechanism* $\mathbf{k}$ *is* ***Bayesian incentive compatible (BIC)*** *if it induces truthful reporting as a* ***BNE*** *for all the agents, i.e., for each* $i \in \mathcal{N}$, $\theta_i \in \Theta_i$, $\theta'_i \in \Theta_i$,

$$\int_{\Theta_{-i}} V_i(\mathbf{k}(\theta), \theta)\mu_{-i}(d\theta_{-i}|\theta_i) \geq \int_{\Theta_{-i}} V_i(\mathbf{k}(\theta'_i, \theta_{-i}), \theta)\mu_{-i}(d\theta_{-i}|\theta_i).$$

A bilateral mechanism $\overline{\mathbf{k}}$ *is* ***BIC*** *if it induces truthful reporting as a* ***BNE*** *for all agents, i.e., for each* $i \in \mathcal{N}$, $\theta_i \in \Theta_i$, $\theta'_i \in \Theta_i$,

$$\int_{\Theta_{-i}} V_i(\overline{\mathbf{k}}(\theta), \theta)\mu_{-i}(d\theta_{-i}|\theta_i) \geq \int_{\Theta_{-i}} V_i(\overline{\mathbf{k}}_i(\theta'_i), \overline{\mathbf{k}}_{-i}(\theta_{-i}), \theta)\mu_{-i}(d\theta_{-i}|\theta_i).$$

Thus, two corresponding the principal's optimization problems address contracting games over Bayesian mechanisms below. The optimal collective BIC mechanism design problem is (**P1**)**:**

$$\begin{aligned} &\max_{\mathbf{k} \in \mathcal{F}(\Theta, \mathcal{K})} \int_{\Theta} u(\mathbf{k}(\theta), \theta)\mu(d\theta) \\ &\text{s.t. } \mathbf{k} \text{ is } \mathit{BIC}. \end{aligned}$$

The optimal bilateral BIC mechanism design problem is (**P2**):

$$\begin{aligned} &\max_{\overline{\mathbf{k}} \in \overline{\mathcal{F}}(\Theta, \mathcal{K})} \int_{\Theta} u(\overline{\mathbf{k}}(\theta), \theta)\mu(d\theta) \\ &\text{s.t. } \overline{\mathbf{k}} \text{ is } \mathit{BIC}. \end{aligned}$$

We temporarily ignore individual rationality constraints in our analysis for simplicity. As we will demonstrate, inclusion of the interim individual rationality constraints (and also ex ante budget balance constraints) does not change our findings, which is discussed in Section 4.1.

3 Main Results

The optimal collective mechanism will clearly render the principal at least as well off as the optimal bilateral mechanism, since the collection of bilateral BIC mechanisms is essentially equivalent to a proper subset of the collection of collective BIC mechanisms. However, this does not completely rule out the possibility of equivalence between bilateral and collective mechanisms. It is desirable to obtain certain economically intuitive conditions *on the primitives* for this equivalence. A key idea is that such simplification of the contracting procedure tends to be offset by a finer information structure (common knowledge) in the contracting environment. We can adopt this idea to identify a class of economically intuitive conditions for the equivalence by introducing interim payoff equivalence in Bayesian mechanism design.

3.1 Interim Payoff Equivalence of Bilateral and Collective Mechanisms

We now explore when "interim payoff equivalence" between bilateral and collective mechanisms holds; namely, when each collective BIC mechanism will bring to all agents the same interim payoffs as some bilateral BIC mechanism does. When comparing bilateral and collective BIC mechanism design, we only need to compare the class of collective BIC mechanisms and its interim-payoff-equivalent class of bilateral BIC mechanisms. Furthermore, when the optimal bilateral mechanism yields as good an objective value (expected payoff) for the principal as any of its interim-payoff-equivalent collective mechanisms, the principal will have incentive to adopt bilateral mechanisms as a more practical contracting procedure than collective mechanisms. Hence, our subsequent discussion boils down to exploring the conditions under which bilateral BIC mechanism design is interim-payoff-equivalent to collective BIC mechanism design.

Definition 3 *Agent i's interim payoff under a collective (respectively, bilateral) mechanism* $\mathbf{k}$ *is defined by*

$$W_i(\mathbf{k}|\theta_i) = \int_{\Theta_{-i}} V_i(\mathbf{k}(\theta), \theta)\mu_{-i}(d\theta_{-i}|\theta_i)$$

(respectively,

$$W_i(\overline{\mathbf{k}}|\theta_i) = \int_{\Theta_{-i}} V_i(\overline{\mathbf{k}}(\theta), \theta)\mu_{-i}(d\theta_{-i}|\theta_i)).$$

A collective mechanism $\mathbf{k} \in \mathcal{F}(\Theta, K)$ and a bilateral mechanism $\overline{\mathbf{k}} \in \overline{\mathcal{F}}(\Theta, \mathcal{K})$ are ***interim-payoff-equivalent (IPE)*** *if for each $i \in \mathcal{N}, \theta_i \in \Theta_i$,*

$$W_i(\mathbf{k}|\theta_i) = W_i(\overline{\mathbf{k}}|\theta_i).$$

Interim payoff equivalence here suggests the situation in which a bilateral mechanism can render all agents as well off (in terms of interim payoffs) as a collective mechanism. Clearly, for each bilateral (respectively, bilateral BIC) mechanism $\overline{\mathbf{k}}$, there exists a collective (respectively, collective BIC) mechanism $\mathbf{k}$ IPE to $\overline{\mathbf{k}}$, since any bilateral mechanism can be regarded as a particular (reduced) form of a collective mechanism. Nevertheless, we are more interested in the converse, that is, for each collective mechanism $\mathbf{k}$, there exists a bilateral mechanism $\overline{\mathbf{k}}$ IPE to $\mathbf{k}$.

To establish interim payoff equivalence, we need to introduce the *quasi-separable environment* as an extension of the *separable environment* introduced by Chung and Ely (2006); it is applicable to a large class of economic scenarios and previous studies. In a quasi-separable environment, interdependent valuations and correlated types are permitted. Each agent can separate his *direct utility from his own contract and type* and *valuation adjustment from all agents' types* in a linearly additive form of his payoff. We will present several examples for quasi-separable environments, together with our subsequent results, in Section 3.2.

Definition 4 *A multi-agency contracting game is played in a* ***quasi-separable environment*** *if for each* $i \in \mathcal{N}$,

$$v_i(x, \theta) \equiv h_i(x_i, \theta_i) w_i(\theta) + q_i(\theta) \tag{1}$$

for some continuous functions $h_i : X_i \times \Theta_i \to \mathbb{R}$, $w_i : \Theta \to \mathbb{R}$ *satisfying* $w_i(\theta)$ *is non-negative, and* $q_i : \Theta \to \mathbb{R}$. *We call* $h_i(x_i, \theta_i)$ *agent* i*'s* ***direct utility*** *from* x_i *and* θ_i, $w_i(\theta)$ *agent* i*'s* ***(interdependent) multiplicative valuation adjustment,*** *and* $q_i(\theta)$ *agent* i*'s* ***(interdependent) additive valuation adjustment****.*

In a quasi-separable environment, we can identify a bilateral mechanism **IPE** to an arbitrary collective mechanism.

Proposition 1 *In a quasi-separable environment, if for each* $i \in \mathcal{N}$, X_i *and* T_i *are connected and closed, then for each collective (respectively, BIC) mechanism* $\mathbf{k}$*, there exists a bilateral (respectively, BIC) mechanism* $\overline{\mathbf{k}}$ ***IPE*** *(with respect to all agents) to* $\mathbf{k}$*. Specifically, for each* i *and* θ_i, $\int_{\Theta_{-i}} v_i(\overline{\mathbf{x}}_i(\theta_i), \theta) \mu_{-i}(d\theta_{-i}|\theta_i) = \int_{\Theta_{-i}} v_i(\mathbf{x}_i(\theta), \theta) \mu_{-i}(d\theta_{-i}|\theta_i)$*, and* $\overline{\mathbf{t}}_i(\theta_i) = \int_{\Theta_{-i}} \mathbf{t}_i(\theta) \mu_{-i}(d\theta_{-i}|\theta_i)$*.*

Proof Sketch: Given i, we first construct two functions and a composition of them. We further show that this composition is a well-defined *function* from Θ_i to X_i and is Borel-measurable; that is, it is a bilateral mechanism. Moreover, under our construction, we show that this function yields the same interim payoff (from the allocation) for agent i as $\mathbf{x}_i$. The interim payoff equivalence (from the transfer) between $\overline{\mathbf{t}}_i$ and $\mathbf{t}_i$ is trivial.

Remark 2 *It is important for Proposition 1 that contract externalities are precluded in a quasi-separable environment. If contract externalities are permitted, it is difficult to define a well-defined function* $\overline{\mathbf{k}}_j$ $(j \neq i)$ *coupled with* $\overline{\mathbf{k}}_i$ *such that* $\overline{\mathbf{k}}$ ***IPE*** *to* $\mathbf{k}$*. The situation free of contract externalities raises the degree of freedom to find well-defined* ***IPE*** *bilateral mechanisms under the interim expectations, since it is more likely to derive each well-defined function* $\overline{\mathbf{k}}_i$ *from* θ_i *to* k_i *independently from the IPE condition that solely involves* k_i *for each* i*.*

Remark 3 *In Proposition 1, interdependent valuations and correlated types are still permitted. They are the main sources of interdependence among the different agents.*

Remark 4 *Connectedness and closedness of the contract sets, combined with the quasi-separable environment, are also important for Proposition 1. In the proof of Proposition 1 in the Appendix, these properties help prevent excessive "jump" between any two available contracts for constructing a bilateral mechanism* ***IPE*** *to a given collective mechanism. Consider a simple counterexample with finite contract sets, as follows.* $\mathcal{N} = \{1,2\}$*.* $\mathcal{K}_1 = \{0,1\}$*.* Θ_1 *is a singleton and therefore can be neglected in the payoffs.* $\Theta_2 = \{L, H\}$*.* θ_2 *are equally distributed. Let* $v_1(0,L) = v_1(1,H) = 1$*, and* $v_1(1,L) = v_1(0,H) = 0$*. Then consider* $\mathbf{k}_1$ *such that* $\mathbf{k}_1(L) = 0$ *and* $\mathbf{k}_1(H) = 1$*.* $\int_{\Theta_2} v_1(\mathbf{k}_1(\theta), \theta)\mu_2(d\theta_2) = \frac{1}{2}(v_1(\mathbf{k}_1(L), L) + v_1(\mathbf{k}_1(H), H)) = 1$*. But it is unlikely to find a (constant) bilateral* $\overline{\mathbf{k}}_1(\theta_1) \in \mathcal{K}_1$ *such that* $\int_{\Theta_2} v_1(\overline{\mathbf{k}}_1(\theta_1), \theta)\mu_2(d\theta_2) = 1$*.*

Corollary 1 *When the agents' types are independent, in a quasi-separable environment, if for each* $i \in \mathcal{N}$*,* X_i *and* T_i *are connected and closed, then for each collective BIC mechanism* $\mathbf{k}$*, there exists a bilateral BIC mechanism* $\overline{\mathbf{k}}$ ***IPE*** *(with respect to all agents) to* $\mathbf{k}$*.*

Corollary 1 implies that in the quasi-separable environment, the collection of bilateral mechanisms IPE to all collective BIC mechanisms is exactly the collection of all bilateral BIC mechanisms. In this respect, Corollary 1 completely characterizes collective BIC mechanisms via interim payoff equivalence with bilateral BIC mechanisms. This hints at a possibility of further equivalence from the principal's viewpoint. From now on, we focus on the case of independent types.

3.2 Equivalence of Optimal Bilateral and Collective Mechanisms

In comparing optimal collective and bilateral BIC mechanism designs, Proposition 1 implies that we can actually compare collective BIC mechanisms and their IPE bilateral mechanisms. The key is to test whether the optimal BIC collective mechanism and at least one bilateral BIC mechanism IPE to collective BIC mechanisms can bring the same expected payoff for the principal. In other words, we need to test whether

$$\max_{\mathbf{k} \text{ is BIC}} \int_{\Theta} u(\mathbf{k}(\theta), \theta)\mu(d\theta) = \max_{\overline{\mathbf{k}} \text{ is IPE to BIC } \mathbf{k}} \int_{\Theta} u(\overline{\mathbf{k}}(\theta), \theta)\mu(d\theta).$$

With additional assumptions on the principal's payoff function related to the agents' payoff functions, Proposition 1 can usher in a further result on the equivalence between optimal collective and bilateral BIC mechanism designs.

Nevertheless, our subsequent analysis will center on the conditions on the primitives for the equivalence. To do this, we first need one more assumption concerning the derivability of the interim beliefs from the prior below.

[**Assumption 1**] for each $i \in \mathcal{N}$ the interim belief $\mu_{-i}(\cdot|\cdot)$ is derived from the prior μ, that is, for any μ-measurable functions $\phi : \Theta \to \mathbb{R}$ satisfying $\int_{\Theta} \phi(\theta)\mu(d\theta)$ exists,

$$\int_{\Theta} \phi(\theta)\mu(d\theta) = \int_{\Theta_i} \int_{\Theta_{-i}} \phi(\theta)\mu_{-i}(d\theta_{-i}|\theta_i)\mu_i(d\theta_i).$$

Based on Proposition 1, if the principal's payoff also exhibits certain relations with separate agents' payoffs, the equivalence between optimal collective and bilateral BIC mechanism designs can be ensured.

Proposition 2 *Under Assumption 1, in a quasi-separable environment with each X_i and T_i being connected and closed, if for each $i \in \mathcal{N}$,*

(i) $u(x,\theta) \equiv \sum_{i=1}^{n} [G_i((h_i(x_i,\theta_i)w_i(\theta)),\theta_i)] + L(\theta)$ for some continuous functions $L : \Theta \to \mathbb{R}$ and $G_i : \mathbb{R} \times \Theta_i \to \mathbb{R}$, and

(ii) either (1) $\int_\Theta G_i((h_i(x_i,\theta_i)w_i(\theta)),\theta_i)\mu(d\theta) = \int_\Theta a_i(\theta_i)[h_i(x_i,\theta_i)w_i(\theta)]\mu(d\theta)$ for some continuous function $a_i : \Theta_i \to \mathbb{R}$, or (2) $G_i(\cdot,\theta_i)$ is a concave transformation for each $\theta_i \in \Theta_i$, and $w_i(\theta) \equiv w_i(\theta_i)$,

then for any optimal collective BIC mechanism $\mathbf{k}^*$, there exists its IPE bilateral mechanism $\overline{\mathbf{k}}^*$ that yields the same expected payoff for the principal . Thus, optimal bilateral BIC mechanism design is equivalent to optimal collective BIC mechanism design.

Hypothesis (i) means that the principal's payoff has some additively separable relation with different agents' payoffs, and each separable component with respect to agent i is a continuous transformation of each agent's payoff given his type θ_i. It would be difficult to find conditions on the primitives for the exact equivalence between a collective mechanism and its IPE bilateral mechanism if we allow *non-separable* relations between the agents' payoffs and the principal's payoff, since $\mathbf{k}_i(\theta)$ and $\mathbf{k}_{-i}(\theta)$ may simultaneously be integrated out with respect to θ_{-i} under μ_{-i}. Nonetheless, we will discuss an asymptotic equivalence result for that case in Section 4.2.

Condition (1) in hypothesis (ii) implies that the principal's payoff exhibits a certain linearly additive separability with the agents' payoffs. More specifically, the principal's payoff component involving the allocation with respect to agent i is a linear transformation of agent i's payoff involving the allocation given his type θ_i (in the sense of expectation). Note that a_i can be either positive or negative. When a_i's take positive signs, this usually involves partnership or social (collective) efficiency. In contrast, when a_i's take negative signs, this reflects conflicts of interest between the principal and the agents, especially in the traditional principal-agent or upstream-downstream relationship. This situation, in which Propositions 1 and 2 are applicable, can be seen in many examples as follow.

Example 1 (Procurement 1) *A buyer (principal) seeks to procure two imperfectly substitutive goods separately from n producers (agents) indexed by $i \in \mathcal{N}$. We consider an interdependent valuation situation as described by Han (2013). Each i knows a segmental informational signal $\theta_i \in [\underline{\theta}_i, \overline{\theta}_i]$ about production.*[15] *Contracts consist of the procurement quantities $x = \prod_{i=1}^{n} x_i \in$*

[15] Han (2013) discusses an example for the procurement with interdependent valuations. Consider that a government needs to procure the construction of a tunnel in a mountainous area. Construction costs will depend on the geological characteristics of the mountain, the composition and distribution of minerals, etc. Different construction companies may receive different signals on construction costs. These signals have interdependent values in the sense that each company's estimate of its construction cost depends on all companies' signals.

$\prod_{i=1}^{n}[0,\infty)$, *where x_i is the quantity of the good purchased from i, and the monetary transfers $t = \prod_{i=1}^{n} t_i \in \prod_{i=1}^{n}[0,\infty)$, where t_i is the monetary payment to i. Each i's payoff is $t_i - c_i(x_i,\theta)$, where $c_i(x_i,\theta) = w_i(\theta)x_i^2$ is the production cost of x_i, and $w_i : \Theta \to \mathbb{R}^+$ is continuous and denotes i's valuation adjustment. The buyer's payoff is $\sum_{i=1}^{n} \alpha_i x_i^2 - \sum_{i=1}^{n} t_i$, where $\alpha_i \geq 0$. Each seller has a convex cost. All of them take the quadratic functional form in the quantities of the goods. Referring to Proposition 2, we can set $h_i(x_i,\theta_i) \equiv -x_i^2$, $a_i(\theta_i) = -\frac{\alpha_i}{\int_{\Theta_{-i}} w_i(\theta)\mu_{-i}(d\theta_{-i})}$, and $L(\theta) \equiv 0$.*

Example 2 (Vertical Contracting 1) *An upstream manufacturer (principal) contracts with n downstream retailers (agents) indexed by $i = 1,\cdots,n$. The manufacturer sells to retailer i a quantity $x_i \in [0,\infty)$ of an essential input, at price $t_i \in [0,\infty)$. Retailer i will transform this input, with a one-to-one Leontief technology, into x_i units of his final good on the downstream market he faces at the positive marginal cost $b_i(\theta)$. Each retailer i privately knows a segmental informational signal $\theta_i \in \Theta_i = [\underline{\theta}_i, \overline{\theta}_i]$ about the transformation technology efficiency. The marginal cost $b_i(\theta)$ depends on all of the private signals. Let i's revenue be $x_iP_i - b_i(\theta)x_i$, where $P_i > 0$ denotes the price that retailer i faces on his product market. Retailer i then has the payoff $v_i(x,t,\theta) = x_iP_i - b_i(\theta)x_i - t_i$. Assume that b_i is continuous and $P_i \geq b_i(\theta)$ for all θ. The manufacturer has a payoff $u(t,x,\theta) \equiv \sum_{i=1}^{n} t_i - c(\sum_{i=1}^{n} x_i)$, where $c(\sum_{i=1}^{n} x_i)$ represents the manufacturing cost for some $c > 0$. Each retailer is risk neutral in the quantities of the goods, and the manufacturer has a linear cost. All of them take the linear functional form in the quantities of the goods. Referring to Proposition 2, we can set $w_i(\theta) = P_i - b_i(\theta)$, $h_i(x_i,\theta_i) \equiv x_i$, $a_i(\theta_i) \equiv -\frac{c}{\int_{\Theta_{-i}} w_i(\theta)\mu_{-i}(d\theta_{-i})}$, and $L(\theta) \equiv 0$.*

Example 3 (Nonlinear Pricing 1) *A seller (principal) sells a consumption good or service to n consumers (agents) indexed by $i = 1,\cdots,n$.[16] The seller sells to consumer i a quantity $x_i \in [0,\infty)$ of the good, at price $t_i \in [0,\infty)$. Consumer i privately knows an informational signal $\theta_i \in \Theta_i = [\underline{\theta}_i, \overline{\theta}_i]$ about other characteristics of the good, such as quality, etc. Consumer i then has the payoff $v_i(x,t,\theta) = w_i(\theta)x_i - t_i$, where $w_i(\theta)$ is continuous in θ and summarizes the overall valuation of the good. The seller has a payoff $u(t,x,\theta) \equiv \sum_{i=1}^{n} t_i - c(\sum_{i=1}^{n} x_i)$, where $c(\sum_{i=1}^{n} x_i)$ represents the production cost for some $c > 0$. Each retailer is risk neutral in the quantities of the goods, and the seller has a linear cost. All of them take the linear functional form in the quantities of the goods. Referring to Proposition 2, we can set $h_i(x_i,\theta_i) \equiv x_i$, $a_i(\theta_i) \equiv -\frac{c}{\int_{\Theta_{-i}} w_i(\theta)\mu_{-i}(d\theta_{-i})}$, and $L(\theta) \equiv 0$.*

Example 4 (Teamwork) *An organization's headquarters (principal) assigns production tasks for a homogeneous good to n branches (agents) indexed by by $i = 1,\cdots,n$. Each branch i*

[16] We do not allow resales.

has an efficiency parameter as its private type $\theta_i \in \Theta_i = [\underline{\theta}_i, \overline{\theta}_i]$. *The units of good branch* i *produces is* $x_i \in [0, \infty)$. *There is no transfer. Contracts consist of the assignment of production quantities. Branch* i *has a profit function* $w_i(\theta)h_i(x_i, \theta_i)$, *where* w_i *is positive and continuous and summarizes the overall teamwork effect of all branches' efficiency on branch* i, *and* h_i *is also continuous. The headquarters has a management cost* $c(\theta)$, *and needs to maximize the full profit* $\sum_{i=1}^{n} w_i(\theta)h_i(x_i, \theta_i) - c(\theta)$. *Referring to Proposition 2, we can set* $a_i(\theta_i) \equiv 1$ *and* $L(\theta) \equiv -c(\theta)$.

Example 5 (Efficient Allocation 1) *Consider a resource allocation context. Contracts consist of the assignment of some divisible resources* $x = \prod_{i=1}^{n} x_i \in \mathbb{R}^n$ *and the monetary transfers (from the agents to the government)* $t = \prod_{i=1}^{n} t_i \in \mathbb{R}^n$. *Each agent* i *has a private evaluation* θ_i *about the his assignment. His payoff function is same as the setting of the quasi-separable environment:*

$$V_i(x, t, \theta) = w_i(\theta)h_i(x_i, \theta_i) - t_i.$$

Then the government's ex post payoff function (social welfare) is $\sum_{i=1}^{n} [w_i(\theta)h_i(x_i, \theta_i)]$,[17] *and she considers ex ante efficient allocation. Referring to Proposition 2, we can set* $a_i(\theta_i) \equiv 1$,*and* $L(\theta) \equiv 0$.

Moreover, condition (2) in hypothesis (ii) permits the nonlinearly additive separability of the principal's payoff with the agents' payoffs, given *private valuations*. In some applications, if the principal's payoff accordingly exhibits a certain nonlinearly additive separability, with each component as a concave transformation of each agent's direct utility (given his own type), the equivalence of optimal bilateral and collective mechanism design can be ensured. We show two more examples to which this situation applies.

Example 6 (Procurement 2) *Recall the Procurement 1 example. Now assume the production cost of* x_i *is* $c_i(x_i, \theta_i) = \theta_i x_i^2$, *yet the buyer's payoff is* $\sum_{i=1}^{n} \ln x_i - \sum_{i=1}^{n} t_i$. $\ln x_i$ *is the logarithm utility the buyer can draw from consumption of* x_i. *Referring to Proposition 2, we can set* $h_i(x_i, \theta_i) = -\theta_i x_i^2$, *and define* $G_i(-\theta_i x_i^2, \theta_i) = \frac{\ln(\frac{-\theta_i x_i^2}{-\theta_i})}{2} = \frac{\ln x_i^2}{2}$. $G_i(\cdot, \theta_i)$ *is a concave transformation of* $h_i(x_i, \theta_i)$ *given* θ_i.

Example 7 (Vertical Contracting 2) *Recall the Vertical Contracting 1 example. Now assume each* $x_i \in [1, \infty)$. *The manufacturer directly sells a final good to retailers. Each retailer* i *privately knows a signal* $\theta_i \in [\underline{\theta}_i, \overline{\theta}_i]$ *about the downstream market he faces. Each retailer* i *has the payoff* $v_i(x, t, \theta) \equiv w_i(\theta_i)P_i(x_i)x_i - t_i$, *where* P_i *and* w_i *are positive continuous functions.* $w_i(\theta_i)P_i(x_i)$ *represents the inverse demand function parameterized by* θ_i *with respect to* x_i. *Let* $P_i(x_i) = \frac{1}{x_i^3}$. *The manufacturer's payoff is* $u(t, x, \theta) \equiv \sum_{i=1}^{n} t_i - (\sum_{i=1}^{n} b_i x_i^2)$,

[17] Suppose that the government also collects the transfers from all agents.

where $b_i x_i^2$ *represents the manufacturing cost of producing* x_i*. Referring to Proposition 2, we can set* $h_i(x_i, \theta_i) = w_i(\theta_i)x_i^{-2}$*, and define* $G_i(w_i(\theta_i)x_i^{-2}, \theta_i) = -b_i(\frac{w_i(\theta_i)x_i^{-2}}{w_i(\theta_i)})^{-1}$*. Note that* $-b_i x_i^2 = -b_i(\frac{w_i(\theta_i)x_i^{-2}}{w_i(\theta_i)})^{-1}$*.* $G_i(\cdot, \theta_i)$ *is a concave transformation of* $w_i(\theta_i)x_i^{-2}$ *given* θ_i*.*

4 Discussion

4.1 Interim or Ex Ante Constraints

Since interim payoff equivalence functions over interim payoffs, it can actually preserve some other interim (or even ex ante) constraints from collective mechanisms to bilateral mechanisms in addition to incentive compatibility constraints. In particular, we consider interim individual rationality constraints and ex ante budget balance constraints. Our results can further incorporate these two constraints and incentive compatibility constraints. This is parallel with Corollary 1.

Interim individual rationality constraints can be formulated as follows. Suppose that the functional form of agent i's reservation utility $r_i : \Theta_i \to \mathbb{R}$ is commonly known. A collective (respectively, bilateral) mechanism $\mathbf{k}$ (respectively, $\overline{\mathbf{k}}$) is **Interim Individual Rational (IR)** if for all $i \in \mathcal{N}$, $\theta_i \in \Theta_i$, $W_i(\mathbf{k}|\theta_i) \geq r_i(\theta_i)$ (respectively, $W_i(\overline{\mathbf{k}}|\theta_i) \geq r_i(\theta_i)$).

From the mathematical perspective, interim IR conditions serve similar roles to corresponding BIC conditions or BNE condition in the constraints of the multi-agency contracting problems. Apparently, given θ_i, the right-hand sides of the IR conditions (the interim payoffs under relevant mechanisms or contract selections) remain the same, and the left-hand sides of the IR conditions ($r_i(\theta_i)$) are simply some constants. Thus, it is not technically difficult to incorporate the individual rationality conditions in all of the aforementioned results. Interim payoff equivalence can easily preserve Bayesian IR constraints.

Ex ante budget balance constraints can be formulated as follows. Suppose there is a budget limitation $b \in \mathbb{R}$ (maybe for the principal). A collective (respectively, bilateral) mechanism $\mathbf{k}$ (respectively, $\overline{\mathbf{k}}$) is **ex ante budget balanced** if $\int_\Theta \sum_{i=1}^{n} \mathbf{t}_i(\theta)\mu(d\theta) \leq b$[18] (respectively, $\int_\Theta \sum_{i=1}^{n} \overline{\mathbf{t}}_i(\theta_i)\mu(d\theta) \leq b$). Again, interim payoff equivalence can clearly preserve ex ante budget balanced constraints due to the form of summation of transfers for different agents in the constraints.

4.2 Asymptotic Equivalence Result

The principal's payoff function may not always have an *additively separable* relation with respective agents' payoff functions, as in Proposition 2, so the exact equivalence may not hold in that situation. Nevertheless, we can show that optimal bilateral mechanism design approaches optimal collective mechanism design from the principal's viewpoint through IPE, as the degrees

[18] The inequality can only take strict equality in the definition in some contexts.

of the strategic interdependence of the allocation of different agents from the perspective of the principal decreases. We first introduce the concept of such degrees of strategic interdependence.

[**Assumption 2**] Let $u(x,\theta) = g((h_i(x_i,\theta_i))_{i\in\mathcal{N}},\theta)$, where $g:\mathbb{R}^n \times \Theta \to \mathbb{R}$ is a function that is continuously second-order differentiable in each of first n arguments[19] and continuous in θ. $g_i(\cdot,\theta)$ denotes the partial derivative with respect to the i-th argument, and $g_{ij}(\cdot,\theta)$ denotes the ij-th second-order derivative with respect to the i-th and j-th arguments.

Second-order derivatives of $g(\cdot,\theta)$ represents *the degrees of the strategic interdependence* (either strategic complementarity or strategic substitutivity) in the principal's payoff between any two agents' direct utilities from individual allocations and types.[20] The polar case is that g is linearly additive in all h_i's. This reflects *strategic independence* in the principal's payoff between any two individual agents' direct utilities, that is, each $g_{ij}(\cdot,\theta)$ is equal to 0. Moreover, if $g_{ij}(\cdot,\theta)$ is nonnegative (respectively, nonpositive), there is strategic complementarity (respectively, strategic substitutivity) between any two individual agents' direct utilities.

Definition 5 *The assignment rule* $\mathbf{x}$ *(respectively,* $\overline{\mathbf{x}}$*) is said to be* ***regular*** *if each* $\mathbf{x}_i$ *(respectively,* $\overline{\mathbf{x}}_i$*) is such that for each* i *and* $\theta, h_i(\mathbf{x}_i(\theta),\theta_i)$ *(respectively,* $h_i(\overline{\mathbf{x}}_i(\theta),\theta_i)$*) is integrable with respect to* μ *over* θ*.*

The regularity of optimal assignment rules can be achieved in many applications. Otherwise, $G(\mathbf{x}^*(\cdot),\cdot)$ or $G(\overline{\mathbf{x}}^*(\cdot),\cdot)$ may not be integrable over θ under μ. For instance, such regularity may be more likely to hold with the inclusion of individual rationality. Moreover, in some applications we need to assume that each X_i is compact, so the regularity of optimal assignment rules can be easily satisfied.

Under such regularities, we can obtain an asymptotic result for the equivalence of optimal bilateral and collective mechanisms as the degrees of the strategic interdependence decrease (or approach zero). Let U^* denote the optimal value of collective BIC mechanism design problem, U^{**} denote the optimal value of bilateral BIC mechanism design problem, and $\mathbf{0}$ denote the n-dimensional vector with all coordinates equal to 0.

Proposition 3 *Under Assumptions 1 and 2, in a quasi-separable environment with each* X_i *and* T_i *being connected and closed, if for each* i*,*

(i) $h_i(x_i^c,\theta_i) = 0$ *for all* θ_i *and some* $x_i^c \in X_i$*,*

(ii) $g_i(\mathbf{0},\theta) = a_i(\theta_i)w_i(\theta)$*, for each* θ *and some continuous function* $a_i:\Theta_i \to \mathbb{R}$*, and*

(iii) $|g_{ij}((h_i(x_i,\theta_i))_{i\in\mathcal{N}},\theta)| \le M$ *for each* j*,* θ*,* x *and some* $M \ge 0$*,*

then $U^* - U^{**} \le M\{\sum_{i=1}^n \sum_{j=1}^n (\alpha_{ij} + \beta_{ij})\}$*, where* $\alpha_{ij} = \int_\Theta \left| h_i(\mathbf{x}_i^*(\theta),\theta_i) h_j(\mathbf{x}_j^*(\theta),\theta_j) \right| \mu(d\theta)$*, and* $\beta_{ij} = \int_\Theta \left| h_i(\overline{\mathbf{x}}_i^*(\theta),\theta_i) h_j(\overline{\mathbf{x}}_j^*(\theta),\theta_j) \right| \mu(d\theta)$*. When each optimal collective assignment rule* $\mathbf{x}_i^*$ *and its IPE bilateral assignment rule* $\overline{\mathbf{x}}_i^*$ *are regular, as* M *approaches zero,* U^{**} *approaches* U^**.*

[19] More rigorously, given θ, g is continuously second-order differentiable in the i-th argument over the interior of the range of $h_i(\cdot,\theta_i)$, and left (respectively, right)-continuously second-order-differentiable at the right (respectively, left) ending point of the range.

[20] Generally speaking, this includes the "interdependence" between any individual agent's direct utility and his direct utility itself, which is reflected by second-order derivatives with respect to h_i itself.

If $u(x,\theta)$ can be expressed as a composite function of $h_i(x_i,\theta_i)$'s, and $g_i(\mathbf{0},\theta)$ can be expressed as a linear transformation of $w_i(\theta)$ by a multiplier $a_i(\theta_i)$ (in the sense of expectation), then optimal collective mechanism design can be approximated arbitrarily close to optimal bilateral mechanism design through IPE, as the degrees of the strategic interdependence approach zero. The weaker the strategic interdependence is, the more likely optimal bilateral mechanism design will approach optimal collective mechanism design. Below are several examples to which Proposition 3 can apply.

Example 8 (Nonliear Pricing 2) *Recall the Nonlinear Pricing 1 example. Now the seller's payoff instead takes a non-separable form in* x, $\sum_{i=1}^{n} t_i - c(x)$, *where* $c(x) = \frac{\sigma}{2}(\sum_{i=1}^{n} x_i)^2$ *is the quadratic cost function of the supplier, where the parameter* $\sigma > 0$. *For each* i, $c_i(x) = \sigma \sum_{i=1}^{n} x_i$. *Referring to Proposition 3,* $h_i(x_i,\theta_i) \equiv x_i$ *here, and then* $g((h_i(x_i,\theta_i))_{i\in\mathcal{N}},\theta) \equiv c(x)$. $c_i(\mathbf{0}) = 0$. *So we can choose* $a_i(\theta_i) \equiv 0$. *For each* i,j,x, $c_{ij}(x) \equiv \sigma$. *Thus,* $U^* - U^{**} \leq \sigma \sum_{i=1}^{n}\sum_{j=1}^{n}(\alpha_{ij} + \beta_{ij})$, *where* $\alpha_{ij} = \int_\Theta \left|\mathbf{x}_i^*(\theta)\mathbf{x}_j^*(\theta)\right| \mu(d\theta)$, *and* $\beta_{ij} = \int_\Theta \left|\overline{\mathbf{x}}_i^*(\theta)\overline{\mathbf{x}}_j^*(\theta)\right| \mu(d\theta)$. *The smaller* σ *is, the closer* U^{**} *gets to* U^*.

Example 9 (Procurement 3) *Recall the Procurement 1 example. Now a buyer needs to procure two* imperfectly substitutive *goods separately from* 2 *producers indexed by* $i = 1,2$. i *receives a production cost signal* $\theta_i > 0$. *Each producer* i'*s payoff is* $t_i - c_i(x_i,\theta_i)$, *where* $c_i(x_i,\theta_i) = \theta_i x_i^2$ *is the production cost of* x_i. *The buyer's payoff is* $B(x) - \sum_{i=1}^{2} t_i$, *where* $B(x) = -e^{-\gamma(x_1^2+x_2^2)}$ *is the benefit the buyer can draw from consumption of* x, *and* $\gamma > 0$. *This takes an exponential utility functional form with constant degree of absolute risk aversion* (γ). *Referring to Proposition 3,* $h_i(x_i,\theta_i) \equiv x_i^2$, *then* $g(x_1^2,x_2^2) = B(x)$.[21] *For each* i, $g_i(x_1^2,x_2^2) = \gamma e^{-\gamma(x_1^2+x_2^2)}$, *and* $g_i(\mathbf{0}) = \gamma$. *We can choose* $a_i(\theta_i) \equiv \frac{\gamma}{\theta_i}$. *Moreover,* $g_{ij}(x_1^2,x_2^2) = -\gamma^2 e^{-\gamma(x_1^2+x_2^2)}$. *Clearly,* $\left|g_{ij}(x_1^2,x_2^2)\right| \leq \gamma^2$. *Thus,* $U^* - U^{**} \leq \gamma^2 \sum_{i=1}^{n}\sum_{j=1}^{n}(\alpha_{ij} + \beta_{ij})$, *where* $\alpha_{ij} = \int_\Theta (\mathbf{x}_i^*(\theta))^2(\mathbf{x}_j^*(\theta))^2\mu(d\theta)$, *and* $\beta_{ij} = \int_\Theta (\overline{\mathbf{x}}_i^*(\theta))^2(\overline{\mathbf{x}}_j^*(\theta))^2\mu(d\theta)$. *The smaller* γ *is, the closer* U^{**} *gets to* U^*.

4.3 Primitive Constraints across Contracts for Different Agents

The aforementioned results do not address the explicit primitive constraints *across* contracts for different agents under which $\mathcal{K}$ is not directly equal to the product of the agents' contract sets, that is, $\mathcal{K} \subseteq \prod_{i=1}^{n} \mathcal{K}_i$, but $\mathcal{K} \neq \prod_{i=1}^{n} \mathcal{K}_i$. This is equivalent to saying that there is some ex post constraint across agents on the mechanisms. For instance, in auction mechanism design, the sum of the probability assignments for different agents must not be greater than 1.

Should the IPE bilateral BIC mechanism $\overline{\mathbf{k}}$ still satisfy the primitive constraint, i.e., $\overline{\mathbf{k}}(\theta) \in \mathcal{K}$ for each θ, our results will still hold. Below is a simple example of this.

Example 10 (Procurement 4) *One producer procures two input goods separately from two input suppliers denoted by* $i = 1,2$. *Contracts are the same as in the Procurement 3 example.*

[21] With a little abuse of notation, g does not depend on θ here.

θ_i's are independently distributed. i's payoff function $v_i(t_i, x_i, \theta_i) = t_i - \theta_i x_i$. The producer's payoff $u(x,t,\theta) = x_1^{\alpha} x_2^{1-\alpha} - t_1 - t_2$. $x_1^{\alpha} x_2^{1-\alpha}$ denotes the Cobb-Douglas (monetary) production function, where $\alpha \in (0,1)$. There is a constraint over $x_i's$: $x_1^{\alpha} x_2^{1-\alpha} \leq \overline{q}$, where $\overline{q}$ denotes the capacity limit. The producer cannot purchase the bundle of (x_1, x_2) beyond the production capacity constraint. For each i, the collective (respectively, bilateral) BIC assignment rule for i is $\mathbf{x}_i : \Theta \to \mathbb{R}$ (respectively, $\overline{\mathbf{x}}_i : \Theta_i \to \mathbb{R}$). Suppose the optimal collective BIC assignment rule is $\mathbf{x}^*$. Then its IPE bilateral assignment rule $\overline{\mathbf{x}}^*$ can be defined by

$$\overline{\mathbf{x}}_i^*(\theta_i) = \int_{\Theta_{-i}} \mathbf{x}_i^*(\theta) \mu_{-i}(d\theta_{-i}), i = 1, 2.$$

Clearly, if for each $\theta, \mathbf{x}_1^{*^{\alpha}}(\theta) \mathbf{x}_2^{*^{1-\alpha}}(\theta) \leq \overline{q}$, then $\overline{\mathbf{x}}_1^{*^{\alpha}}(\theta_1) \overline{\mathbf{x}}_2^{*^{1-\alpha}}(\theta_2) \leq \overline{q}$.

In some cases, IPE bilateral mechanisms may not necessarily preserve some primitive constraints across contracts for different agents, especially for those linear combination inequality constraints, such as the natural requirement for probabilistic assignments in auction design, and proper equality constraints, such as the natural requirement for common allocations in public good provision design. It is generally difficult to provide conditions on the primitives for such preservation. Such preservation may need some requirement on the properties of the optimal collective mechanism per se, and then we need to check whether its IPE bilateral mechanism satisfies the primitive constraints. For instance, the symmetric mechanism design with ex ante identical agents may help in this regard, especially in auction contexts. It could be more tractable to establish the equivalence and/or its approximation of the (optimal) bilateral and collective mechanisms in more concrete applications.

4.4 Mathematical Generalizations

We can permit some mathematical generalizations of our model and analysis, which will not change any of our findings. In the baseline model, we assume that for each i the sets Θ_i and $\mathcal{K}_i$ are close subsets of Euclidean spaces. In fact, in our model, they can be generalized to be Polish spaces, i.e., complete separable metric spaces.[22] A familiar example of Polish space is any Euclidean space, $\mathbb{R}^n$. Moreover, in Proposition 1 and the subsequent results, each X_i and T_i can be generalized to be connected, locally compact Polish spaces. A typical example of a locally compact Polish space is any closed or open subset of a Euclidean space.

The setting of Polish space beyond Euclidean space can render our findings applicable to broader contexts. For instance, if there is no transfer, and $\mathcal{K}_i = X_i$ is some general Polish space, we can allow a *state-contingent contract* set for it, as follows.[23] The state (or outcome) is $\omega \in \Omega \subseteq \mathbb{R}$. η is a probability measure over Ω. Assume that all contracts are outcome-contingent.

[22] Note that any open or closed subset of a Polish space is still Polish, and a finite product of Polish spaces is still Polish. Any compact metric space is also a Polish space.

[23] Page and Monteiro (2003) present a similar example under common agency.

If for each i, (1) $\mathcal{K}_i$ is a subset of the collection of all Borel-measurable functions from Ω to $[L, H] \subseteq \mathbb{R}$ indexed by a compact metric space I, that is, $\mathcal{K}_i = \{f(\cdot, \gamma) : \Omega \to [L, H] | \gamma \in I$, and f is Borel measurable in ω and continuous in $\gamma\}$, where I is a compact metric space, and (2) $\mathcal{K}_i$ contains no redundant contracts, that is, if for any two k_i and k'_i in $\mathcal{K}_i$ satisfying $k_i(\omega') \neq k'_i(\omega')$ for some $\omega' \in \Omega$, $\eta(\{\omega \in \Omega : k_i(\omega) \neq k'_i(\omega)\}) > 0$, then $\mathcal{K}_i$ is a compact metric space by Proposition 1 in Tulcea (1973) and therefore a locally compact Polish spaces. So is $\mathcal{K}$. We can also impose the connectedness on $\mathcal{K}_i$ for applying our results.

Appendix: Proofs

Proof of Proposition 1. First note that each X_i is a locally compact metric space, as we discuss in the last paragraph of Section 4.4. For each i, given θ_i, define $\Phi_i : X_i \to \mathbb{R}$ by $\Phi_i(x_i) = \int_{\Theta_{-i}} v_i(x_i, \theta)\mu_{-i}(d\theta_{-i}|\theta_i)$. Since v_i is continuous, Φ_i is also continuous. Thus, the connectedness of X_i implies that the range of Φ_i is also connected in $\mathbb{R}$, and therefore should be an interval I_{i,θ_i} taking the form of $[a_i(\theta_i), b_i(\theta_i)]$, $[a_i(\theta_i), b_i(\theta_i))$, $(a_i(\theta_i), b_i(\theta_i)]$, or $(a_i(\theta_i), b_i(\theta_i))$. Moreover, Φ_i is clearly onto from X_i to I_{i,θ_i}.

Then we define the inverse set-valued function of Φ_i as $\rho^{-1}_{i,\theta_i} : I_{i,\theta_i} \twoheadrightarrow X_i$ by

$$\rho^{-1}_{i,\theta_i}(y) = \{x_i \in X_i : \Phi_i(x_i) = y\}$$

By a variant of the closed map lemma,[24] the local compactness of X_i[25] implies that Φ_i is a continuous closed map. Thus, ρ^{-1}_{i,θ_i} must be nonempty closed-valued and is a measurable set-valued function. The Kuratowski-Ryll-Nardzewski Selection Theorem (Aliprantis and Border, 2006, page 600) implies that ρ^{-1}_{i,θ_i} must admit a Borel-measurable selector, say $\varphi_{i,\theta_i} : I_{i,\theta_i} \to X_i$.

Next, given $\mathbf{x}_i$, we define $\phi_{i,\mathbf{x}_i} : \Theta_i \to \mathbb{R}$ by

$$\phi_{i,\mathbf{x}_i}(\theta_i) = \int_{\Theta_{-i}} v_i(\mathbf{x}_i(\theta), \theta)\mu_{-i}(d\theta_{-i}|\theta_i).$$

Obviously, $\phi_{i,\mathbf{x}_i}$ is a Borel-measurable function of θ_i due to the assumption on μ_{-i} in the model primitives.

Moreover, for all θ, $v_i(\mathbf{x}_i(\theta), \theta)$ will be contained in an interval taking the same form as I_{i,θ_i} but with two boundary points as $\inf_{x_i \in X_i} v_i(x_i, \theta)$ and $\sup_{x_i \in X_i} v_i(x_i, \theta)$. Thus, $\phi_{i,\mathbf{x}_i}(\theta_i)$ will be contained in another interval taking the same form as I_{i,θ_i}, but with two boundary points as $\int_{\Theta_{-i}} \inf_{x_i \in X_i} v_i(x_i, \theta)\mu_{-i}(d\theta_{-i}|\theta_i)$ and $\int_{\Theta_{-i}} \sup_{x_i \in X_i} v_i(x_i, \theta)\mu_{-i}(d\theta_{-i}|\theta_i)$.

[24] A continuous function between locally compact Hausdorff spaces is closed.

[25] X, as a finite product of X_i's, is also locally compact.

Now we fix θ_i. Due to the non-negativity of $w_i(\theta)$, we can have

$$\begin{aligned}
& \int_{\Theta_{-i}} \sup_{x_i \in X_i} [h_i(x_i,\theta_i)w_i(\theta)]\mu_{-i}(d\theta_{-i}|\theta_i) \\
= & \int_{\Theta_{-i}} [\sup_{x_i \in X_i} h_i(x_i,\theta_i)]w_i(\theta)\mu_{-i}(d\theta_{-i}|\theta_i) \\
= & [\sup_{x_i \in X_i} h_i(x_i,\theta_i)] \int_{\Theta_{-i}} w_i(\theta)\mu_{-i}(d\theta_{-i}|\theta_i) \\
= & \sup_{x_i \in X_i} [h_i(x_i,\theta_i) \int_{\Theta_{-i}} w_i(\theta)\mu_{-i}(d\theta_{-i}|\theta_i)] \\
= & \sup_{x_i \in X_i} \int_{\Theta_{-i}} h_i(x_i,\theta_i)w_i(\theta)\mu_{-i}(d\theta_{-i}|\theta_i).
\end{aligned}$$

Thus, in a quasi-separable environment,

$$\sup_{x_i \in X_i} \int_{\Theta_{-i}} v_i(x_i,\theta)\mu_{-i}(d\theta_{-i}|\theta_i) = \int_{\Theta_{-i}} \sup_{x_i \in X_i} v_i(x_i,\theta)\mu_{-i}(d\theta_{-i}|\theta_i) \tag{2}$$

By a similar argument,

$$\inf_{x_i \in X_i} \int_{\Theta_{-i}} v_i(x_i,\theta)\mu_{-i}(d\theta_{-i}|\theta_i) = \int_{\Theta_{-i}} \inf_{x_i \in X_i} v_i(x_i,\theta)\mu_{-i}(d\theta_{-i}|\theta_i). \tag{3}$$

(1) and (2) imply $\phi_{i,\mathbf{x}_i}(\theta_i) \in I_{i,\theta_i}$ for all θ_i in a quasi-separable environment.

Therefore, we can define a function $\overline{\mathbf{x}}_i : \Theta_i \to X_i$ by

$$\overline{\mathbf{x}}_i(\theta_i) = \varphi_{i,\theta_i}(\phi_{i,\mathbf{x}_i}(\theta_i)).$$

Hence $\overline{\mathbf{x}}_i$ is clearly a Borel-measurable function, and then $\overline{\mathbf{x}}$ is a well-defined bilateral mechanism. By the definitions above, it is easy to see that for each θ_i, $W_i(\mathbf{x}_i|\theta_i) = W_i(\overline{\mathbf{x}}_i|\theta_i)$.

By a similar argument, it is trivial to show that given $\mathbf{t}_i$ there must exist some $\overline{\mathbf{t}}_i : \Theta_i \to T_i$ such that $\int_{\Theta_{-i}} \mathbf{t}_i(\theta)\mu_{-i}(d\theta_{-i}|\theta_i) = \int_{\Theta_{-i}} \overline{\mathbf{t}}_i(\theta_i)\mu_{-i}(d\theta_{-i}|\theta_i)$. In sum, $\overline{\mathbf{k}}_i$ is IPE to $\mathbf{k}_i$. Moreover, it is clear that interim payoff equivalence can preserve Bayesian incentive compatibility from collective mechanisms to bilateral mechanisms. □

Proof of Proposition 2. First note that

$$\int_{\Theta_{-i}} G_i(h_i(x_i,\theta_i)w_i(\theta),\theta_i)\mu_{-i}(d\theta_{-i}) = G_i(\int_{\Theta_{-i}} (h_i(x_i,\theta_i)w_i(\theta))\mu_{-i}(d\theta_{-i}),\theta_i)$$

if hypothesis (ii) holds; this is straightforward by the conditions under which the equality holds in the Jensen's inequality.

By Proposition 1, we can always find a bilateral BIC mechanism $\overline{\mathbf{k}}$ interim-payoff-equivalent to $\mathbf{k}^*$. Hence,

$\int_{\Theta} u(\mathbf{k}^*(\theta), \theta)\mu(d\theta)$
$= \int_{\Theta}\{\sum_{i=1}^{n}[G_i((h_i(\mathbf{x}_i^*(\theta), \theta_i)w_i(\theta)), \theta_i)] + L(\theta) + \mathbf{t}_i^*(\theta)\}\mu(d\theta)$
$= \sum_{i=1}^{n}[(\int_{\Theta_i}\int_{\Theta_{-i}} G_i((h_i(\mathbf{x}_i^*(\theta), \theta_i)w_i(\theta)), \theta_i)\mu_{-i}(d\theta_{-i})\mu_i(d\theta_i))] + \int_{\Theta} L(\theta)\mu(d\theta) + \int_{\Theta}\mathbf{t}_i^*(\theta)\mu(d\theta)$
$\leq \sum_{i=1}^{n}[\int_{\Theta_i} G_i(\int_{\Theta_{-i}}(h_i(\mathbf{x}_i^*(\theta), \theta_i)w_i(\theta))\mu_{-i}(d\theta_{-i}), \theta_i)\mu_i(d\theta_i)] + \int_{\Theta} L(\theta)\mu(d\theta) \; + \int_{\Theta}\mathbf{t}_i^*(\theta)\mu(d\theta)$
(By Jensen's inequality)
$= \sum_{i=1}^{n}[\int_{\Theta_i} G_i(\int_{\Theta_{-i}}(h_i(\overline{\mathbf{x}}_i^*(\theta_i), \theta_i)w_i(\theta))\mu_{-i}(d\theta_{-i}), \theta_i)\mu_i(d\theta_i)] + \int_{\Theta} L(\theta)\mu(d\theta) + \int_{\Theta}\mathbf{t}_i^*(\theta)\mu(d\theta)$
(By IPE)
$= \sum_{i=1}^{n}[\int_{\Theta_i}\int_{\Theta_{-i}} G_i((h_i(\overline{\mathbf{x}}_i^*(\theta_i), \theta_i)w_i(\theta)), \theta_i)\mu_{-i}(d\theta_{-i})\mu_i(d\theta_i)] + \int_{\Theta} L(\theta)\mu(d\theta) + \int_{\Theta}\overline{\mathbf{t}}_i^*(\theta)\mu(d\theta)$
(By hypothesis (ii))
$= \int_{\Theta} u(\overline{\mathbf{k}}^*(\theta), \theta)\mu(d\theta)$.

Since $\mathbf{k}^*$ is the optimal solution to **(P1),** $\int_{\Theta} u(\mathbf{k}^*(\theta), \theta)\mu(d\theta) = \int_{\Theta} u(\overline{\mathbf{k}}^*(\theta), \theta)\mu(d\theta)$. Thus, **P1** is strategically equivalent to **P2**. □

Proof of Proposition 3. Clearly, in the quasi-linear environment, there exists some $(\overline{\mathbf{x}}^*, \overline{\mathbf{t}}^*)$ IPE to $(\mathbf{x}^*, \mathbf{t}^*)$. Thus, the difference between U^* and U^{**},

$U^* - U^{**} \leq \int_{\Theta} g((h_i(\mathbf{x}_i^*(\theta), \theta_i))_{i\in\mathcal{N}}, \theta)\mu(d\theta) - \int_{\Theta} g((h_i(\overline{\mathbf{x}}^*(\theta), \theta_i))_{i\in\mathcal{N}}, \theta)\mu(d\theta)$
$= \frac{1}{2}\int_{\Theta} | \sum_{i=1}^{n}\sum_{j=1}^{n} g_{ij}(o^{c\theta}, \theta)h_i(\mathbf{x}_i^*(\theta), \theta_i)h_j(\mathbf{x}_j^*(\theta), \theta_j)$
$- \sum_{i=1}^{n}\sum_{j=1}^{n} g_{ij}(o^{d\theta}, \theta)h_i(\overline{\mathbf{x}}_i^*(\theta), \theta_i)h_j(\overline{\mathbf{x}}_j^*(\theta), \theta_i)|\mu(d\theta)$,

for some $o^{c\theta}$ between $(h_i(\mathbf{x}_i^*(\theta), \theta_i))_{i\in\mathcal{N}}$ and $\mathbf{0}$ and some $o^{d\theta}$ between $(h_i(\overline{\mathbf{x}}_i^*(\theta), \theta_i))_{i\in\mathcal{N}}$ and $\mathbf{0}$ for each θ. The equality holds by Taylor Expansion Theorem, hypotheses (i) and (ii), and Proposition 2. Thus,

$U^* - U^{**}$
$\leq \frac{1}{2}\int_{\Theta}\{\left|\sum_{i=1}^{n}\sum_{j=1}^{n} g_{ij}(o^{c\theta}, \theta)h_i(\mathbf{x}_i^*(\theta), \theta_i)h_j(\mathbf{x}_j^*(\theta), \theta_j)\right|$
$+ \left|\sum_{i=1}^{n}\sum_{j=1}^{n} g_{ij}(o^{d\theta}, \theta)h_i(\overline{\mathbf{x}}_i^*(\theta), \theta_i)h_i(\overline{\mathbf{x}}_j^*(\theta), \theta_i)\right|\}\mu(d\theta)$
$\leq \frac{1}{2}\int_{\Theta}\{\sum_{i=1}^{n}\sum_{j=1}^{n}\left|g_{ij}(o^{c\theta}, \theta)h_i(\mathbf{x}_i^*(\theta), \theta_i)h_j(\mathbf{x}_j^*(\theta), \theta_j)\right|$
$+ \sum_{i=1}^{n}\sum_{j=1}^{n}|g_{ij}(o^{d\theta}, \theta)h_i(\overline{\mathbf{x}}_i^*(\theta), \theta_i)h_i(\overline{\mathbf{x}}_j^*(\theta), \theta_i)|\}\mu(d\theta)$
$\leq \frac{1}{2}\int_{\Theta}\{\sum_{i=1}^{n}\sum_{j=1}^{n} M\left|h_i(\mathbf{x}_i^*(\theta), \theta_i)h_j(\mathbf{x}_j^*(\theta), \theta_j)\right| + \sum_{i=1}^{n}\sum_{j=1}^{n} M|h_i(\overline{\mathbf{x}}_i^*(\theta), \theta_i)h_i(\overline{\mathbf{x}}_j^*(\theta), \theta_i)|\}\mu(d\theta)$
(By hypothesis (iii))
$= M\{\sum_{i=1}^{n}\sum_{j=1}^{n}(\alpha_{ij} + \beta_{ij})\}$,

where $\alpha_{ij} = \int_{\Theta}\left|h_i(\mathbf{x}_i^*(\theta), \theta_i)h_j(\mathbf{x}_j^*(\theta), \theta_j)\right|\mu(d\theta)$, and $\beta_{ij} = \int_{\Theta}\left|h_i(\overline{\mathbf{x}}_i^*(\theta), \theta_i)h_j(\overline{\mathbf{x}}_j^*(\theta), \theta_j)\right|\mu(d\theta)$. α_{ij} and β_{ij} will be nonnegative and finite, as each $\mathbf{x}_i^*$ and $\overline{\mathbf{x}}_i^*$ are regular. Furthermore, as M goes to zero, U^{**} clearly approaches U^*. □

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