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# **Financial integration and house price dynamics in equilibrium modeling of intra-EMU and global trade imbalances**

**Karl Farmer<sup>1</sup>**

## **Abstract**

A dramatic decline of EMU periphery's and the US saving rate, house price booms and huge global and intra-EMU trade imbalances characterize the evolution of the world economy since Euro-related intra-EMU and global financial integration after the East-Asian currency crisis up to the onset of the global financial crisis. While the intra-EMU and global trade imbalances and the huge level differences between Asian and US saving rates can be explained by means of Farmer and Mihaiescu's (2016) three-country, three-good OLG model with financial constraints and growth rate differentials, the pronounced decline in EMU periphery's and in the US saving rate cannot. It is natural to suggest that house price booms in EMU periphery and in the US boosted consumption and reduced savings. Thus, this paper introduces house price dynamics à la Arce and López-Salido (2011) and Basco (2014) into Farmer and Mihaiescu's (2016) intertemporal equilibrium model, and finds that house price increases let EMU periphery's and US saving rates indeed decline more quickly – in line with empirical facts.

JEL: F36

**Keywords:** Trade Imbalances, Financial Integration, House Price Dynamics, Overlapping Generations, Three-Country Model

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## **Introduction and Motivation**

A dramatic decline of EMU periphery's and US saving rate, house price booms and huge global and intra-EMU trade imbalances characterize the evolution of the world economy between Euro-related intra-EMU and global financial integration after the East-Asian currency crisis and the onset of the global financial crisis. These three empirical facts are obviously not unconnected as several economists observed. There is first the relationship between global imbalances, i.e. current account surpluses of emerging South-Asia and of oil-exporting countries and US current account deficits, and US housing prices: The increase in US current account and trade balance deficits was associated with a boom in US housing (and stock) markets (Bernanke 2005, Obstfeld and Rogoff 2009, Rajan 2011, Stiglitz 2012). Second, the boom in house prices and other asset prices was related to the boom of US economic activities as consumption and credit (Bernanke 2009, Geanakoplos 2010, Martin and Ventura 2012). As a consequence, it is tempting to suggest that the above mentioned decline of US saving rate can be attributed to the house price boom and enlarged as a consequence the US current account and trade deficit.

The household saving rate declined and the current account deficit popped up not only in the USA, but also in the so-called periphery of the European Economic and Monetary Union (EMU) after Euro-related financial integration and up to the onset of the global financial crisis. The saving rate decline in EMU periphery was even more pronounced than in the US contributing by itself to the rather dramatic increase of the current account and trade balance deficit in EMU periphery.

Acknowledging the similarity in the pre-crisis development of the household saving rate and the trade imbalance for US and EMU periphery it seems to be natural to address the issues of US and EMU periphery's saving rate decline and trade deficit increase within a unified framework capable to investigate also the trade and financial interlinkages and spillovers among EMU, Asia and the USA. To the best of this author's knowledge, as yet only Farmer and Mihaiescu (2016) developed a three-country (EMU, Asia, USA), two-region (EMU core, EMU periphery) intertemporal equilibrium model à la Buitier (1981) which accounts explicitly for the trade linkages between EMU sub-areas and extra-EMU trading partners forcefully pointed out by Chen et al. (2013). Global and intra-EMU external imbalances emerge in Farmer and Mihaiescu's model due to real interest rate convergence both within the EMU, and also between Asia and the USA. Moreover, temporary growth differentials between Asia and the other countries together with internationally differently biting credit constraints generate a decline in world average real interest rate and allow for the replication of the empirically observed level differences between the Asian and the US saving rate. However, while these authors are able to attribute both global and intra-EMU external imbalances, the world-wide interest rate decline and Asian-US saving rate level discrepancies to intra-EMU and global financial integration as well as to the larger Asian

productivity growth rate and the associated increase in Asian's share in global GDP, the authors could not satisfactorily explain the observed decline of EMU periphery's and US household saving rates.

Acknowledging the positive relationship between US consumption and US house prices mentioned above it is natural to suggest that the decline in the US household saving rate was induced by the US house price boom. First because of the negative relation between savings and consumption and second, because larger house prices enabled US households to get more mortgage loans since the value of the collateral increased with rising house prices. Thus, it seems likely to suggest that the US house price dynamics is indeed decisive for the decline for the US saving rate. What is true for the US seems also to be applicable to EMU periphery. While in the former case house price booms are fueled by inflow of Asian loanable funds (Bernanke 2005), higher yield seeking EMU core's loanable funds fueled periphery's house price boom. In both cases house price booms accelerated consumption and depressed savings.

Thus, it is apt to introduce housing into Farmer and Ban's (2015) three-country, two-region OLG model in order to be able to replicate the empirically observed, pronounced saving rate decline in EMU periphery and US. However, since in this model there is no explicit housing market the relevant literature was consulted, and main elements of Arce and López-Salido's (2011) closed-economy and Basco's (2014) two-country open economy modeling of the housing market were adopted. While in López-Salido (2011) the housing stock is exogenously given and constant over time, in Basco (2014) the housing stock changes over time due to depreciation and the production of new houses which, however, are manufactured by means of labor input only. This simplifying assumption enables a closed-form steady state solution, but precludes both capital services as a second input into goods production and the accumulation of a second capital good (besides houses) serving as an alternative asset in households' portfolios. Extending López-Salido's (2011) seminal closed-economy and Basco's (2014) two-country housing market model by including a second produced capital good is not only more in line with empirical reality but also facilitates the integration of house price dynamics into Farmer and Ban's (2015) three-country model with capital accumulation but without housing markets.

In order to be able to replicate in this model extended by housing the intra-EMU and Asian-US trade imbalances, interest rate convergence and saving rate declines in EMU periphery and US under financial integration the author assumes in line with Arce and López-Salido (2011) and Basco (2014) three-period lived overlapping generations, subjected to credit constraints on discounted future house wealth in all countries respective regions. This novel model will be used to see whether the empirically observed trade imbalances, the decline of the world interest rate and of EMU periphery's and US saving rates can be better replicated than in the model without house price dynamics.

In order to motivate the set-up of the extended model we recall key stylized facts regarding intra-EMU and global trade imbalances between the late 1990s and the onset of the global financial crisis in 2007. First, while euro-related financial integration and convergence expectations (Blanchard and Giavazzi, 2002; Lane, 2006; Japelli and Pagano, 2008; Spiegel, 2009; Kalemli-Ozcan et al., 2010, Schmitz and von Hagen, 2011) contributed without doubt to the evolution of intra-EMU trade imbalances, Chen et al. (2013) emphasize extra-EMU factors as (1) the increasing competitive advantage of Asian exporters vis-à-vis EMU periphery exporters, (2) the rising demand for EMU core capital goods by Asia and oil exporters and (3) the US demand for EMU core financial assets. As a consequence, EMU periphery's current account deficit, while financed mostly by capital inflows from the EMU core, did not increase that much vis-à-vis the core but vis-à-vis Asia and oil exporters. Similarly, EMU core's current account surplus after the Euro launch resulted not from EMU periphery's imports but from rising Asian imports of EMU core (capital) goods.

As mentioned above, financial integration occurred not only in the EMU after Euro inception but also between Asia and the USA – albeit under different institutional ramifications than those existing in the Eurozone. The interest convergence between initially higher US and lower Asian rates is seen as a major driver for the emergence of US current account deficits and Asian current account surpluses (Mendoza et al. 2009; Angeletos and Panousi, 2009). As a consequence of high Asian saving rates, Asians accumulated huge amounts of save US government bonds inducing the convergence of nominal and real Asian and US interest rates.

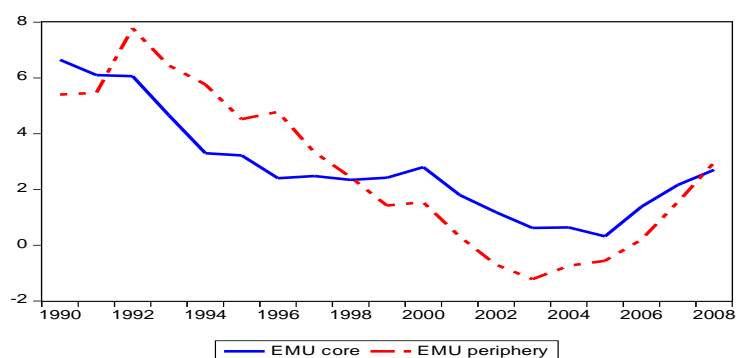
In line with Buiter's (1981) seminal one-good, two-country OLG model we depict the pre-Euro and pre-East-Asian currency crisis situation with relatively larger real interest rates in EMU periphery and in the USA as financial autarky and the convergence of (real) interest rates within the Eurozone and between US and Asia after the 1998 currency crisis as financial integration. The higher EMU periphery's and US autarky interest rates are attributed to EMU periphery's and US relatively higher future (utility) discount factors, to EMU periphery's and Asia's relatively higher capital production shares, to EMU periphery's and US relatively larger public debt to GDP's ratios. These national differences in structural and policy parameters translate under financial autarky via the national housing markets into substantial level divergences between national saving rates. Through financial integration and the associated interest rate convergence EMU core and US house prices increase inducing rising household wealth and lower saving rates. In this vein external imbalances represent equilibrium responses to international differences in national fundamentals as in the seminal Buiter (1981) one-good, two country OLG model.

The paper is organized as follows. The next section presents key stylized international macroeconomic facts characterizing both the financial autarky situation of 1980s and 1990s and the financial integration period between 1999 and 2008. Then, the financial autarky variant of the

three-country, two-region OLG model with housing markets in all countries is presented. Thereafter we show the household budget constraints and the market clearing conditions under financial integration, derive the intertemporal equilibrium dynamics and investigate how key international macro indicators as the net foreign asset position, the trade imbalance and the aggregate saving rate are related to the main variables of the intertemporal equilibrium dynamics. In the following section our model economy is numerically specified for the financial autarky case, and the numerical solution for net foreign asset positions, trade imbalances and aggregate saving rates in the integration period is compared to main stylized macroeconomic facts, related to intra-EMU and Asia-US financial integration up to the onset of the global financial crisis in 2007. Concluding remarks in the final section summarize key results.

### Stylized Macroeconomic Facts: Financial Autarky versus Financial Integration

In order to illuminate the design of the three-good, three-country and two-region OLG model, some of the relevant stylized facts are now presented in this section. These relate to the macroeconomic performance of the EMU members, Asia and the USA, and the evolution of the trade balance and the net foreign asset positions in the EMU, Asia and the USA before the launch of the euro in 1999 and up to 2008. Following Fagan and Gaspar (2008, p. 9), the EMU countries are separated into two groups based on the differences in short-term real interest rates in the late 1990s, i.e. before the euro launch. The first group, usually denoted as the “core” countries, comprises the low interest rate countries Austria, Belgium, France, Germany and the Netherlands.<sup>2</sup> The second group, denoted as “periphery” or converging countries, consists of Ireland, Italy, Spain and Portugal which had relatively high interest rates before the introduction of the euro (see Fig. 1).

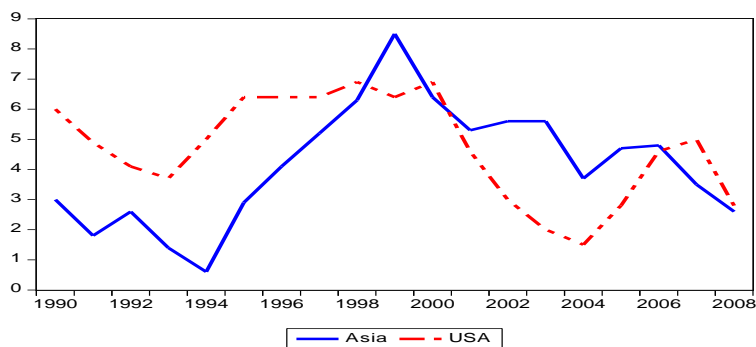


**Fig. 1** Real short-term interest rates in EMU core and periphery 1990-2008. Source: AMECO

Figure 1 reveals that in contrast to the pre-EMU situation (before 1999), EMU periphery’s higher real interest rates decreased towards lower rates in EMU core.<sup>3</sup>

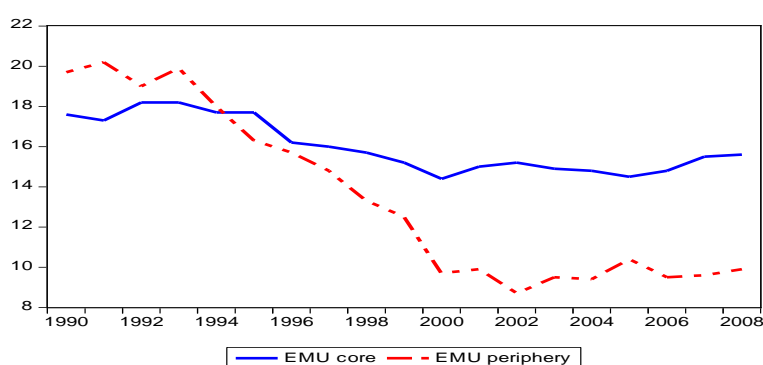
<sup>2</sup> Nowadays Finland is included within core countries. In line with Fagan and Gaspar (2008) we exclude Finland from core countries since in the 1990s the Finnish economy was distorted by special factors after the collapse of the Soviet Union.

<sup>3</sup> Remaining differences in the real interest rates are due to inflation rate differences across EMU core and periphery.



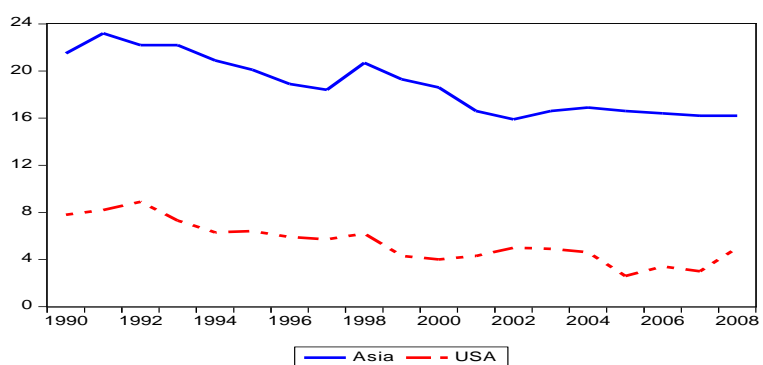
**Fig. 2** Real short-term interest rates in USA and Asia 1990-2008. Source: World Bank: World Development Indicators

Figure 2 shows a similar convergence of higher US short term real interest rates in the late 1990s towards the lower Asian<sup>4</sup> rates, following the 1990s East-Asian currency crises (= at the time of euro inception).



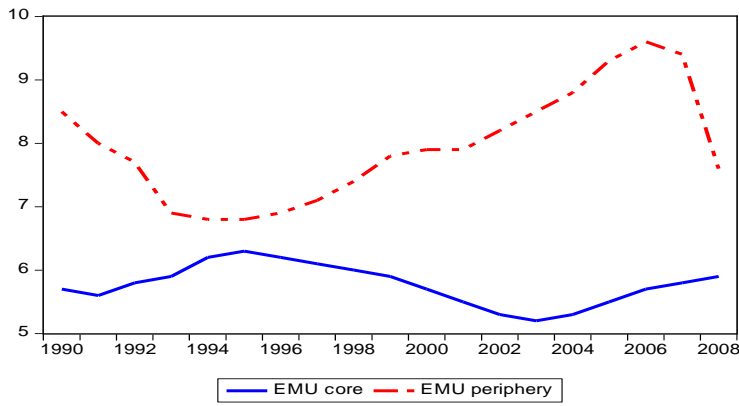
**Fig. 3** Personal saving rates in EMU core and periphery 1990-2008. Source: AMECO.

Figure 3 portrays for the pre-euro period of the 1990s on average rather similar personal saving ratios (rates) (= gross household savings as percent of gross disposable income) for EMU core and EMU periphery, and a significant decline of the saving rate in the EMU periphery after the euro launch. In contrast, Figure 4 reveals that the US personal saving rate is persistently and substantially lower than the Asian rate, both in the 1990s and 2000s.



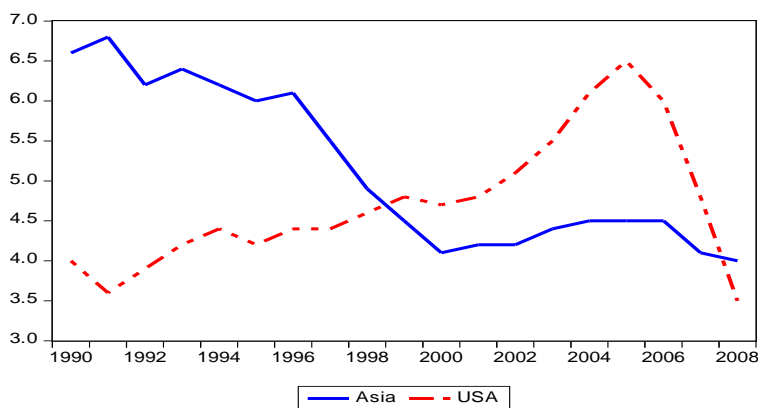
**Fig. 4** Asian and US personal saving rates 1990-2008. Source: Source: China Statistical Yearbook Database, CEIC, AMECO, Economic Statistics System, Bank of Korea, FRED.

<sup>4</sup> The Asian magnitudes result from the unweighted averages of Chinese, Indian, Japanese, Hong Kong and South Korean magnitudes. All remaining Asian macro variables are comprised by these country values.



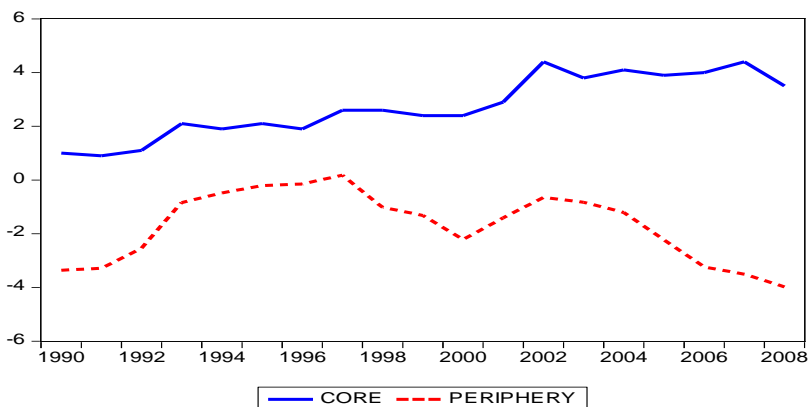
**Fig. 5** Housing investment (as percent of GDP) in EMU periphery and core 1990-2008. Source: AMECO

Figures 5 and 6 portray housing investment (as percent of GDP) in EMU periphery and core, and in Asia and the USA, respectively. While housing investment rose significantly in the EMU periphery and in the USA, it first declined and then slightly rose in the EMU core and in Asia.



**Fig. 6** Housing investment (as percent of GDP) in Asia and in USA 1990-2008. Source: BEA National Economic Accounts

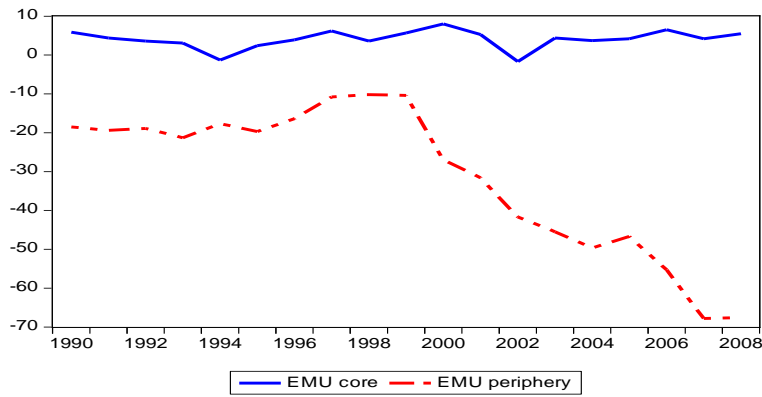
Through the sharp increase in private domestic expenditures in the periphery, and the muted response of output (Fagan and Gaspar 2008), trade balances in the periphery significantly deteriorated (see Fig. 7).



**Fig. 7:** Trade balance on goods and services (as percent of GDP) in EMU core and periphery 1990-2008. Source: World Bank Indicators.

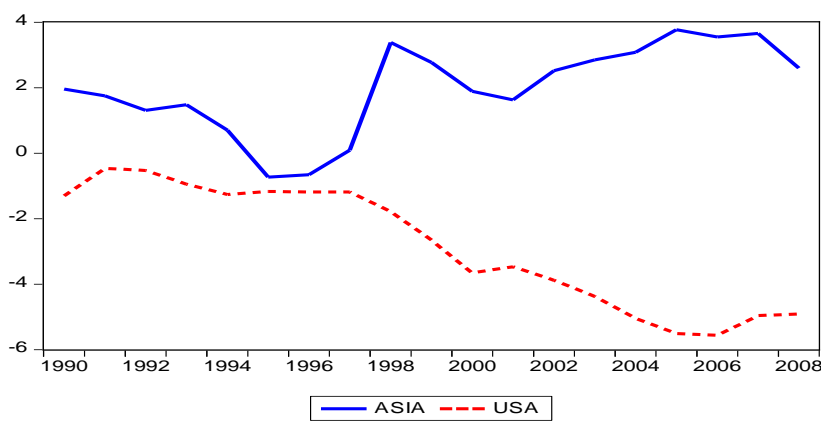
Not surprisingly, EMU periphery's trade balance deficits led to the accumulation of a significant net foreign debtor position. See Figure 8 below.



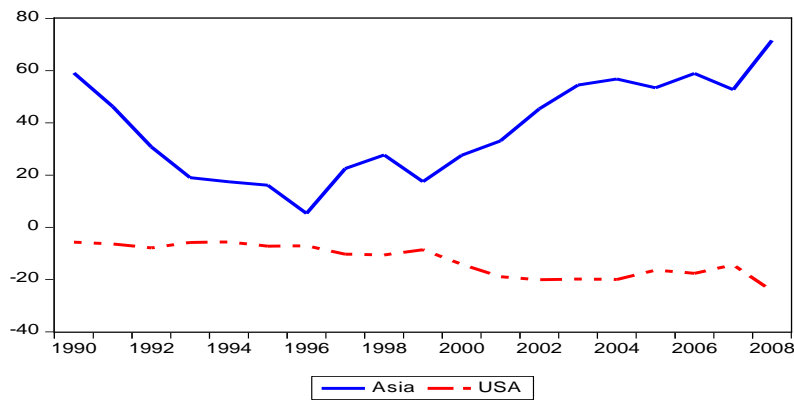


**Fig. 8:** Net foreign assets (as percent of GDP) in EMU core and periphery 1990-2008. Source: updated and extended version of a dataset constructed by Lane and Milesi-Ferretti (2007)

The differences with respect to the evolution of trade balance and net foreign asset positions in Asia and the USA are again similar to those in Europe. See figures 9 and 10.



**Fig. 9:** Trade balance on goods and services (as percent of GDP) in Asia and USA 1990-2008. Source: World Bank Indicators.



**Fig. 10** Net foreign asset positions (in percent to GDP) of Asia and USA 1990-2008. Source: updated and extended version of a dataset constructed by Lane and Milesi-Ferretti (2007)

### Model Setting and the Production Side of the Model Economy

Consider an infinite-horizon model economy consisting of three areas (“countries”) of the world economy, namely (i) the EMU, comprising two regions, named North (indexed by  $N$ ) representing EMU’s core, and South (indexed by  $S$ ), representing EMU’s periphery countries, (ii) the countries characterized by a current account surplus outside the EMU (indexed by  $A$ ) representing Asia and oil-exporting countries, and (iii) the current-account deficit countries (indexed by  $U$ ) representing mainly the USA. In each country two final goods are produced: first a tradable, general-purpose

good comprising the aggregate of thousands of non-housing goods and services and second non-tradable houses are produced. The first goods aggregate can be used for the purpose of consumption as well as for investment in real capital. The EMU specializes completely in the production of the non-housing good  $X$ , with  $X^S$  in EMU South and  $X^N$  in EMU North, in the production of houses in EMU South,  $H^S$ , and houses in EMU North,  $H^N$ , Asia in the production of good  $Y$  and of houses in Asia,  $H^A$ , and US in the production of good  $Z$  and houses in US,  $H^U$ . Perfectly competitive firms in EMU's South (North) employ in every period  $t = 1, 2, \dots$  labor services  $N_t^{X,S}$  ( $N_t^{X,N}$ ) and capital services  $K_t^{X,S}$  ( $K_t^{X,N}$ ) using the Cobb-Douglas (CD) production function  $M^{X,S} (a_t^S N_t^{X,S})^{1-\alpha^{X,S}} (K_t^{X,S})^{\alpha^{X,S}}$  ( $M^{X,N} (a_t^N N_t^{X,N})^{1-\alpha^{X,N}} (K_t^{X,N})^{\alpha^{X,N}}$ ) to produce southern (northern) EMU aggregate output  $X_t^S$  ( $X_t^N$ ) and employ in every period  $t = 1, 2, \dots$  labor services  $N_t^{H,S}$  ( $N_t^{H,N}$ ) and capital services  $K_t^{H,S}$  ( $K_t^{H,N}$ ) using the Cobb-Douglas (CD) production function  $M^{H,S} (a_t^S N_t^{H,S})^{1-\alpha^{H,S}} (K_t^{H,S})^{\alpha^{H,S}}$  ( $M^{H,N} (a_t^N N_t^{H,N})^{1-\alpha^{H,N}} (K_t^{H,N})^{\alpha^{H,N}}$ ) to produce southern (northern) EMU houses  $O_t^S$  ( $O_t^N$ ). Similarly, Asia employs in every period  $t = 1, 2, \dots$  labor services  $N_t^{Y,A}$  and capital services  $K_t^{Y,A}$  using the Cobb-Douglas (CD) production function  $M^{Y,A} (a_t^A N_t^{Y,A})^{1-\alpha^{Y,A}} (K_t^{Y,A})^{\alpha^{Y,A}}$  to produce Asian non-housing output  $Y_t$  and employs in every period  $t = 1, 2, \dots$  labor services  $N_t^{H,A}$  and capital services  $K_t^{H,A}$  using the Cobb-Douglas (CD) production function  $M^{H,A} (a_t^A N_t^{H,A})^{1-\alpha^{H,A}} (K_t^{H,A})^{\alpha^{H,A}}$  to produce southern (northern) EMU houses  $O_t^A$ . Finally, the USA employs in every period  $t = 1, 2, \dots$  labor services  $N_t^{Z,U}$  and capital services  $K_t^{Z,U}$  using the Cobb-Douglas (CD) production function  $M^{Z,U} (a_t^U N_t^{Z,U})^{1-\alpha^{Z,U}} (K_t^{Z,U})^{\alpha^{Z,U}}$  to produce US non-housing output  $Z_t$  and employs in every period  $t = 1, 2, \dots$  labor services  $N_t^{H,U}$  and capital services  $K_t^{H,U}$  using the Cobb-Douglas (CD) production function  $M^{H,U} (a_t^U N_t^{H,U})^{1-\alpha^{H,U}} (K_t^{H,U})^{\alpha^{H,U}}$  to produce US houses  $O_t^U$ . Hereby,  $M^{X,i} > 0$ ,  $M^{H,i}$ ,  $i = S, N$  denote total factor productivity in EMU's South (North) non-housing (housing) production sector,  $M^{Y,A}$  ( $M^{H,A}$ ) is total factor productivity in Asia's non-housing (housing) production sector and  $M^{Z,U}$  ( $M^{H,U}$ ) represents total factor productivity in US non-housing (housing) production sector.  $a_t^i$ ,  $i = S, N, A, U$  is the country-specific labor productivity and  $0 < \alpha^{X,S} < 1$ ,  $0 < \alpha^{X,N} < 1$ ,  $0 < \alpha^{Y,A} < 1$ ,  $0 < \alpha^{Z,U} < 1$ ,  $0 < \alpha^{H,S} < 1$ ,  $0 < \alpha^{H,N} < 1$ ,  $0 < \alpha^{H,A} < 1$ ,  $0 < \alpha^{H,U} < 1$  denote the capital production shares in the production function for the general purpose-good and housing output, respectively, in EMU South, EMU North, Asia and in the USA with  $\alpha^{Z,U} \approx \alpha^{X,N} < \alpha^{X,S} < \alpha^{Y,A}$  and  $\alpha^{H,S} > \alpha^{X,S}$ ,  $\alpha^{H,N} > \alpha^{X,N}$ ,  $\alpha^{H,A} > \alpha^{Y,A}$ ,  $\alpha^{H,U} > \alpha^{Z,U}$ .

One-period profit maximization by firms in EMU's South implies the following FOCs:

$$(1 - \alpha^{X,S}) M^{X,S} a_t^S (K_t^{X,S} / a_t^S N_t^{X,S})^{\alpha^{X,S}} = w_t^S = p_t^S (1 - \alpha^{H,S}) M^{H,S} a_t^S (K_t^{H,S} / a_t^S N_t^{H,S})^{\alpha^{H,S}}, \quad (1a)$$

$$\alpha^{X,S} M^{X,S} (K_t^{X,S} / a_t^S N_t^{X,S})^{\alpha^{X,S}-1} = q_t^S = p_t^S \alpha^{H,S} M^{H,S} (K_t^{H,S} / a_t^S N_t^{H,S})^{\alpha^{H,S}-1}, \quad (2a)$$

whereby  $w_t^S$  denotes southern EMU real wage rate,  $q_t^S$  denotes southern EMU real unit capital user costs and  $p_t^S$  signifies the relative (to non-housing output) house price in southern EMU.

Similarly, one-period profit maximization by firms in EMU's North implies the following FOCs:

$$(1 - \alpha^{X,N})M^{X,N}a_t^N (K_t^{X,N}/a_t^N N_t^{X,N})^{\alpha^{X,N}} = w_t^N = p_t^N (1 - \alpha^{H,N})M^{H,N}a_t^N (K_t^{H,N}/a_t^N N_t^{H,N})^{\alpha^{H,N}}, \quad (1b)$$

$$\alpha^{X,N}M^{X,N}(K_t^{X,N}/a_t^N N_t^{X,N})^{\alpha^{X,N}-1} = q_t^N = p_t^N \alpha^{H,N}M^{H,N}(K_t^{H,N}/a_t^N N_t^{H,N})^{\alpha^{H,N}-1}, \quad (2b)$$

whereby  $w_t^N$  denotes northern EMU real wage rate,  $q_t^N$  denotes northern EMU real unit capital user costs and  $p_t^N$  signifies the relative (to non-housing output) house price in northern EMU.

Analogously, one-period profit maximization by Asian firms implies the following FOCs:

$$(1 - \alpha^{Y,A})M^{Y,A}a_t^A (K_t^{Y,A}/a_t^A N_t^{Y,A})^{\alpha^{Y,A}} = w_t^A = p_t^A (1 - \alpha^{H,A})M^{H,A}a_t^A (K_t^{H,A}/a_t^A N_t^{H,A})^{\alpha^{H,A}}, \quad (1c)$$

$$\alpha^{Y,A}M^{Y,A}(K_t^{Y,A}/a_t^A N_t^{Y,A})^{\alpha^{Y,A}-1} = q_t^A = p_t^A \alpha^{H,A}M^{H,A}(K_t^{H,A}/a_t^A N_t^{H,A})^{\alpha^{H,A}-1}, \quad (2c)$$

whereby  $w_t^A$  denotes Asian real wage rate,  $q_t^A$  denotes Asian real unit capital user costs and  $p_t^A$  signifies the relative (to non-housing output) house price in Asia.

Finally, one-period profit maximization by US firms implies the following FOCs:

$$(1 - \alpha^{Z,U})M^{Z,U}a_t^U (K_t^{Z,U}/a_t^U N_t^{Z,U})^{\alpha^{Z,U}} = w_t^U = p_t^U (1 - \alpha^{H,U})M^{H,U}a_t^U (K_t^{H,U}/a_t^U N_t^{H,U})^{\alpha^{H,U}}, \quad (1d)$$

$$\alpha^{Z,U}M^{Z,U}(K_t^{Z,U}/a_t^U N_t^{Z,U})^{\alpha^{Z,U}-1} = q_t^U = p_t^U \alpha^{H,U}M^{H,U}(K_t^{H,U}/a_t^U N_t^{H,U})^{\alpha^{H,U}-1}, \quad (2d)$$

whereby  $w_t^U$  denotes US real wage rate,  $q_t^U$  denotes US real unit capital user costs and  $p_t^U$  signifies the relative (to non-housing output) house price in US.

Following Japelli and Papano (1994), three generations of homogeneous individuals overlap in each period  $t$ . At date  $t$ , a new generation of size  $L_t^{i,1}$  enters the economy of country (region)  $i = S, N, A, U$ . For simplicity we assume that the population growth factors of all countries (regions) are identical and are constant over time, equal to  $G^L$ . On the other hand, empirical (productivity) GDP growth rates for 1990s and 2000s differ both across intra-EMU regions and between Asia and USA, the former lesser than the latter. Thus, we assume in line with Fagan and Gaspar (2008) that the southern time-stationary EMU growth factor of labor productivity  $G^S$  is a little bit larger than the corresponding northern time-stationary productivity growth factor  $G^N$ . Moreover, we assume that the substantially larger Asian GDP growth rates in the data are also due to a time-dependent higher productivity growth factor  $G_t^A$  than in USA and in EMU.

Each generation lives for three periods, borrowing and working as young-aged in the first period, enjoying housing services, repaying the incurred debt and accumulating wealth during the second period when middle-aged, and consuming the returns on her savings in the third when old. The choice variables of each generation, when young, are denoted by superscript 1, when middle-aged, they are denoted by superscript 2, and when old, they are depicted by superscript 3. The proceeds from issuing debenture bonds in the first period are used to finance both young-age goods consumption expenditures and the outlays for house purchases needed to enjoy housing

services in middle age. Borrowing is constrained by a fraction of the discounted resale value of the undepreciated house. For each member of the young-aged generation, the supply of labor to firms is wage-inelastic since households attribute no value to leisure.

In order to describe the optimization problems of households more specifically the institutional framework regarding international transactions across the three countries and across EMU core and periphery is now addressed. Regarding the three countries, we assume that each country has its own currency and that before the inception of the EMU, the southern and northern EMU member countries had their own currency, too. To mimic the period before the introduction of the common currency in 1999 we follow Gourinchas and Jeanne (2006) as well as Fagan and Gaspar (2008), and assume that before 1999, EMU's South and North were financially autarkic. In contrast to the de-facto financial relationships between subsequent EMU countries, Asia and the USA which existed before euro inception, we also assume financial autarky for Asia and the USA in the pre-euro period. In difference to financial autarky, we do, however, allow for trade relations between later EMU, Asia and US during the pre-euro period, albeit on a moderate and balanced scale, thus mimicking the fact of mainly Japanese trade linkages vis-à-vis later EMU countries and the US. China and India did not play any important role in international trade during the pre-euro period.

Complete nominal, and to a lesser extent, real interest convergence across EMU's South and North after the euro launch signifies financial integration across EMU's South and North. This stylized fact is portrayed in our intertemporal equilibrium model in line with Fagan and Gaspar (2008) as an equality of real interest rates of southern and northern EMU countries along the intertemporal equilibrium path. While by no means as complete as that within EMU, there is also some real interest convergence or financial integration across Asia and USA in the early 2000s. We take this stylized fact as support for our rather strong modeling assumption that after the euro launch an uncovered parity condition, in terms of real interest rates, holds across both Asia and the USA. In line with the empirical fact (Chen et al. 2013) that investors from outside EMU invested their wealth in northern EMU financial assets we also assume an uncovered real interest parity condition between USA and EMU. In other words: after euro inception financial integration prevails worldwide but not as strictly as within the EMU.

In order to work out the consequences of intra-EMU, Asian and US financial integration and the trade developments of EMU vis-à-vis non-EMU countries as clearly as possible, the optimization problems of households and firms as well as the market clearing conditions are first described for financial autarky and then for intra-EMU, Asian and US financial integration.

## **Pre-Euro and Asian-US Financial Autarky**

In order to facilitate the modeling of the pre-euro situation as financial autarky, we first recall that before euro launch southern EMU real interest rates were sizeable larger than the corresponding northern rates (see Figure 1 above). Second, in the 1990s EMU South (with the exception of Portugal) did not run large current account deficits (as ratio of GDP). Hence, our modeling assumes that in the pre-euro period both the trade balance and the net foreign asset position of EMU South and North were zero. In contrast, in the 1990s Asia (including oil exporters) ran current account surpluses (as percent of GDP) roughly equivalent in size to the current account deficit of the USA (Engler 2009, p. 2). However, since at this time the US net foreign asset position was only moderately negative and China and other emerging Asian countries did not contribute much to the imbalance, we assume that the USA and Asia were financially autarkic, just as the later EMU South and North were. Third, in contrast to the tremendous post-crisis accumulation of public debt, particularly in southern EMU, in Japan and in the USA, in the 1990s and 2000s (up to 2008), the average debt to GDP ratios for EMU periphery, EMU core and the USA remained constant over time or even receded slightly. In Asia (with the exception of Japan) public debt to GDP ratios decreased, and remained far below the EMU and US ratios. We also assume that the (un-weighted) average of government debt to GDP ratios in Asia (including Japan) remains constant over time. Additionally, as Figure 3 above shows, after 1993 the personal saving rate in EMU South was lower than in EMU North. From Figure 4 we know that Asia's personal saving rate is significantly higher than the corresponding northern EMU rate, while the US personal saving rate is slightly below the southern EMU personal saving rate (see Figures 3 and 4).

In line with Chen et al. (2013) we accept the stylized fact of merchandise trade between EMU's North and South being relatively modest after euro launch, and we assume that there is no intra-temporal trade between EMU South and North in the model economy before and after euro launch. For the sake of simplicity we assume that the same composite, non-housing good is produced in North and South. Thus, while younger households in EMU South (North) cannot choose between consumption of the domestic and of the northern (southern) non-housing good they can buy Asian and US goods in addition to the domestically produced good even before the euro launch.

Against this empirical background of stylized facts and simplifying modeling assumptions the intertemporal utility maximization problem in later EMU's South before euro inception (= financial autarky) reads as follows:

$$\begin{aligned}
\max \rightarrow & \frac{\left\{ \left[ \left( \zeta^x \right)^{\frac{1}{\eta}} \left( x_t^{S,1} \right)^{\frac{\eta-1}{\eta}} + \left( \zeta^y \right)^{\frac{1}{\eta}} \left( y_t^{S,1} \right)^{\frac{\eta-1}{\eta}} + \left( \zeta^z \right)^{\frac{1}{\eta}} \left( z_t^{S,1} \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \right\}^{\frac{\sigma-1}{\sigma}} - 1}{\frac{\sigma-1}{\sigma}} \\
& + \beta^S \frac{\left\{ \left[ \left( \zeta^x \right)^{\frac{1}{\eta}} \left( x_{t+1}^{S,2} \right)^{\frac{\eta-1}{\eta}} + \left( \zeta^y \right)^{\frac{1}{\eta}} \left( y_{t+1}^{S,2} \right)^{\frac{\eta-1}{\eta}} + \left( \zeta^z \right)^{\frac{1}{\eta}} \left( z_{t+1}^{S,2} \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \right\}^{\frac{\sigma-1}{\sigma}} - 1 + \left( H_{t+1}^{S,1} / L_t^{S,1} \right)^{\frac{\sigma-1}{\sigma}} - 1}{\frac{\sigma-1}{\sigma}} \\
& + \left( \beta^S \right)^2 \frac{\left\{ \left[ \left( \zeta^x \right)^{\frac{1}{\eta}} \left( x_{t+2}^{S,3} \right)^{\frac{\eta-1}{\eta}} + \left( \zeta^y \right)^{\frac{1}{\eta}} \left( y_{t+2}^{S,3} \right)^{\frac{\eta-1}{\eta}} + \left( \zeta^z \right)^{\frac{1}{\eta}} \left( z_{t+2}^{S,3} \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \right\}^{\frac{\sigma-1}{\sigma}} - 1}{\frac{\sigma-1}{\sigma}}
\end{aligned}$$

s. t.:

$$\begin{aligned}
(i) \quad & x_t^{S,1} + (1/e_t^A) y_t^{S,1} + (1/e_t^U) z_t^{S,1} + p_t^S H_{t+1}^{S,1} / L_t^{S,1} = b_{t+1}^{S,1} + w_t^S (1 - \tau_t^S), \quad b_{t+1}^{S,1} \leq \frac{\theta^S (1 - \delta^{H,S}) p_{t+1}^S H_{t+1}^{S,1}}{(1 + i_{t+1}^S) L_t^{S,1}}, \\
(ii) \quad & x_{t+1}^{S,2} + \frac{y_{t+1}^{S,2}}{e_{t+1}^A} + \frac{z_{t+1}^{S,2}}{e_{t+1}^U} + s_{t+1}^{S,2} = (1 - \delta^{H,S}) p_{t+1}^S H_{t+1}^{S,1} / L_t^{S,1} - (1 + i_{t+1}^S) b_{t+1}^{S,1}, \\
& \text{with } s_{t+1}^{S,2} = [(I_{t+1}^S + (1 - \delta^S) K_{t+1}^{S,S}) / L_{t+1}^{S,2}] + (B_{t+2}^{S,S} / L_{t+1}^{S,2}) + b_{t+2}^{SS,2}, \\
(iii) \quad & x_{t+2}^{S,3} + \frac{y_{t+2}^{S,3}}{e_{t+2}^A} + \frac{z_{t+2}^{S,3}}{e_{t+2}^U} = \frac{(q_{t+2}^S + 1 - \delta^S) [I_{t+1}^S + (1 - \delta^S) K_{t+1}^{S,S}]}{L_{t+1}^{S,2}} + \frac{(1 + i_{t+2}^S) B_{t+2}^{S,S}}{L_{t+1}^{S,2}} + (1 + i_{t+2}^S) b_{t+2}^{SS,2}.
\end{aligned}$$

Here  $0 < \beta^S \leq 1$  denotes the time discount factor of (later) EMU's southern younger generation,  $\sigma \leq 1$  is the elasticity of intertemporal substitution in consumption,  $0 < \eta$  denotes the elasticity of intra-temporal substitution in consumption of the domestic and foreign goods,  $\zeta^k$ ,  $k = x, y, z$  with  $\zeta^x + \zeta^y + \zeta^z = 1$  represents the utility elasticity of the consumption of good  $k$ ,  $x_t^{S,1}$  is the per capita consumption of the commodity produced in EMU's South acquired at unit relative price,  $y_t^{S,1}$  is South's per capita consumption of the Asian good bought at the relative price of  $1/e_t^A$ , and  $z_t^{S,1}$  is southern EMU per capita consumption of the US good acquired at the relative price of  $1/e_t^U$ .  $e_t^A$  denotes the units of the Asian good per unit of EMU good (= EMU terms of trade vis-à-vis Asia), while  $e_t^U$  portrays the units of the US good per unit of EMU good (= EMU terms of trade vis-à-vis USA). Budget constraint (i) demands that first-period consumption expenditures and the expenditures for houses,  $p_t^S H_{t+1}^{S,1} / L_t^{S,1}$ , with  $p_t^S$  denoting the relative price of houses in South and  $H_{t+1}^{S,1} / L_t^{S,1}$  featuring how much housing is wanted by southern young-aged household, are covered by the revenues from private bond selling,  $b_{t+1}^{S,1}$ , and by the proceeds from wages,  $w_t^S$  net of the flat wage tax rate  $\tau_t^S$  in South. Southern younger household's borrowing in the first period is constrained by a fraction  $\theta^S$  of the present value of future market

value of undepreciated houses (see Arce and López-Salido 2011, p. 215, footnote 1). Second-period consumption expenditures and second-period savings  $s_{t+1}^{S,2}$  are to be equal to the revenues from selling undepreciated houses,  $p_{t+1}^S(1-\delta^{H,S})h_{t+1}^{S,1}$  with exogenously fixed depreciation rate  $\delta^{H,S}$  for southern housing minus private credit repayments (inclusive of interest payments),  $(1+i_{t+1}^S)b_{t+1}^{S,1}$  (see budget constraint (ii)). Southern middle-age savings  $s_{t+1}^{S,2}$  are used to buy southern real capital per capita,  $K_{t+2}^{S,S}/L_{t+1}^{S,2}$ , southern government bonds per capita  $B_{t+2}^{S,S}/L_{t+1}^{S,2}$ , and southern private bonds  $b_{t+2}^{SS,2}$ . Budget constraint (iii) demands that old-age consumption expenditures are financed by the revenues from renting real capital services to southern firms,  $q_{t+2}^S K_{t+2}^{S,S}/L_{t+1}^{S,2}$ , and from repaid public and private bonds (inclusive of interest)  $(1+i_{t+2}^S)B_{t+2}^{S,S}/L_{t+1}^{S,2} + (1+i_{t+2}^S)b_{t+2}^{SS,2}$ . In line with pre-financial crisis experience in EMU South the interest rates on public and private bonds are assumed to be equal.

In order to solve step by step the intertemporal utility maximization problem of southern young household, we define southern real consumption per capita in period  $t + \tau$ ,  $\tau = 0, 1, 2$  as  $c_{t+\tau}^{S,\tau+1} \equiv [(\zeta^x)^{1/\eta}(x_{t+\tau}^{S,\tau+1})^{(\eta-1)/\eta} + (\zeta^y)^{1/\eta}(y_{t+\tau}^{S,\tau+1})^{(\eta-1)/\eta} + (\zeta^z)^{1/\eta}(z_{t+\tau}^{S,\tau+1})^{(\eta-1)/\eta}]^{\eta/(\eta-1)}$  and calculate the expenditure-minimizing price indices of these consumption bundles denoted by  $\pi_{t+\tau}^S, \tau = 0, 1, 2$  as follows:  $\pi_{t+\tau}^S = [\zeta^x + \zeta^y(e_{t+\tau}^A)^{\eta-1} + \zeta^z(e_{t+\tau}^U)^{\eta-1}]^{1/(1-\eta)}, \tau = 0, 1, 2$ . Using these definitions we find the following structure of the optimal solution for the southern younger household:

$$s_t^{S,1} = -b_{t+1}^{S,1} = -[\theta^S(1-\delta^{H,S})p_{t+1}^S H_{t+1}^{S,1}] / L_t^{S,1}(1+i_{t+1}^S), \quad s_{t+1}^{S,2} = (\beta^S)^\sigma [\pi_{t+2}^S / \pi_{t+1}^S(1+i_{t+2}^S)]^{1-\sigma} \pi_{t+1}^S c_{t+1}^{S,2},$$

whereby  $\pi_t^S c_t^{S,1}, \pi_{t+1}^S c_{t+1}^{S,2}, \pi_{t+2}^S c_{t+2}^{S,3}$  and  $H_{t+1}^{S,1} / L_t^{S,1}$  result from solving equations (3.a)-(3.d):

$$\pi_t^S c_t^{S,1} + [p_t^S(1+i_{t+1}^S) - \theta^S(1-\delta^{H,S})p_{t+1}^S] / (1+i_{t+1}^S) = w_t^S(1-\tau_t^S), \quad (3.a)$$

$$\{1 + (\beta^S)^\sigma [\pi_{t+2}^S / \pi_{t+1}^S(1+i_{t+2}^S)]^{1-\sigma}\} \pi_{t+1}^S c_{t+1}^{S,2} = (1-\delta^{H,S})(1-\theta^S)p_{t+1}^S H_{t+1}^{S,1} / L_t^{S,1}, \quad (3.b)$$

$$\beta^S / (H_{t+1}^{S,1} / L_t^{S,1})^{1/\sigma} = [p_t^S(1+i_{t+1}^S) - \theta^S(1-\delta^{H,S})p_{t+1}^S] / [(1+i_{t+1}^S)\pi_t^S(c_t^{S,1})^{1/\sigma}] - [\beta^S(1-\delta^{H,S})(1-\theta^S)p_{t+1}^S] / [\pi_{t+1}^S(c_{t+1}^{S,2})^{1/\sigma}], \quad (3.c)$$

$$\pi_{t+2}^S c_{t+2}^{S,3} / (1+i_{t+2}^S) = (\beta^S)^\sigma [\pi_{t+2}^S / \pi_{t+1}^S(1+i_{t+2}^S)]^{1-\sigma} \pi_{t+1}^S c_{t+1}^{S,2}. \quad (3.d)$$

$$x_{t+\tau}^{S,\tau+1} = \zeta^x (\pi_{t+\tau}^S)^{\eta-1} \pi_{t+\tau}^S c_{t+\tau}^{S,\tau+1}, \quad y_{t+\tau}^{S,\tau+1} = \zeta^y (\pi_{t+\tau}^S)^{\eta-1} (e_{t+\tau}^A)^\eta \pi_{t+\tau}^S c_{t+\tau}^{S,\tau+1}, \quad (4)$$

$$z_{t+\tau}^{S,\tau+1} = \zeta^z (\pi_{t+\tau}^S)^{\eta-1} (e_{t+\tau}^U)^\eta \pi_{t+\tau}^S c_{t+\tau}^{S,\tau+1}, \quad \tau = 0, 1, 2.$$

The intertemporal optimization calculus of the northern EMU younger household reads as follows:

$$\begin{aligned} \max \rightarrow & \frac{\left\{ \left[ (\zeta^x)^{\frac{1}{\eta}} (x_t^{N,1})^{\frac{\eta-1}{\eta}} + (\zeta^y)^{\frac{1}{\eta}} (y_t^{N,1})^{\frac{\eta-1}{\eta}} + (\zeta^z)^{\frac{1}{\eta}} (z_t^{N,1})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \right\}^{\frac{\sigma-1}{\sigma}} - 1}{\frac{\sigma-1}{\sigma}} \\ & + \beta^N \frac{\left\{ \left[ (\zeta^x)^{\frac{1}{\eta}} (x_{t+1}^{N,2})^{\frac{\eta-1}{\eta}} + (\zeta^y)^{\frac{1}{\eta}} (y_{t+1}^{N,2})^{\frac{\eta-1}{\eta}} + (\zeta^z)^{\frac{1}{\eta}} (z_{t+1}^{N,2})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \right\}^{\frac{\sigma-1}{\sigma}} - 1 + (H_{t+1}^{N,1} / L_t^{N,1})^{\frac{\sigma-1}{\sigma}} - 1}{\frac{\sigma-1}{\sigma}} \end{aligned}$$

$$+(\beta^N)^2 \frac{\left\{ \left[ (\zeta^x)^{\frac{1}{\eta}} (x_{t+2}^{N,3})^{\frac{\eta-1}{\eta}} + (\zeta^y)^{\frac{1}{\eta}} (y_{t+2}^{N,3})^{\frac{\eta-1}{\eta}} + (\zeta^z)^{\frac{1}{\eta}} (z_{t+2}^{N,3})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \right\}^{\frac{\sigma-1}{\sigma}}}{\frac{\sigma-1}{\sigma}}$$

s.t.:

$$(i) \quad x_t^{N,1} + (1/e_t^A) y_t^{N,1} + (1/e_t^U) z_t^{N,1} + p_t^N H_{t+1}^{N,1} / L_t^{N,1} = b_{t+1}^{N,1} + w_t^N (1 - \tau_t^N), \quad b_{t+1}^{N,1} \leq \frac{\theta^N (1 - \delta^{H,N}) p_{t+1}^N H_{t+1}^{N,1}}{(1 + i_{t+1}^N) L_t^{N,1}},$$

$$(ii) \quad x_{t+1}^{N,2} + \frac{y_{t+1}^{N,2}}{e_{t+1}^A} + \frac{z_{t+1}^{N,2}}{e_{t+1}^U} + s_{t+1}^{N,2} = (1 - \delta^{H,N}) p_{t+1}^N H_{t+1}^{N,1} / L_t^{N,1} - (1 + i_{t+1}^N) b_{t+1}^{N,1},$$

$$\text{with } s_{t+1}^{N,2} = [(I_{t+1}^N + (1 - \delta^N) K_{t+1}^{N,N}) / L_{t+1}^{N,2}] + (B_{t+2}^{N,N} / L_{t+1}^{N,2}) + b_{t+2}^{NN,2},$$

$$(iii) \quad x_{t+2}^{N,3} + \frac{y_{t+2}^{N,3}}{e_{t+2}^A} + \frac{z_{t+2}^{N,3}}{e_{t+2}^U} = \frac{(q_{t+2}^N + 1 - \delta^N) [I_{t+1}^N + (1 - \delta^N) K_{t+1}^{N,N}]}{L_{t+1}^{N,2}} + \frac{(1 + i_{t+2}^N) B_{t+2}^{N,N}}{L_{t+1}^{N,2}} + (1 + i_{t+2}^N) b_{t+2}^{NN,2}.$$

The structure of the solution of the northern intertemporal optimization problem is exactly equal to that of southern young household presented above save to substitute  $S$  for  $N$ .

The same holds true mutatis mutandis for the intertemporal utility function of the typical Asian younger household. Since, however, the budget constraints of the Asian younger household differ from the budget constraints of EMU households it is in order to state them explicitly as follows:

$$(i) \quad e_t^A x_t^{A,1} + y_t^{A,1} + (e_t^A / e_t^U) z_t^{A,1} + p_t^A \frac{H_{t+1}^{A,1}}{L_t^{A,1}} = w_t^A (1 - \tau_t^A) + b_{t+1}^{A,1}, \quad b_{t+1}^{A,1} \leq \theta^A \frac{p_{t+1}^A (1 - \delta^{H,A}) H_{t+1}^{A,1}}{(1 + i_{t+1}^A) L_t^{A,1}},$$

$$(ii) \quad e_{t+1}^A x_{t+1}^{A,2} + y_{t+1}^{A,2} + (e_{t+1}^A / e_{t+1}^U) z_{t+1}^{A,2} + s_{t+1}^{A,2} = p_{t+1}^A (1 - \delta^{H,A}) H_{t+1}^{A,1} / L_t^{A,1} - (1 + i_{t+1}^A) b_{t+1}^{A,1},$$

$$\text{with } s_{t+1}^{A,2} = [(I_{t+1}^A + (1 - \delta^A) K_{t+1}^{A,A}) / L_{t+1}^{A,2}] + B_{t+2}^{A,A} / L_{t+1}^{A,2} + b_{t+2}^{AA,2},$$

$$(iii) \quad e_{t+2}^A x_{t+2}^{A,3} + y_{t+2}^{A,3} + (e_{t+2}^A / e_{t+2}^U) z_{t+2}^{A,3} = (q_{t+2}^A + 1 - \delta^A) [(I_{t+1}^A + (1 - \delta^A) K_{t+1}^{A,A}) / L_{t+1}^{A,2}]$$

$$+ (1 + i_{t+2}^A) (B_{t+2}^{A,A} / L_{t+1}^{A,2}) + (1 + i_{t+2}^A) b_{t+2}^{AA,2}.$$

Here,  $x_t^{A,1}$  stands for the purchases (= consumption) of later EMU goods by the Asian young household at the relative price of  $e_t^A$ , while the purchase of the US product by the Asian young household occurs at the relative price  $e_t^A / e_t^U$ , i.e. units of the Asian product per unit of the US good. All other variables may be interpreted similarly to those in EMU South's young household optimization problem.

Analogously to above we define Asian real consumption per capita in period  $t + \tau$ ,  $\tau = 0, 1, 2$  as  $c_{t+\tau}^{A,\tau+1} \equiv [(\zeta^x)^{1/\eta} (x_{t+\tau}^{A,\tau+1})^{(\eta-1)/\eta} + (\zeta^y)^{1/\eta} (y_{t+\tau}^{A,\tau+1})^{(\eta-1)/\eta} + (\zeta^z)^{1/\eta} (z_{t+\tau}^{A,\tau+1})^{(\eta-1)/\eta}]^{\eta/(\eta-1)}$  and calculate the expenditure-minimizing price indices of these consumption bundles denoted by  $\pi_{t+\tau}^A$ ,  $\tau = 0, 1, 2$  as follows:  $\pi_{t+\tau}^A = [\zeta^x (e_{t+\tau}^A)^{1-\eta} + \zeta^y + \zeta^z (e_{t+\tau}^A / e_{t+\tau}^U)^{1-\eta}]^{1/(1-\eta)}$ ,  $\tau = 0, 1, 2$ . Using these definitions we find the following structure of the optimal solution for the Asian younger household:



$$s_t^{A,1} = -b_{t+1}^{A,1} = -[\theta^A(1 - \delta^{H,A})p_{t+1}^A H_{t+1}^{A,1}] / (1 + i_{t+1}^A) L_t^{A,1}, \quad s_{t+1}^{A,2} = (\beta^A)^\sigma [\pi_{t+2}^A / \pi_{t+1}^A (1 + i_{t+2}^A)]^{1-\sigma} \pi_{t+1}^A c_{t+1}^{A,2},$$

whereby

$$\begin{aligned} \pi_t^A c_t^{A,1} + [p_t^A(1 + i_{t+1}^A) - \theta^A(1 - \delta^{H,A})p_{t+1}^A] / (1 + i_{t+1}^A) &= w_t^A(1 - \tau_t^A), \\ \{1 + (\beta^A)^\sigma [\pi_{t+2}^A / \pi_{t+1}^A (1 + i_{t+2}^A)]^{1-\sigma}\} \pi_{t+1}^A c_{t+1}^{A,2} &= (1 - \delta^{H,A})(1 - \theta^A)p_{t+1}^A H_{t+1}^{A,1} / L_t^{A,1}, \\ \beta^A / (H_{t+1}^{A,1} / L_t^{A,1})^{1/\sigma} &= [p_t^A(1 + i_{t+1}^A) - \theta^A(1 - \delta^{H,A})p_{t+1}^A] / [(1 + i_{t+1}^A)\pi_t^A (c_t^{A,1})^{1/\sigma}] \\ &\quad - [\beta^A(1 - \delta^{H,A})(1 - \theta^A)p_{t+1}^A] / [\pi_{t+1}^A (c_{t+1}^{A,2})^{1/\sigma}], \\ \pi_{t+2}^A c_{t+2}^{A,3} / (1 + i_{t+2}^A) &= (\beta^A)^\sigma [\pi_{t+2}^A / \pi_{t+1}^A (1 + i_{t+2}^A)]^{1-\sigma} \pi_{t+1}^A c_{t+1}^{A,2}. \end{aligned} \quad (5)$$

$$\begin{aligned} x_{t+\tau}^{A,\tau+1} &= \zeta^x (e_{t+\tau}^A)^{-\eta} (\pi_{t+\tau}^A)^{\eta-1} \pi_{t+\tau}^A c_{t+\tau}^{A,\tau+1}, \quad y_{t+\tau}^{A,\tau+1} = \zeta^y (\pi_{t+\tau}^A)^{\eta-1} \pi_{t+\tau}^A c_{t+\tau}^{A,\tau+1}, \\ z_{t+\tau}^{A,\tau+1} &= \zeta^z (e_{t+\tau}^A / e_{t+\tau}^U)^{-\eta} (\pi_{t+\tau}^A)^{\eta-1} \pi_{t+\tau}^A c_{t+\tau}^{A,\tau+1}, \quad \tau = 0, 1, 2. \end{aligned} \quad (6)$$

Finally, the typical US younger household faces the following budget constraints:

$$\begin{aligned} (i) \quad e_t^U x_t^{U,1} + (e_t^U / e_t^A) y_t^{U,1} + z_t^{U,1} + p_t^U \frac{H_{t+1}^{U,1}}{L_t^{U,1}} &= w_t^U (1 - \tau_t^U) + b_{t+1}^{U,1}, \quad b_{t+1}^{U,1} = \theta^U \frac{p_{t+1}^U (1 - \delta^{H,U}) H_{t+1}^{U,1}}{(1 + i_{t+1}^U) L_t^{U,1}}, \\ (ii) \quad e_{t+1}^U x_{t+1}^{U,2} + (e_{t+1}^U / e_{t+1}^A) y_{t+1}^{U,2} + z_{t+1}^{U,2} + s_{t+1}^{U,2} &= p_{t+1}^U (1 - \delta^{H,U}) H_{t+1}^{U,1} / L_t^{U,1} - (1 + i_{t+1}^U) b_{t+1}^{U,1}, \\ &\quad \text{with } s_{t+1}^{U,2} = [(I_{t+1}^U + (1 - \delta^U) K_{t+1}^{U,U}) / L_{t+1}^{U,2}] + B_{t+2}^{U,U} / L_{t+1}^{U,2} + b_{t+2}^{UU,2}, \\ (iii) \quad e_{t+2}^U x_{t+2}^{U,3} + (e_{t+2}^U / e_{t+2}^A) y_{t+2}^{U,3} + z_{t+2}^{U,3} &= (q_{t+2}^U + 1 - \delta^U) [(I_{t+1}^U + (1 - \delta^U) K_{t+1}^{U,U}) / L_{t+1}^{U,2}] \\ &\quad + (1 + i_{t+2}^U) B_{t+2}^{U,U} / L_{t+1}^{U,2} + (1 + i_{t+2}^U) b_{t+2}^{UU,2}. \end{aligned}$$

Again, we define US real consumption per capita in period  $t + \tau$ ,  $\tau = 0, 1, 2$  as  $c_{t+\tau}^{U,\tau+1} \equiv [(\zeta^x)^{1/\eta} (x_{t+\tau}^{U,\tau+1})^{(\eta-1)/\eta} + (\zeta^y)^{1/\eta} (y_{t+\tau}^{U,\tau+1})^{(\eta-1)/\eta} + (\zeta^z)^{1/\eta} (z_{t+\tau}^{U,\tau+1})^{(\eta-1)/\eta}]^{\eta/(\eta-1)}$  and calculate the expenditure-minimizing price indices of these consumption bundles denoted by  $\pi_{t+\tau}^U$ ,  $\tau = 0, 1, 2$  as follows:  $\pi_{t+\tau}^U = [\zeta^x (e_{t+\tau}^U)^{1-\eta} + \zeta^y (e_{t+\tau}^U / e_{t+\tau}^A)^{1-\eta} + \zeta^z]^{1/(1-\eta)}$ ,  $\tau = 0, 1, 2$ . Using these definitions we find the following structure of the optimal solution for the US younger household:

$$s_t^{U,1} = -b_{t+1}^{U,1} = -[\theta^U(1 - \delta^{H,U})p_{t+1}^U H_{t+1}^{U,1}] / (1 + i_{t+1}^U) L_t^{U,1}, \quad s_{t+1}^{U,2} = (\beta^U)^\sigma [\pi_{t+2}^U / \pi_{t+1}^U (1 + i_{t+2}^U)]^{1-\sigma} \pi_{t+1}^U c_{t+1}^{U,2},$$

whereby

$$\begin{aligned} \pi_t^U c_t^{U,1} + [p_t^U(1 + i_{t+1}^U) - \theta^U(1 - \delta^{H,U})p_{t+1}^U] / (1 + i_{t+1}^U) &= w_t^U(1 - \tau_t^U), \\ \{1 + (\beta^U)^\sigma [\pi_{t+2}^U / \pi_{t+1}^U (1 + i_{t+2}^U)]^{1-\sigma}\} \pi_{t+1}^U c_{t+1}^{U,2} &= (1 - \delta^{H,U})(1 - \theta^U)p_{t+1}^U H_{t+1}^{U,1} / L_t^{U,1}, \\ \beta^U / (H_{t+1}^{U,1} / L_t^{U,1})^{1/\sigma} &= [p_t^U(1 + i_{t+1}^U) - \theta^U(1 - \delta^{H,U})p_{t+1}^U] / [(1 + i_{t+1}^U)\pi_t^U (c_t^{U,1})^{1/\sigma}] \\ &\quad - [\beta^U(1 - \delta^{H,U})(1 - \theta^U)p_{t+1}^U] / [\pi_{t+1}^U (c_{t+1}^{U,2})^{1/\sigma}], \\ \pi_{t+2}^U c_{t+2}^{U,3} / (1 + i_{t+2}^U) &= (\beta^U)^\sigma [\pi_{t+2}^U / \pi_{t+1}^U (1 + i_{t+2}^U)]^{1-\sigma} \pi_{t+1}^U c_{t+1}^{U,2}. \end{aligned} \quad (7)$$

$$\begin{aligned} x_{t+\tau}^{U,\tau+1} &= \zeta^x (e_{t+\tau}^U)^{-\eta} (\pi_{t+\tau}^U)^{\eta-1} \pi_{t+\tau}^U c_{t+\tau}^{U,\tau+1}, \quad y_{t+\tau}^{U,\tau+1} = \zeta^y (e_{t+\tau}^U / e_{t+\tau}^A)^{-\eta} (\pi_{t+\tau}^U)^{\eta-1} \pi_{t+\tau}^U c_{t+\tau}^{U,\tau+1}, \\ z_{t+\tau}^{U,\tau+1} &= \zeta^z (\pi_{t+\tau}^U)^{\eta-1} \pi_{t+\tau}^U c_{t+\tau}^{U,\tau+1}, \quad \tau = 0, 1, 2. \end{aligned} \quad (8)$$

The government of each country (region)  $i = S, N, A, U$  taxes labor income and uses the revenues from additional borrowing to finance the interest costs on existing government debt and government expenditures. The government budget constraint of country (region)  $i$  reads as follows:

$$B_{t+1}^i - B_t^i + \tau_t^i w_t^i L_t^{i,1} = i_t^i B_t^i + \Gamma_t^i, \quad i = S, N, A, U, \quad t = 0, 1, 2, \dots, \quad (9)$$

where  $\Gamma_t^i$  denotes real government expenditures and  $B_t^i$  is the level of real government debt in country (region)  $i = S, N, A, U$  at the beginning of period  $t$ . In line with Diamond (1965), we assume that government expenditures are unproductive.

The aggregate stock of houses in each country (region),  $H_t^i, i = S, N, A, U$ , accumulates over time as follows:

$$H_{t+1}^i = O_t^i + (1 - \delta^{H,i})H_t^i, i = S, N, A, U, t = 0, 1, 2, \dots \quad (10)$$

In addition to the restrictions imposed by household and firm optimization, by the above government budget constraints and the housing stock accumulation equations, markets for the production factors labor and capital services as well as housing stock markets have to clear in all countries (regions) and in all periods.

$$N_t^{X,i} + N_t^{H,i} = L_t^{i,1}, i = S, N, N_t^{Y,A} + N_t^{H,A} = L_t^{A,1}, N_t^{Z,U} + N_t^{H,U} = L_t^{U,1}, t = 0, 1, 2, \dots, \quad (11)$$

$$K_t^{X,i} + K_t^{H,i} = K_t^i, i = S, N, K_t^{Y,A} + K_t^{H,A} = K_t^A, K_t^{Z,U} + K_t^{H,U} = K_t^U, t = 0, 1, 2, \dots, \quad (12)$$

$$H_{t+1}^i = H_{t+1}^{i,1}, i = S, N, A, U, t = 0, 1, 2, \dots \quad (13)$$

Since the asset markets are competitive, transaction and adjustment costs do not occur, no risk (aversion) prevails, the following no-arbitrage condition (= national Fisher equation) holds in all countries (regions):

$$1 + i_{t+1}^i = q_{t+1}^i + 1 - \delta^i, i = S, N, A, U, t = 0, 1, 2, \dots, \quad (14)$$

whereby  $0 < \delta^i \leq 1, i = S, N, A, U$  depicts the fixed depreciation rate of private capital (period by period) in country (region)  $i$ .

The asset market clearing conditions in all countries (regions) read as follows:

$$L_t^{i,1} s_t^{i,1} + L_t^{i,2} s_t^{i,2} = K_{t+1}^i + B_{t+1}^i, i = S, N, A, U, t = 0, 1, 2, \dots, \quad (15)$$

$$B_t^i = B_{t+1}^i, K_{t+1}^i = K_{t+1}^{i,i}, i = S, N, A, U, t = 0, 1, 2, \dots \quad (16)$$

$$L_t^{S,1} b_{t+1}^{S,1} = L_t^{S,2} b_{t+1}^{SS,2}, L_t^{N,1} b_{t+1}^{N,1} = L_t^{N,2} b_{t+1}^{NN,2}, L_t^{A,1} b_{t+1}^{A,1} = L_t^{A,2} b_{t+1}^{AA,2}, L_t^{U,1} b_{t+1}^{U,1} = L_t^{U,2} b_{t+1}^{UU,2}. \quad (17)$$

Finally, the following clearing conditions for the product markets hold:

$$X_t^S + X_t^N = L_t^{S,1} x_t^{S,1} + L_t^{S,2} x_t^{S,2} + L_t^{S,3} x_t^{S,3} + \Gamma_t^S + (K_{t+1}^S - (1 - \delta^S)K_t^S) + L_t^{N,1} x_t^{N,1} + L_t^{N,2} x_t^{N,2} + L_t^{N,3} x_t^{N,3} + \Gamma_t^N + (K_{t+1}^N - (1 - \delta^N)K_t^N) + L_t^{A,1} x_t^{A,1} + L_t^{A,2} x_t^{A,2} + L_t^{A,3} x_t^{A,3} + L_t^{U,1} x_t^{U,1} + L_t^{U,2} x_t^{U,2} + L_t^{U,3} x_t^{U,3}, \quad (18.a)$$

$$Y_t = L_t^{A,1} y_t^{A,1} + L_t^{A,2} y_t^{A,2} + L_t^{A,3} y_t^{A,3} + \Gamma_t^A + (K_{t+1}^A - (1 - \delta^A)K_t^A) + L_t^{S,1} y_t^{S,1} + L_t^{S,2} y_t^{S,2} + L_t^{S,3} y_t^{S,3} + L_t^{N,1} y_t^{N,1} + L_t^{N,2} y_t^{N,2} + L_t^{N,3} y_t^{N,3} + L_t^{U,1} y_t^{U,1} + L_t^{U,2} y_t^{U,2} + L_t^{U,3} y_t^{U,3}, \quad (18.b)$$

$$Z_t = L_t^{U,1} z_t^{U,1} + L_t^{U,2} z_t^{U,2} + L_t^{U,3} z_t^{U,3} + \Gamma_t^U + (K_{t+1}^U - (1 - \delta^U)K_t^U) + L_t^{S,1} z_t^{S,1} + L_t^{S,2} z_t^{S,2} + L_t^{S,3} z_t^{S,3} + L_t^{N,1} z_t^{N,1} + L_t^{N,2} z_t^{N,2} + L_t^{N,3} z_t^{N,3} + L_t^{A,1} z_t^{A,1} + L_t^{A,2} z_t^{A,2} + L_t^{A,3} z_t^{A,3}. \quad (18.c)$$

In order to be able to model the fact of time-stationarity of country (region)  $i$ 's public debt and government expenditures to GDP ratios between 1999 and 2008 we transform total outstanding government debt and government expenditures in country (region)  $i$ 's government budget

constraint into debt respective expenditures to GDP ratios. This is achieved by dividing both sides of (9) by  $X_t^i + p_t^i O_t^i$  for  $i = S, N$ , by  $Y_t + p_t^A O_t^A$  for  $i = A$ , by  $Z_t + p_t^U O_t^U$  for  $i = U$  and by defining the debt to GDP ratios as  $b_t^i = B_t^i / (X_t^i + p_t^i O_t^i)$ ,  $i = S, N$ ,  $b_t^A = B_t^A / (Y_t + p_t^A O_t^A)$ ,  $b_t^U = B_t^U / (Z_t + p_t^U O_t^U)$  and the expenditure to GDP ratios as  $\gamma_t^i \equiv \Gamma_t^i / (X_t^i + p_t^i O_t^i)$ ,  $i = S, N$ ,  $\gamma_t^A \equiv \Gamma_t^A / (Y_t + p_t^A O_t^A)$ ,  $\gamma_t^U \equiv \Gamma_t^U / (Z_t + p_t^U O_t^U)$ , we obtain for country  $i$ :

$$G_{t+1}^{GP,i} b_{t+1}^i = (1 + i_t^i) b_t^i + \gamma_t^i - \frac{w_t^i \tau_t^i L_t^{i,1}}{X_t^i + p_t^i O_t^i}, \quad G_{t+1}^{GP,i} \equiv \frac{X_{t+1}^i + p_{t+1}^i O_{t+1}^i}{X_t^i + p_t^i O_t^i}, \quad i = S, N, t = 0, 1, 2, \dots \quad (19.a)$$

$$G_{t+1}^{GP,A} b_{t+1}^A = (1 + i_t^A) b_t^A + \gamma_t^A - \frac{w_t^A \tau_t^A L_t^{A,1}}{Y_t + p_t^A O_t^A}, \quad G_{t+1}^{GP,A} \equiv \frac{Y_{t+1} + p_{t+1}^A O_{t+1}^A}{Y_t + p_t^A O_t^A}, \quad t = 0, 1, 2, \dots, \quad (19.b)$$

$$G_{t+1}^{GP,U} b_{t+1}^U = (1 + i_t^U) b_t^U + \gamma_t^U - \frac{w_t^U \tau_t^U L_t^{U,1}}{Z_t + p_t^U O_t^U}, \quad G_{t+1}^{GP,U} \equiv \frac{Z_{t+1} + p_{t+1}^U O_{t+1}^U}{Z_t + p_t^U O_t^U}, \quad t = 0, 1, 2, \dots \quad (19.c)$$

The gross domestic products  $X_t^i + p_t^i O_t^i \equiv GP_t^i$ ,  $Y_t + p_t^A O_t^A \equiv GP_t^A$  and  $Z_t + p_t^U O_t^U \equiv GP_t^U$  in (19) – (21) can be rewritten by using the sectoral production functions  $X_t^i = M^{X,i} a_t^i N_t^{X,i} (k_t^{X,i})^{\alpha^{X,i}}$ ,  $k_t^{X,i} \equiv K_t^{X,i} / (a_t^i N_t^{X,i})$ ,  $O_t^i = M^{H,i} a_t^i N_t^{H,i} (k_t^{H,i})^{\alpha^{H,i}}$ ,  $k_t^{H,i} \equiv K_t^{H,i} / (a_t^i N_t^{H,i})$ ,  $i = S, N$ ,  $Y_t = M^{Y,A} a_t^A N_t^{Y,A} (k_t^{Y,A})^{\alpha^{Y,A}}$ ,  $k_t^{Y,A} \equiv K_t^{Y,A} / (a_t^A N_t^{Y,A})$ ,  $O_t^A = M^{H,A} a_t^A N_t^{H,A} (k_t^{H,A})^{\alpha^{H,A}}$ ,  $k_t^{H,A} \equiv K_t^{H,A} / (a_t^A N_t^{H,A})$ ,  $Z_t = M^{Z,U} a_t^U N_t^{Z,U} (k_t^{Z,U})^{\alpha^{Z,U}}$ ,  $k_t^{Z,U} \equiv K_t^{Z,U} / (a_t^U N_t^{Z,U})$ ,  $O_t^U = M^{H,U} a_t^U N_t^{H,U} (k_t^{H,U})^{\alpha^{H,U}}$ ,  $k_t^{H,U} \equiv K_t^{H,U} / (a_t^U N_t^{H,U})$ , the first-order conditions for maximum profits (1) – (4) and factor market clearing conditions (10) – (11) as follows:

$$\begin{aligned} GP_t^i &= a_t^i N_t^{X,i} M^{X,i} (k_t^{X,i})^{\alpha^{X,i}} + p_t^i a_t^i N_t^{H,i} M^{H,i} (k_t^{H,i})^{\alpha^{H,i}} = a_t^i N_t^{X,i} M^{X,i} (k_t^{X,i})^{\alpha^{X,i}} \\ &+ \frac{(1 - \alpha^{X,i}) M^{X,i} (k_t^{X,i})^{\alpha^{X,i}}}{(1 - \alpha^{H,i}) M^{H,i} (k_t^{H,i})^{\alpha^{H,i}}} a_t^i N_t^{H,i} M^{H,i} (k_t^{H,i})^{\alpha^{H,i}} = a_t^i M^{X,i} (k_t^{X,i})^{\alpha^{X,i}} \left[ N_t^{X,i} + \frac{(1 - \alpha^{X,i})}{(1 - \alpha^{H,i})} N_t^{H,i} \right] \\ &= L_t^{i,1} a_t^i M^{X,i} (k_t^{X,i})^{\alpha^{X,i}} \left[ \frac{N_t^{X,i}}{L_t^{i,1}} + \frac{(1 - \alpha^{X,i})}{(1 - \alpha^{H,i})} \frac{N_t^{H,i}}{L_t^{i,1}} \right] = L_t^{i,1} a_t^i M^{X,i} (k_t^{X,i})^{\alpha^{X,i}} \left[ l_t^{X,i} + \frac{(1 - \alpha^{X,i})}{(1 - \alpha^{H,i})} (1 - l_t^{X,i}) \right] \\ &= \frac{L_t^{i,1} a_t^i M^{X,i} (k_t^{X,i})^{\alpha^{X,i}}}{(1 - \alpha^{H,i})} [1 - \alpha^{X,i} + (\alpha^{X,i} - \alpha^{H,i}) l_t^{X,i}], \quad l_t^{X,i} \equiv \frac{N_t^{X,i}}{L_t^{i,1}}, \quad l_t^{H,i} \equiv \frac{N_t^{H,i}}{L_t^{i,1}}, \quad i = S, N. \end{aligned} \quad (20.a)$$

$$GP_t^A = \frac{L_t^{A,1} a_t^A M^{Y,A} (k_t^{Y,A})^{\alpha^{Y,A}}}{(1 - \alpha^{H,A})} [1 - \alpha^{Y,A} + (\alpha^{Y,A} - \alpha^{H,A}) l_t^{Y,A}], \quad l_t^{Y,A} \equiv \frac{N_t^{Y,A}}{L_t^{A,1}}, \quad l_t^{H,A} \equiv \frac{N_t^{H,A}}{L_t^{A,1}}, \quad (20.b)$$

$$GP_t^U = \frac{L_t^{U,1} a_t^U M^{Z,U} (k_t^{Z,U})^{\alpha^{Z,U}}}{(1 - \alpha^{H,U})} [1 - \alpha^{Z,U} + (\alpha^{Z,U} - \alpha^{H,U}) l_t^{Z,U}], \quad l_t^{Z,U} \equiv \frac{N_t^{Z,U}}{L_t^{U,1}}, \quad l_t^{H,U} \equiv \frac{N_t^{H,U}}{L_t^{U,1}}. \quad (20.c)$$

By proceeding analogously to deriving equation (20.a), we are able to derive the respective growth factors of domestic gross products as follows:

$$G_{t+1}^{GP^i} \equiv G^i \left( \frac{k_{t+1}^{X,i}}{k_t^{X,i}} \right)^{\alpha^{X,i}} \left[ \frac{1 - \alpha^{X,i} + (\alpha^{X,i} - \alpha^{H,i}) l_{t+1}^{X,i}}{1 - \alpha^{X,i} + (\alpha^{X,i} - \alpha^{H,i}) l_t^{X,i}} \right], \quad i = S, N, \quad (21.a)$$

$$G_{t+1}^{GP^A} \equiv G_{t+1}^A \left( \frac{k_{t+1}^{Y,A}}{k_t^{Y,A}} \right)^{\alpha^{Y,A}} \left[ \frac{1 - \alpha^{Y,A} + (\alpha^{Y,A} - \alpha^{H,A}) l_{t+1}^{Y,A}}{1 - \alpha^{Y,A} + (\alpha^{Y,A} - \alpha^{H,A}) l_t^{Y,A}} \right], \quad (21.b)$$

$$G_{t+1}^{GP^U} \equiv G_{t+1}^U \left( \frac{k_{t+1}^{Z,U}}{k_t^{Z,U}} \right)^{\alpha^{Z,U}} \left[ \frac{1 - \alpha^{Z,U} + (\alpha^{Z,U} - \alpha^{H,U}) l_{t+1}^{Z,U}}{1 - \alpha^{Z,U} + (\alpha^{Z,U} - \alpha^{H,U}) l_t^{Z,U}} \right]. \quad (21.c)$$

The tax revenues as a ratio of gross domestic product can be written as follows:

$$\frac{L_t^{i,1} w_t^i \tau_t^i}{GP_t^i} = \frac{w_t^i \tau_t^i (1 - \alpha^{H,i})}{a_t^i M^{X,i} (k_t^{X,i})^{\alpha^{X,i}} [1 - \alpha^{X,i} + (\alpha^{X,i} - \alpha^{H,i}) l_t^{X,i}]}, \quad i = S, N, t = 0, 1, 2, \dots, \quad (22.a)$$

$$\frac{L_t^{A,1} w_t^A \tau_t^A}{GP_t^A} = \frac{w_t^A \tau_t^A (1 - \alpha^{H,A})}{a_t^A M^{Y,A} (k_t^{Y,A})^{\alpha^{Y,A}} [1 - \alpha^{Y,A} + (\alpha^{Y,A} - \alpha^{H,A}) l_t^{Y,A}]}, \quad t = 0, 1, 2, \dots, \quad (22.b)$$

$$\frac{L_t^{U,1} w_t^U \tau_t^U}{GP_t^U} = \frac{w_t^U \tau_t^U (1 - \alpha^{H,U})}{a_t^U M^{Z,U} (k_t^{Z,U})^{\alpha^{Z,U}} [1 - \alpha^{Z,U} + (\alpha^{Z,U} - \alpha^{H,U}) l_t^{Z,U}]}, \quad t = 0, 1, 2, \dots \quad (22.c)$$

In line with empirical observations for the 1990s we henceforth assume that the public debt to GDP ratios,  $b_t^i$ ,  $i = S, N, A, U$ , and the government expenditures as a ratio of GDP,  $\gamma_t^i$ ,  $i = S, N, A, U$ , remain constant over time, i.e.  $b_t^i = b^i$  and  $\gamma_t^i = \gamma^i$ ,  $i = S, N, A, U$  for  $t = 0, 1, 2, \dots$ . As a consequence, the government budget constraints in terms of per-GDP variables read as follows:

$$[G_{t+1}^{GP,i} - (1 + i_t^i)] b^i = \gamma^i - \frac{w_t^i \tau_t^i (1 - \alpha^{H,i})}{a_t^i M^{X,i} (k_t^{X,i})^{\alpha^{X,i}} [1 - \alpha^{X,i} + (\alpha^{X,i} - \alpha^{H,i}) l_t^{X,i}]}, \quad i = S, N, t = 0, 1, 2, \dots \quad (23.a)$$

$$[G_{t+1}^{GP,A} - (1 + i_t^A)] b^A = \gamma^A - \frac{w_t^A \tau_t^A (1 - \alpha^{H,A})}{a_t^A M^{Y,A} (k_t^{Y,A})^{\alpha^{Y,A}} [1 - \alpha^{Y,A} + (\alpha^{Y,A} - \alpha^{H,A}) l_t^{Y,A}]}, \quad t = 0, 1, 2, \dots \quad (23.b)$$

$$[G_{t+1}^{GP,U} - (1 + i_t^U)] b^U = \gamma^U - \frac{w_t^U \tau_t^U (1 - \alpha^{H,U})}{a_t^U M^{Z,U} (k_t^{Z,U})^{\alpha^{Z,U}} [1 - \alpha^{Z,U} + (\alpha^{Z,U} - \alpha^{H,U}) l_t^{Z,U}]}, \quad t = 0, 1, 2, \dots \quad (23.c)$$

In transforming the aggregate stock of houses in each country into per-GDP magnitudes, i.e.  $h_t^i \equiv H_t^i / GP_t^i$ ,  $i = S, N$ , we obtain from the accumulation equation (10):

$$G_{t+1}^{GP^i} h_{t+1}^i = \frac{O_t^i}{GP_t^i} + (1 - \delta^{H,i}) h_t^i, \quad i = S, N, t = 0, 1, 2, \dots, \quad (24.a)$$

$$G_{t+1}^{GP^A} h_{t+1}^A = \frac{O_t^A}{GP_t^A} + (1 - \delta^{H,A}) h_t^A, \quad t = 0, 1, 2, \dots, \quad (24.b)$$

$$G_{t+1}^{GP^U} h_{t+1}^U = \frac{O_t^U}{GP_t^U} + (1 - \delta^{H,U}) h_t^U, \quad t = 0, 1, 2, \dots \quad (24.c)$$

Using once more the housing production functions housing production per GDP reads as follows:

$$\frac{O_t^i}{GP_t^i} = \frac{M^{H,i} a_t^i N_t^{H,i} (k_t^{H,i})^{\alpha^{H,i}} (1 - \alpha^{H,i})}{a_t^i L_t^{i,1} M^{X,i} (k_t^{X,i})^{\alpha^{X,i}} [1 - \alpha^{X,i} + (\alpha^{X,i} - \alpha^{H,i}) l_t^{X,i}]} = \frac{M^{H,i} (1 - l_t^{X,i}) (k_t^{H,i})^{\alpha^{H,i}} (1 - \alpha^{H,i})}{M^{X,i} (k_t^{X,i})^{\alpha^{X,i}} [1 - \alpha^{X,i} + (\alpha^{X,i} - \alpha^{H,i}) l_t^{X,i}]}, \quad (25.a)$$

$$i = S, N, t = 0, 1, 2, \dots$$

$$\frac{O_t^A}{GP_t^A} = \frac{M^{H,A}(1-l_t^{Y,A})(k_t^{H,A})^{\alpha^{H,A}}(1-\alpha^{H,A})}{M^{Y,A}(k_t^{Y,A})^{\alpha^{Y,A}}[1-\alpha^{Y,A}+(\alpha^{Y,A}-\alpha^{H,A})l_t^{Y,A}]}, t=0,1,2,\dots, \quad (25.b)$$

$$\frac{O_t^U}{GP_t^U} = \frac{M^{H,U}(1-l_t^{Z,U})(k_t^{H,U})^{\alpha^{H,U}}(1-\alpha^{H,U})}{M^{Z,U}(k_t^{Z,U})^{\alpha^{Z,U}}[1-\alpha^{Z,U}+(\alpha^{Z,U}-\alpha^{H,U})l_t^{Z,U}]}, t=0,1,2,\dots \quad (25.c)$$

Writing the variables in the clearing conditions for the factor markets as ratios of labor supply, i.e. of  $L_t^{i,1}, i = S, N, A, U$ , we get the following well-known relations:

$$l_t^{X,i} + l_t^{H,i} = 1 \wedge k_t^{X,i} l_t^{X,i} + k_t^{H,i} l_t^{H,i} = k_t^i \Rightarrow l_t^{X,i} = \frac{(k_t^i - k_t^{H,i})}{(k_t^{X,i} - k_t^{H,i})}, l_t^{H,i} = \frac{-(k_t^i - k_t^{X,i})}{(k_t^{X,i} - k_t^{H,i})}, i = S, N, \forall t \quad (26.a)$$

$$l_t^{Y,A} + l_t^{H,A} = 1 \wedge k_t^{Y,A} l_t^{Y,A} + k_t^{H,A} l_t^{H,A} = k_t^A \Rightarrow l_t^{Y,A} = \frac{(k_t^A - k_t^{H,A})}{(k_t^{Y,A} - k_t^{H,A})}, l_t^{H,A} = \frac{-(k_t^A - k_t^{Y,A})}{(k_t^{Y,A} - k_t^{H,A})}, \forall t \quad (26.b)$$

$$l_t^{Z,U} + l_t^{H,U} = 1 \wedge k_t^{Z,U} l_t^{Z,U} + k_t^{H,U} l_t^{H,U} = k_t^U \Rightarrow l_t^{Z,U} = \frac{(k_t^U - k_t^{H,U})}{(k_t^{Z,U} - k_t^{H,U})}, l_t^{H,U} = \frac{-(k_t^U - k_t^{Z,U})}{(k_t^{Z,U} - k_t^{H,U})}, \forall t \quad (26.c)$$

Dividing the asset market clearing condition (15) on both sides by  $a_t^i L_t^{i,1}, i = S, N, A, U, \forall t$ , and using the definition of the debt output ratios, (15) can be rewritten as follows:

$$\frac{K_{t+1}^i}{a_{t+1}^i L_{t+1}^{i,1}} + \frac{B_{t+1}^i}{a_{t+1}^i L_{t+1}^{i,1}} = \frac{S_t^{i,1}}{a_t^i} + \frac{L_t^{i,2} S_t^{i,2}}{a_t^i L_t^{i,1}} \Leftrightarrow G_{t+1}^i (k_{t+1}^i + \tilde{b}_{t+1}^i) = \frac{S_t^{i,1}}{a_t^i} + \frac{L_t^{i,1} S_t^{i,2}}{a_t^i L_t^{i,1}},$$

$$\tilde{b}_{t+1}^i \equiv \frac{B_{t+1}^i}{a_{t+1}^i L_{t+1}^{i,1}} = \begin{cases} \frac{b^i M^{X,i} (k_{t+1}^{X,i})^{\alpha^{X,i}} [1-\alpha^{X,i} + (\alpha^{X,i} - \alpha^{H,i}) l_{t+1}^{X,i}]}{(1-\alpha^{H,i})}, i = S, N, \\ \frac{b^A M^{Y,A} (k_{t+1}^{Y,A})^{\alpha^{Y,A}} [1-\alpha^{Y,A} + (\alpha^{Y,A} - \alpha^{H,A}) l_{t+1}^{Y,A}]}{(1-\alpha^{H,A})} \\ \frac{b^U M^{Z,U} (k_{t+1}^{Z,U})^{\alpha^{Z,U}} [1-\alpha^{Z,U} + (\alpha^{Z,U} - \alpha^{H,U}) l_{t+1}^{Z,U}]}{(1-\alpha^{H,U})} \end{cases} \quad (27)$$

$$G_{t+1}^i \equiv \frac{a_{t+1}^i L_{t+1}^{i,1}}{a_t^i L_t^{i,1}}, i = S, N, A, U, \forall t.$$

The three market clearing conditions for final product markets can be reduced to the following two intertemporal equilibrium conditions by dividing market clearing condition (18.b) respective (18.c) on both sides by market clearing condition (18.a) and transforming main quantity variables into variable to GDP and effective labor ratios.

$$(e_t^A)^\eta = \zeta^x \left[ (1-\gamma^A) M^{Y,A} l_t^{Y,A} (k_t^{Y,A})^{\alpha^{Y,A}} - G_{t+1}^A k_{t+1}^A + (1-\delta^A) k_t^A \right] / \zeta^y \left\{ \frac{a_t^S L_t^{S,1}}{a_t^A L_t^{A,1}} \left[ (1-\gamma^S) M^{X,S} l_t^{X,S} (k_t^{X,S})^{\alpha^{X,S}} - G_{t+1}^S k_{t+1}^S + (1-\delta^S) k_t^S \right] \right. \\ \left. + \frac{a_t^N L_t^{N,1}}{a_t^A L_t^{A,1}} \left[ (1-\gamma^N) M^{X,N} l_t^{X,N} (k_t^{X,N})^{\alpha^{X,N}} - G_{t+1}^N k_{t+1}^N + (1-\delta^N) k_t^N \right] \right\} \quad (28)$$

$$(e_t^U)^\eta = \zeta^x \left[ (1-\gamma^U) M^{Z,U} l_t^{Z,U} (k_t^{Z,U})^{\alpha^{Z,U}} - G_{t+1}^U k_{t+1}^U + (1-\delta^U) k_t^U \right] / \zeta^z \left\{ \frac{a_t^S L_t^{S,1}}{a_t^A L_t^{A,1}} \left[ (1-\gamma^S) M^{X,S} l_t^{X,S} (k_t^{X,S})^{\alpha^{X,S}} - G_{t+1}^S k_{t+1}^S + (1-\delta^S) k_t^S \right] \right. \\ \left. + \frac{a_t^N L_t^{N,1}}{a_t^A L_t^{A,1}} \left[ (1-\gamma^N) M^{X,N} l_t^{X,N} (k_t^{X,N})^{\alpha^{X,N}} - G_{t+1}^N k_{t+1}^N + (1-\delta^N) k_t^N \right] \right\} \quad (29)$$

with  $a_{t+1}^i = G_{t+1}^i a_t^i, L_{t+1}^{i,1} = G^L L_t^{i,1}, i = S, N, A, U, t = 0, 1, 2, \dots$

For exogenously given structural and policy parameters  $\beta^i, \theta^i, b^i, \gamma^i, G_t^i, \delta^i, M^{H,i} (i = S, N, A, U, t = 0, 1, 2, \dots), M^{X,i}, \alpha^{X,i} (i = S, N), M^{Y,A}, \alpha^{Y,A}, M^{Z,U}, \alpha^{Z,U}, \eta, \sigma, \zeta^x, \zeta^y, \zeta^z$  and initial values of the variables  $k_0^i, a_0^i, L_0^{i,1} (i = S, N, A, U)$ , the difference equation system (23)-(24) and (27) – (29) together with market clearing condition (13) represents a determinate dynamic system.

Before switching to intra-EMU and global financial integration, it is worth noting that in all countries (regions) foreign trade in goods is balanced under financial autarky. To see this we need the following definitions of country- (region-) specific trade balances in period  $t$  in country  $i$  denoted by  $TB_t^i, i = S, N, A, U$  which equals the difference between exports and imports in terms of the country-specific good:

$$TB_t^S = X_t^S - \Gamma_t^S - (K_{t+1}^S - (1-\delta)K_t^S) - L_t^{S,1} x_t^{S,1} - L_t^{S,2} x_t^{S,2} - L_t^{S,3} x_t^{S,3} \\ - (1/e_t^A) L_t^{S,1} y_t^{S,1} - (1/e_t^A) L_t^{S,2} y_t^{S,2} - (1/e_t^A) L_t^{S,3} y_t^{S,3} \\ - (1/e_t^U) L_t^{S,1} z_t^{S,1} - (1/e_t^U) L_t^{S,2} z_t^{S,2} - (1/e_t^U) L_t^{S,3} z_t^{S,3}, \quad (30.a)$$

$$TB_t^N = X_t^N - \Gamma_t^N - (K_{t+1}^N - (1-\delta)K_t^N) - L_t^{N,1} x_t^{N,1} - L_t^{N,2} x_t^{N,2} - L_t^{N,3} x_t^{N,3} \\ - (1/e_t^A) L_t^{N,1} y_t^{N,1} - (1/e_t^A) L_t^{N,2} y_t^{N,2} - (1/e_t^A) L_t^{N,3} y_t^{N,3} \\ - (1/e_t^U) L_t^{N,1} z_t^{N,1} - (1/e_t^U) L_t^{N,2} z_t^{N,2} - (1/e_t^U) L_t^{N,3} z_t^{N,3}, \quad (30.b)$$

$$TB_t^A = Y_t - \Gamma_t^A - (K_{t+1}^A - (1-\delta)K_t^A) - L_t^{A,1} y_t^{A,1} - L_t^{A,2} y_t^{A,2} - L_t^{A,3} y_t^{A,3} \\ - e_t^A L_t^{A,1} x_t^{A,1} - e_t^A L_t^{A,2} x_t^{A,2} - e_t^A L_t^{A,3} x_t^{A,3} \\ - (e_t^A / e_t^U) L_t^{A,1} z_t^{A,1} - (e_t^A / e_t^U) L_t^{A,2} z_t^{A,2} - (e_t^A / e_t^U) L_t^{A,3} z_t^{A,3}, \quad (30.c)$$

$$TB_t^U = Z_t - \Gamma_t^U - (K_{t+1}^U - (1-\delta)K_t^U) - L_t^{U,1} z_t^{U,1} - L_t^{U,2} z_t^{U,2} - L_t^{U,3} z_t^{U,3} \\ - e_t^U L_t^{U,1} x_t^{U,1} - e_t^U L_t^{U,2} x_t^{U,2} - e_t^U L_t^{U,3} x_t^{U,3} \\ - (e_t^U / e_t^A) L_t^{U,1} y_t^{U,1} - (e_t^U / e_t^A) L_t^{U,2} y_t^{U,2} - (e_t^U / e_t^A) L_t^{U,3} y_t^{U,3}. \quad (30.d)$$

Using the market clearing conditions (11)-(15), the budget constraints for all private households and the governments in period  $t$  as well as the zero-profit and no-arbitrage conditions, it is not difficult to show that the following equality holds in all periods of the intertemporal equilibrium:

$$TB_t^i = L_t^{i,1} s_t^{i,1} + L_t^{i,2} s_t^{i,2} - K_{t+1}^i - B_{t+1}^i, i = S, N, A, U, t = 0, 1, 2, \dots \quad (31)$$

Asset market clearing condition (15) immediately implies  $TB_t^i = 0, i = S, N, A, U, t = 0, 1, 2, \dots$ , i.e. balanced trade. It also implies that financial openness represents a necessary condition for

trade imbalances. Only if national savings are not only invested in national real capital and national government bonds, trade imbalances can occur. While international capital mobility being a necessary condition for the emergence of trade imbalances, it remains an open question whether international financial integration is sufficient to explain the signs and the sizes of trade imbalances observed empirically between EMU core and periphery since euro-related financial integration, and between Asia and the USA since global financial integration. To provide an answer to this question is the objective of the next section.

### International Equilibrium under intra-EMU and Global Financial Integration

To mimic the financial integration arising through the set-up of the EMU and the Asian-US financial integration we assume in line with stylized facts (Chen et al. 2013) that northern EMU invests its savings also in southern physical capital and government bonds, that Asia buys US government bonds, and that the USA purchases northern EMU government bonds without incurring any transaction costs. However, also in line with empirical data, we assume that the southern EMU young household buys neither northern real capital, nor northern government bonds nor Asian or US assets.

While the intertemporal optimization of southern EMU household in international equilibrium remains as under autarky, the optimization constraints of northern EMU households change as follows:

$$\begin{aligned}
s_{t+1}^{N,2} &\equiv \frac{I_{t+1}^{N,N} + (1-\delta)K_{t+1}^{N,N}}{L_{t+1}^{N,2}} + \frac{I_{t+1}^{S,N} + (1-\delta)K_{t+1}^{S,N}}{L_{t+1}^{N,2}} + \frac{B_{t+2}^{N,N}}{L_{t+1}^{N,2}} + \frac{B_{t+2}^{S,N}}{L_{t+1}^{N,2}} + b_{t+2}^{NN,2} + b_{t+2}^{SN,2}, \\
\text{(iii)} \quad x_{t+2}^{N,3} + \frac{y_{t+2}^{N,3}}{e_{t+2}^A} + \frac{z_{t+2}^{N,3}}{e_{t+2}^U} &= (q_{t+2}^N + 1 - \delta) \frac{[I_{t+1}^{N,N} + (1-\delta)K_{t+1}^{N,N}]}{L_{t+1}^{N,2}} + (q_{t+2}^S + 1 - \delta) \frac{[I_{t+1}^{S,N} + (1-\delta)K_{t+1}^{S,N}]}{L_{t+1}^{N,2}} \\
&\quad + (1+i_{t+2}^N) \left( \frac{B_{t+2}^{N,N}}{L_{t+1}^{N,2}} \right) + (1+i_{t+2}^S) \left( \frac{B_{t+2}^{S,N}}{L_{t+1}^{N,2}} \right) + (1+i_{t+2}^N) b_{t+2}^{NN,2} + (1+i_{t+2}^S) b_{t+2}^{SN,2}.
\end{aligned} \tag{32}$$

Here,  $I_{t+1}^{S,N}/L_{t+1}^{N,2}$  denotes the per-capita investment in southern real capital in period  $t+1$ ,  $B_{t+2}^{S,N}/L_{t+1}^{N,2}$  is the per-capita stock of southern government bonds which the northern EMU young household wants to hold at the beginning of period  $t+2$ , and  $b_{t+2}^{SN,2}$  is the stock of private bonds emitted by southern young households which the northern EMU young household wants to hold at the beginning of period  $t+2$ . The northern younger household buys in middle-age not only private bonds issued by northern EMU young ager but also the private bonds issued by the southern young ager. Since for the time period between euro inception and the onset of the global financial crisis physical capital, government and private bonds in each EMU region can be assumed to be perfectly substitutable, and since within the monetary union these assets can be assumed to be perfectly mobile across South and North, the following international Fisher equation (= real international interest parity condition) holds in addition to the national Fisher equations (14):

$$1 + i_{t+1}^S = 1 + i_{t+1}^N. \tag{33}$$

The typical Asian young household solves under financial integration essentially the same problem as under financial autarky with the exception of the use of per-capita savings and the old-age budget constraint which read now as follows:

$$s_{t+1}^{A,2} = [I_{t+1}^{A,A} + (1-\delta)K_{t+1}^{A,A}] / L_{t+1}^{A,2} + B_{t+2}^{A,A} / L_{t+1}^{A,2} + (e_{t+1}^A / e_{t+1}^U) B_{t+2}^{U,A} / L_{t+1}^{A,2} + b_{t+2}^{AA,2} + (e_{t+1}^A / e_{t+1}^U) b_{t+2}^{UA,2},$$

$$(iii) e_{t+2}^A x_{t+2}^{A,3} + y_{t+2}^{A,3} + (e_{t+2}^A / e_{t+2}^U) z_{t+2}^{A,3} = [q_{t+2}^A + (1-\delta)][I_{t+1}^{A,A} + (1-\delta)K_{t+1}^{A,A}] / L_{t+1}^{A,2} + (1+i_{t+2}^A)(B_{t+2}^{A,A} / L_{t+1}^{A,2})$$

$$+ (1+i_{t+2}^U)(e_{t+2}^A / e_{t+2}^U)(B_{t+2}^{U,A} / L_{t+1}^{A,2}) + (1+i_{t+2}^A) b_{t+2}^{AA,2} + (1+i_{t+2}^U)(e_{t+2}^A / e_{t+2}^U) b_{t+2}^{UA,2}.$$

Here  $B_{t+2}^{U,A} / L_{t+1}^{A,2}$  ( $b_{t+2}^{UA,2}$ ) denotes the stock of US government (private) bonds which the Asian young household wants to hold at the beginning of period  $t+2$ . In line with pre-crisis reality the Asian young household does not hold EMU government bonds.

Analogously, savings per capita and the old-age budget constraint of the typical US young household are as follows:

$$s_{t+1}^{U,2} = [I_{t+1}^{U,U} + (1-\delta)K_{t+1}^{U,U}] / L_{t+1}^{U,2} + B_{t+2}^{U,U} / L_{t+1}^{U,2} + e_{t+1}^U B_{t+2}^{N,U} / L_{t+1}^{U,2} + b_{t+2}^{UU,2} + e_{t+1}^U b_{t+2}^{NU,2} + p_{t+1}^U h_{t+2}^{U,2},$$

$$(iii) e_{t+2}^U x_{t+2}^{U,3} + (e_{t+2}^U / e_{t+2}^A) y_{t+2}^{U,3} + z_{t+2}^{A,3} = [q_{t+2}^U + (1-\delta)][I_{t+1}^{U,U} + (1-\delta)K_{t+1}^{U,U}] / L_{t+1}^{U,2}$$

$$+ (1+i_{t+2}^U) B_{t+2}^{U,U} / L_{t+1}^{U,2} + (1+i_{t+2}^U) b_{t+2}^{UU,2} + (1+i_{t+2}^N) e_{t+2}^U B_{t+2}^{N,U} / L_{t+1}^{U,2} + (1+i_{t+2}^U) e_{t+2}^U b_{t+2}^{NU,2} + p_{t+2}^U h_{t+2}^{U,2}.$$

Again in line with pre-crisis empirical reality, the US young household does hold from abroad only northern EMU government bonds.

In order to ensure arbitrage-free terms of trade, the following international real interest parity conditions in addition to (33) ought to hold:

$$1 + i_{t+1}^A = \frac{e_{t+1}^A}{e_t^A} (1 + i_{t+1}^N), \quad \forall t = 0, 1, 2, \dots, \quad (36)$$

$$1 + i_{t+1}^U = \frac{e_{t+1}^U}{e_t^U} (1 + i_{t+1}^N), \quad \forall t = 0, 1, 2, \dots. \quad (37)$$

The markets for southern and northern EMU, Asian and US real capital clear according to:

$$K_{t+1}^S = K_{t+1}^{S,S} + K_{t+1}^{S,N}, \quad K_{t+1}^N = K_{t+1}^{N,N} + K_{t+1}^{N,U}, \quad K_{t+1}^A = K_{t+1}^{A,A}, \quad K_{t+1}^U = K_{t+1}^{U,U}, \quad t = 0, 1, 2, \dots \quad (38)$$

The markets for southern and northern EMU, Asian and US government bonds clear now according to:

$$B_{t+1}^S = B_{t+1}^{S,S} + B_{t+1}^{S,N}, \quad B_{t+1}^N = B_{t+1}^{N,N} + B_{t+1}^{N,U}, \quad B_{t+1}^A = B_{t+1}^{A,A}, \quad B_{t+1}^U = B_{t+1}^{U,U} + B_{t+1}^{U,A}, \quad t = 0, 1, 2, \dots \quad (39)$$

While the clearing conditions for the labor markets and good markets remain as under financial autarky, the clearing conditions for the following private bond markets read now as follows:

$$L_t^{S,1} b_{t+1}^{S,1} = L_t^{S,2} b_{t+1}^{SS,2} + L_t^{N,2} b_{t+1}^{SN,2}, \quad L_t^{N,1} b_{t+1}^{N,1} = L_t^{N,2} b_{t+1}^{NN,2}, \quad L_t^{A,1} b_{t+1}^{A,1} = L_t^{A,2} b_{t+1}^{AA,2}, \quad L_t^{U,1} b_{t+1}^{U,1} = L_t^{U,2} b_{t+1}^{UU,2} + L_t^{A,2} b_{t+1}^{UA,2}. \quad (40)$$

Financial integration affects mostly the asset market clearing condition (15). Now, the worldwide amount of savings ought to be equal to the worldwide supply of assets from southern and northern EMU, Asia and the US. Thus:

$$L_t^{S,1} s_t^{S,1} + L_t^{S,2} s_t^{S,2} + L_t^{N,1} s_t^{N,1} + L_t^{N,2} s_t^{N,2} + (L_t^{A,1} s_t^{A,1} + L_t^{A,2} s_t^{A,2}) / e_t^A + (L_t^{U,1} s_t^{U,1} + L_t^{U,2} s_t^{U,2}) / e_t^U$$

$$= K_{t+1}^S + K_{t+1}^N + B_{t+1}^S + B_{t+1}^N + (K_{t+1}^A + B_{t+1}^A) / e_t^A + (K_{t+1}^U + B_{t+1}^U) / e_t^U. \quad (41)$$



Having described the optimization problems of households and firms as well as the market clearing conditions, the intertemporal equilibrium dynamics can now be derived.

From the FOCs for profit maximization and from the international Fisher equations (33), (36) and (37) the following relationships between southern EMU, Asian, US and northern EMU aggregate capital intensities (= factor proportions) and the terms of trade result:

$$\alpha^{X,S} M^{X,S} (k_{t+1}^{X,S})^{\alpha^{X,S}-1} + 1 - \delta^S = \alpha^{X,N} M^{X,N} (k_{t+1}^{X,N})^{\alpha^{X,N}-1} + 1 - \delta^N, \quad t = 0, 1, 2, \dots, \quad (42)$$

$$e_{t+1}^A = e_t^A \frac{[\alpha^{Y,A} M^{Y,A} (k_{t+1}^{Y,A})^{\alpha^{Y,A}-1} + 1 - \delta^A]}{[\alpha^{X,N} M^{X,N} (k_{t+1}^{X,N})^{\alpha^{X,N}-1} + 1 - \delta^N]}, \quad t = 0, 1, 2, \dots, \quad (43)$$

$$e_{t+1}^U = e_t^U \frac{[\alpha^{Z,U} M^{Z,U} (k_{t+1}^{Z,U})^{\alpha^{Z,U}-1} + 1 - \delta^U]}{[\alpha^{X,N} M^{X,N} (k_{t+1}^{X,N})^{\alpha^{X,N}-1} + 1 - \delta^N]}, \quad t = 0, 1, 2, \dots \quad (44)$$

Dividing (41) on both sides by the world efficiency weighted labor defined as  $AL_t^W \equiv a_t^S L_t^{S,1} + a_t^N L_t^{N,1} + a_t^A L_t^{A,1} + a_t^U L_t^{U,1}$  and introducing the definitions of debt output ratios, the world-wide asset market clearing condition (41) can be rewritten as follows:

$$\begin{aligned} & \frac{L_t^{S,1} s_t^{S,1} + L_t^{S,2} s_t^{S,2}}{a_t^S L_t^{S,1}} \frac{a_t^S L_t^{S,1}}{AL_t^W} + \frac{L_t^{N,1} s_t^{N,1} + L_t^{N,2} s_t^{N,2}}{a_t^N L_t^{N,1}} \frac{a_t^N L_t^{N,1}}{AL_t^W} + \frac{L_t^{A,1} s_t^{A,1} + L_t^{A,2} s_t^{A,2}}{e_t^A a_t^A L_t^{A,1}} \frac{a_t^A L_t^{A,1}}{AL_t^W} + \frac{L_t^{U,1} s_t^{U,1} + L_t^{U,2} s_t^{U,2}}{e_t^U a_t^U L_t^{U,1}} \frac{a_t^U L_t^{U,1}}{AL_{t+1}^W} \\ & = G_{t+1}^S \{k_{t+1}^S + b^S M^{X,S} (k_{t+1}^{X,S})^{\alpha^{X,S}} [1 - \alpha^{X,S} + (\alpha^{X,S} - \alpha^{H,S}) l_{t+1}^{X,S}]\} \frac{a_t^S L_t^{S,1}}{AL_t^W} \\ & \quad + G_{t+1}^N \{k_{t+1}^N + b^N M^{X,N} (k_{t+1}^{X,N})^{\alpha^{X,N}} [1 - \alpha^{X,N} + (\alpha^{X,N} - \alpha^{H,N}) l_{t+1}^{X,N}]\} \frac{a_t^N L_t^{N,1}}{AL_t^W} \\ & \quad + G_{t+1}^A \{k_{t+1}^A + b^A M^{Y,A} (k_{t+1}^{Y,A})^{\alpha^{Y,A}} [1 - \alpha^{Y,A} + (\alpha^{Y,A} - \alpha^{H,A}) l_{t+1}^{Y,A}]\} \frac{a_t^A L_t^{A,1}}{e_t^A AL_t^W} \\ & \quad + G_{t+1}^U \{k_{t+1}^U + b^U M^{Z,U} (k_{t+1}^{Z,U})^{\alpha^{Z,U}} [1 - \alpha^{Z,U} + (\alpha^{Z,U} - \alpha^{H,U}) l_{t+1}^{Z,U}]\} \frac{a_t^U L_t^{U,1}}{e_t^U AL_t^W}, \quad t = 0, 1, 2, \dots \end{aligned} \quad (45)$$

Acknowledging the optimal savings functions resulting from household's utility maximization problems in (45), the resulting equation together with equations (42) - (44) and the unchanged dynamic equations from financial autarky represent the difference equation system of the intertemporal international equilibrium under financial integration.

### National Net Foreign Asset Position, Trade Imbalance and Aggregate Savings

Since national savings of young and middle-aged households need no longer be equal to the sum of national real capital and real government bonds, trade imbalances are now feasible along an intertemporal equilibrium path. Since these trade imbalances represent the other side of the divergence between national savings and national asset accumulation it is usual in the international macroeconomics literature (e.g. Zee 1987, p. 609) to introduce an appropriate variable denoted by  $\Phi_{t+1}^i$ ,  $i = S, N, A, U$  which is defined as:  $\Phi_{t+1}^i = L_t^{i,1} s_t^{i,1} + L_t^{i,2} s_t^{i,2} - K_{t+1}^i - B_{t+1}^i$ ,  $i = S, N, A, U, t = 0, 1, 2, \dots$

Acknowledging the use of aggregate savings within the period- $t$  budget constraints of young, middle-aged and old-aged households and the market clearing conditions for real capital and

government bonds, it turns out that  $\Phi_{t+1}^i$  equals the *net foreign asset position* of country  $i = S, N, A, U$ . In particular, it follows:

$$\Phi_{t+1}^S = -K_{t+1}^{S,N} - B_{t+1}^{S,N} - L_t^{N,2} b_{t+1}^{SN,2}, \quad t = 0, 1, 2, \dots, \quad (46.a)$$

$$\Phi_{t+1}^N = K_{t+1}^{S,N} + B_{t+1}^{S,N} + L_t^{N,2} b_{t+1}^{SN,2} - B_{t+1}^{N,U}, \quad t = 0, 1, 2, \dots, \quad (46.b)$$

$$\Phi_{t+1}^A = (e_t^A / e_t^U) B_{t+1}^{U,A} + (e_t^A / e_t^U) L_t^{A,2} b_{t+1}^{UA,2}, \quad t = 0, 1, 2, \dots, \quad (46.c)$$

$$\Phi_{t+1}^U = e_t^U B_{t+1}^{N,U} + e_t^U L_t^{U,2} b_{t+1}^{NU,2} - L_t^{A,2} b_{t+1}^{UA,2} - B_{t+1}^{U,A}, \quad t = 0, 1, 2, \dots. \quad (46.d)$$

Using again the budget constraints of young, middle-aged and old households plus the zero-profit and no-arbitrage conditions as well as the market clearing conditions one can show that the trade balance of country  $i = S, N, A, U$  is related to the respective net foreign asset position as follows:

$$TB_t^S = -K_{t+1}^{S,N} - B_{t+1}^{S,N} - L_t^{N,2} b_{t+1}^{SN,2} - (1+i_t^S)(-K_t^{S,N} - B_t^{S,N} - L_t^{N,3} b_t^{SN,2}) = \Phi_{t+1}^S - (1+i_t^S)\Phi_t^S, \quad (47.a)$$

$$TB_t^N = K_{t+1}^{S,N} + B_{t+1}^{S,N} + L_t^{N,2} b_{t+1}^{SN,2} - B_{t+1}^{N,U} - (1+i_t^N)(K_t^{S,N} + B_t^{S,N} + L_t^{N,2} b_t^{SN,2} - B_t^{N,U}) = \Phi_{t+1}^N - (1+i_t^N)\Phi_t^N, \quad (47.b)$$

$$TB_t^A = (e_t^A / e_t^U)(B_{t+1}^{U,A} + L_t^{A,2} b_{t+1}^{UA,2}) - (1+i_t^A)(e_{t-1}^A / e_{t-1}^U)(B_t^{U,A} + L_{t-1}^{A,2} b_t^{UA,2}) = \Phi_{t+1}^A - (1+i_t^A)\Phi_t^A, \quad (47.c)$$

$$TB_t^U = e_t^U B_{t+1}^{N,U} + e_t^U L_t^{U,2} b_{t+1}^{NU,2} - L_t^{A,2} b_{t+1}^{UA,2} - B_{t+1}^{U,A} - (1+i_t^U)[e_{t-1}^U (B_t^{N,U} + L_{t-1}^{U,2} b_t^{NU,2}) - B_t^{U,A} - L_{t-1}^{A,2} b_t^{UA,2}] \\ = \Phi_{t+1}^U - (1+i_t^U)\Phi_t^U. \quad (47.d)$$

Dividing the  $TB_t^i$ 's in equations (47.a)-(47.d) by the respective GDPs, we obtain the trade balance to GDP ratios denoted by  $tb_t^i$  and defined as  $tb_t^S \equiv TB_t^S / GP_t^S, tb_t^N \equiv TB_t^N / GP_t^N, tb_t^A \equiv TB_t^A / GP_t^A, tb_t^U \equiv TB_t^U / GP_t^U$ . The relation of the trade balance to GDP ratios to the corresponding net foreign asset to GDP ratios defined as  $\phi_t^S \equiv \Phi_t^S / GP_t^S, \phi_t^N \equiv \Phi_t^N / GP_t^N, \phi_t^A \equiv \Phi_t^A / GP_t^A, \phi_t^U \equiv \Phi_t^U / GP_t^U$  follows immediately from equations (47.a)-(47.d):

$$tb_t^i = \phi_{t+1}^i G_t^{Gp^i} - (1+i_t^i)\phi_t^i, \quad i = S, N, A, U. \quad (48)$$

Finally, we define aggregate savings of country (region)  $i = S, N, A, U$  as gross national product less public and private consumption as follows:

$$S_t^S \equiv X_t^S + i_t \Phi_t^S - \Gamma_t^S - L_t^{S,1} x_t^{S,1} - L_t^{S,2} x_t^{S,2} - L_t^{S,3} x_t^{S,3} - (1/e_t^A) L_t^{S,1} y_t^{S,1} - (1/e_t^A) L_t^{S,2} y_t^{S,2} - (1/e_t^A) L_t^{S,3} y_t^{S,3} \\ - (1/e_t^U) L_t^{S,1} z_t^{S,1} - (1/e_t^U) L_t^{S,2} z_t^{S,2} - (1/e_t^U) L_t^{S,3} z_t^{S,3}, \quad (49.a)$$

$$S_t^N \equiv X_t^N + i_t \Phi_t^N - \Gamma_t^N - L_t^{N,1} x_t^{N,1} - L_t^{N,2} x_t^{N,2} - L_t^{N,3} x_t^{N,3} - (1/e_t^A) L_t^{N,1} y_t^{N,1} - (1/e_t^A) L_t^{N,2} y_t^{N,2} \\ - (1/e_t^A) L_t^{N,3} y_t^{N,3} - (1/e_t^U) L_t^{N,1} z_t^{N,1} - (1/e_t^U) L_t^{N,2} z_t^{N,2} - (1/e_t^U) L_t^{N,3} z_t^{N,3}, \quad (49.b)$$

$$S_t^A \equiv Y_t + i_t \Phi_t^A - \Gamma_t^A - e_t^A L_t^{A,1} x_t^{A,1} - e_t^A L_t^{A,2} x_t^{A,2} - e_t^A L_t^{A,3} x_t^{A,3} - L_t^{A,1} y_t^{A,1} - L_t^{A,2} y_t^{A,2} \\ - L_t^{A,3} y_t^{A,3} - (e_t^A / e_t^U) L_t^{A,1} z_t^{A,1} - (e_t^A / e_t^U) L_t^{A,2} z_t^{A,2} - (e_t^A / e_t^U) L_t^{A,3} z_t^{A,3}, \quad (49.c)$$

$$S_t^U \equiv Z_t + i_t \Phi_t^U - \Gamma_t^U - e_t^U L_t^{U,1} x_t^{U,1} - e_t^U L_t^{U,2} x_t^{U,2} - e_t^U L_t^{U,3} x_t^{U,3} - (e_t^U / e_t^A) L_t^{U,1} y_t^{U,1} - (e_t^U / e_t^A) L_t^{U,2} y_t^{U,2} \\ - (e_t^U / e_t^A) L_t^{U,3} y_t^{U,3} - L_t^{U,1} z_t^{U,1} - L_t^{U,2} z_t^{U,2} - L_t^{U,3} z_t^{U,3}. \quad (49.d)$$

Upon using again the zero-profit property, the no-arbitrage conditions and the budget constraints of all households it can be shown that  $S_t^i$ ,  $i = S, N, A, U$  can be equivalently written as follows:

$$S_t^i = L_t^{i,1} s_t^{i,1} + L_t^{i,2} s_t^{i,2} - i_t B_t^i - \Gamma_t^i + L_t^{i,2} w_t^i \tau_t^i - L_{t-1}^{i,1} s_{t-1}^{i,1} - L_{t-1}^{i,2} s_{t-1}^{i,2} + \delta K_t^i, i = S, N, A, U, t = 0, 1, 2, \dots \quad (50)$$

By dividing  $S_t^i$  through the respective GDP we obtain the national saving ratios denoted by  $sr_t^i$  and defined as:  $sr_t^S \equiv S_t^S / GP_t^S$ ,  $sr_t^N \equiv S_t^N / GP_t^N$ ,  $sr_t^A \equiv S_t^A / GP_t^A$ ,  $sr_t^U \equiv S_t^U / GP_t^U$ . Inserting the optimal individual saving functions from solutions of the household optimization problems and using the GDP relations above we obtain finally the national saving ratios in terms of model variables.

### Numerical Specification and Model Generated versus Stylized Macro Facts

The numerical specification of our three-country, three-good OLG model with endogenous house price is used to investigate whether a small number of shocks at the start of the new millennium – intra-EMU and Asian-US capital market integration and the rapid growth of Emerging Asia – can generate the decline in the common world interest rate, the emergence of intra-EMU and Asian-US trade imbalances and the divergence of saving rates observed in the data between 2000 and the onset of the global financial crisis. Structural and policy parameters are specified such that the differences in real interest rates between EMU core and periphery and between Asia and USA, their approximately balanced current accounts and zero net foreign asset positions in the 1990s (before euro inception and the East-Asian currency crises) and the international differences in national saving rate levels are replicated by the financial autarky version of the model.

In order to highlight the significance of the interaction between capital market integration and productivity growth we do start from financial autarky steady states but assume internationally unequal productivity growth rates in line with Coeurdacier et al. (2015). As these authors we assume that one period (= 20 calendar years) before financial integration Asia exhibits a significantly higher labor productivity growth rate than the USA, and continues higher productivity two periods (= 40 calendar years) after financial integration. Regarding the EMU, we assume in line with empirical reality of 1990s (Fagan and Gaspar, 2008), that the productivity growth rate of EMU periphery is a little bit higher than that of EMU core - a scenario which continues under EMU financial integration. We assume the following numerical values for the respective national growth factors (yearly growth rates):  $G^S = 1.6$  (2.4% yearly),  $G^N = 1.5$  (2%),  $G_0^A = 2.1$  (3.8%),  $G_1^A = G_2^A = 2.5$  (4.7%),  $G_t^A = 1.6$ ,  $t = 3, \dots$ ,  $G_t^U = 1.6$  (2.4%),  $t = 1, 2, 3, \dots$ .

As mentioned above public debt and government expenditure ratios are assumed to be time-stationary and the magnitudes being in line with 1990s averages as follows:  $\gamma^S = 0.2$ ,  $\gamma^N = 0.22$ ,  $\gamma^A = 0.15$ ,  $\gamma^U = 0.2$ ,  $b^S = 0.025$ ,  $b^N = 0.02$ ,  $b^A = 0.015$ ,  $b^U = 0.02$ . Given the above specified productivity growth factors, the just mentioned policy parameters, the consumption expenditure shares  $\zeta^x = 0.4$ ,  $\zeta^y = 0.3$ ,  $\zeta^z = 0.3$ , the total factor productivity numbers  $M^{X,S} = 11$ ,  $M^{H,S} = 12$ ,  $M^{X,N} = 13$ ,  $M^{H,N} = 14$ ,  $M^{Y,A} = 11$ ,  $M^{H,A} = 12$ ,  $M^{Z,U} = 13$ ,  $M^{H,U} = 14$ , the non-housing

depreciation rates  $\delta^i = 1, i = S, N, A, U$ , the housing depreciation rates  $\delta^{H,i} = 0.1, i = S, N, A, U$  and the following subjective utility discount factors taken from the literature  $\beta^S = 0.7, \beta^N = 0.8, \beta^A = 0.9, \beta^U = 0.35$ , we fix the sectoral capital production shares  $\alpha^{X,i}, \alpha^{H,i} (i = S, N), \alpha^{Y,A}, \alpha^{H,A}, \alpha^{Z,U}, \alpha^{H,U}$  such that the country-specific net foreign asset positions are zero, the associated real interest rates correspond roughly to the 1990s country averages and, most importantly, that the aggregate saving rates accord to 1990s data:  $\alpha^{X,S} = 0.29, \alpha^{H,S} = 0.1, \alpha^{X,N} = 0.26, \alpha^{H,N} = 0.1, \alpha^{Y,A} = 0.32, \alpha^{H,A} = 0.1, \alpha^{Z,U} = 0.2, \alpha^{H,U} = 0.1$ .

For the calibration exercise we assume the following values for the inter- and intra-temporal elasticity in consumption, respectively:  $\sigma = 1, \eta = 1$ . The intertemporal elasticity in consumption is within the range of empirically estimated values but much higher than in Coeurdacier et al. (2015).

Before presenting the results for financial autarky under these parameter specifications it is important to notice that the calculation is performed under the assumption that none of the financial constraints in household optimization problems is binding. The reason for this assumption is that we could not find values for the credit constraint parameters such that the empirically observed values for the real interest rates and net foreign asset position ratios could be replicated by the model solution under financial autarky.

Table 1 reports the steady state values of main endogenous variables in EMU South, EMU North, in Asia and in USA under financial autarky.

**Table 1** Main endogenous variables (as ratio to GDP) in EMU South, EMU North, in Asia and in USA (calculated on a yearly basis) under *financial autarky*

	Capital output ratio	Real interest rate (in %)	House price	Net foreign assets to GDP	EMU terms of trade relative to	Saving rates (in %)	Housing investment rate (in %)	Housing stock ratio (in %)
EMU South	2.51	3.85	0.92	0		20.00	13.00	14.50
EMU North	2.69	3.07	0.98	0		16.70	8.57	17.74
Asia	2.59	4.24	0.92	0	0.62	27.87	10.85	11.93
USA	1.52	4.70	0.90	0	1.33	12.63	11.21	12.20

Source: Own calculations

Note that the yearly real interest rate in EMU South, North, in Asia and in USA exhibited in Table 1 is not too far from the real interest rates in the late 1990s portrayed in Fig. 1 and Fig. 2.

Table 1 shows personal saving rates in EMU South and EMU North in the 1990s rather similar to those portrayed in Figure 3. Although somewhat too high the Asian and US saving rates, exhibited in Table 1 are also rather similar to those portrayed in Figure 4 for the 1990s.

Housing investment ratios featured in Table 1 are roughly in line with those presented in Figures 5 and 6 for the 1990s except for the fact that the US housing investment in the model is higher than the Asian rate. Better calibration could probably remedy this failure to reproduce the data.

Finally, Table 1 presents zero net foreign asset positions in all countries a fact which is not exactly true as Figure 8 for the EMU and Figure 10 for the Asian-US case show.

Given the structural and policy parameters calibrated to the financial autarky situation, the first-period results for main variables under financial integration read as presented in Table 2.

**Table 2** Main endogenous variables in EMU South, EMU North, in Asia and in USA (calculated on a yearly basis) under *financial integration and endogenous house prices* in shock period.

	Capital output ratio	Real interest rate (in %)	Net foreign assets to GDP (%)	Trade balance to GDP ratio (%) (Shock period)	House price	Saving rates (in %)	EMU terms of trade relative to	Housing investment ratio (in %)	Housing stock ratio (in %)
EMU South	2.78	3.37	-69.12	-5.76	0.94	17.62		11.47	15.68
EMU North	2.51	3.37	+12.26	+0.09	0.97	22.00		10.56	16.76
Asia	2.10	5.26	+ 42.35	+4.79	0.86	26.30	0.89	14.53	9.55
USA	1.78	3.86	-69.68	-5.80	0.92	9.31	1.46	11.00	13.32

Source: Own calculation

Starting from the financial autarky solution Table 2 reports the results of financial integration with endogenous house prices and exogenous productivity growth. Comparing the results in Table 1 to those in Table 2 we see that the capital output ratio in EMU South and in the USA increases while it decreases in EMU North and in Asia which parallels the development in housing investment in EMU, Asia and USA, as the comparison of the last columns in Table 1 and 2 shows. The comparison of the housing investment ratios in Table 2 and 1 to the data presented in Figures 5 and 6 reveals some divergences mainly with direction of the changes of the housing investment ratio in the shock period. While in the data the housing investment ratio increased both in EMU South and in US (see Figure 5 and 6), it decreased in the model as the comparison of the results in Table 1 to Table 2 shows. On the other hand, the slight increase of the housing investment ratio in EMU North and in Asia reported in Figures 5 and 6 is replicated by the model albeit on a somewhat too large extent.

The comparison of the interest rate column in Table 1 and Table 2, respectively, reveals that the interest rate in EMU South decreases and in EMU North increases featuring the approximate<sup>5</sup> convergence of real interest rates within EMU reported in Figure 1 above. As Figure 2 shows the Asian real interest rate increases sharply around 2000 and then declines but on average it remains above the autarky level a development which is replicated by our model. The model replicates also the decline of the US real interest rate portrayed in Figure 2 as the comparison of the respective columns in Table 1 and Table 2 shows. The rise of the real interest rate of Asia as a consequence of the much higher productivity growth in Asia than in EMU North is accompanied by a relatively large increase in the EMU terms of trade relative to Asia in line with the real interest parity condition (36) and shown in the terms of trade column in Table 1 and 2, respectively. Regarding the development of the net foreign asset position we see in Table 2 a

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<sup>5</sup> While the convergence of nominal (short-term) interest rates within EMU is perfect (see chart 4 in Fagan and Gaspar, 2008, p. 34), real interest rates did not converge due to higher inflation rates in EMU periphery than in EMU core as mentioned above.

massive worsening of EMU periphery's position and a slight improvement of EMU core's net foreign asset position in line with the development of net foreign asset to GDP ratios reported in Figure 8. Table 2 shows also the pronounced accumulation of Asian net foreign assets, and the deterioration of the US position.<sup>6</sup> Important for the research results reported in this paper are the model generated signs and sizes of the trade imbalance to GDP ratios presented in the fifth column of Table 2 as compared to the empirical values shown in Figure 7 for the EMU and in Figure 9 for Asia and the US. While the right signs of the trade imbalance to GDP ratios can also be replicated in Farmer and Ban's (2014, 2015) three-good, three-country OLG model without house price dynamics, the model generated sizes are generally too small compared to the empirical values. Here the inclusion of house prices and their wealth effects definitively help to improve the empirical fit of the model. However, Table 2 also shows that the model under the present parameter calibration generates too small EMU core trade surpluses. Further research should show whether a better parameter calibration can help to remedy this problem.

Besides the inadequate sizes of model generated trade imbalances the empirically inadequate levels of aggregate saving ratios and their development in the integration period were another driver for the present research. While the inclusion of household credit constraints in Farmer and Mihaiescu (2016) definitively helped to replicate the international differences in saving rates levels, in particular between Asia and the US, it failed to reproduce the pronounced decline in the EMU South and US saving rates exhibited in Figure 3 and Figure 4 above. In line with our first expectations that rising house prices and their positive wealth effect will consumption increase and savings decrease we are now able to report satisfactory results. Not only that internationally diverging housing stock ratios and accompanied house price differences enabled us to calibrate the parameters of the financial autarky variant of the model such that the huge difference in saving rate levels between Asia and US can be replicated. But also, this level difference remains during the financial integration period as the respective column in Table 2 shows. Without over-interpreting the empirical fit of the model, we see in Figure 3 a sizeable drop of the EMU periphery's saving rate and a slight increase in EMU core's saving rate immediately after 2000 developments which are corroborated by the model generated results presented in the saving rate column of Table 2. Similarly, as the comparison of the saving rate column between Table 1 and Table 2 shows, the model is also able to replicate the pronounced decline of the US saving rate between 2000 and 2007 and the slight decline in the Asian saving rate in the after integration period reported in Figure 4.

## **Conclusion**

Farmer and Mihaiescu (2016) investigated in a three-country, three-good OLG model with household credit constraints simultaneously the financial integration and differential productivity growth effects on both intra-EMU and global external imbalances. While the authors were able to

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<sup>6</sup> Table 2 reports a larger negative US net foreign asset position than shown in Figure 10. Gourinchas and Rey (2014) estimate that approximately 20 % of the US net foreign asset position is due to capital gains on its net foreign assets. Eugeni (2015) suggests that even 50% of the US net foreign asset position is due to capital gains on its net foreign assets. Correcting the US net foreign asset position reported in Figure 10 by these percentage points we see that the model generated US net foreign asset position diverges from the empirically observed value not that much.

show how EMU core and Asian trade surpluses as well as EMU periphery and US trade deficits in sign and size can be attributed to intra-EMU and global financial integration, the pronounced decline of EMU periphery's and US saving rates after Euro-related intra-EMU and global financial integration between Asia and US this model was unable to address.

This paper investigates whether Farmer and Mihaiescu's (2016) disappointing saving rate results can be improved by the incorporation of housing into the optimization problems of three-period lived younger households in line with the intertemporal equilibrium housing market models of Arce and López-Salido (2011) and Basco (2014). It finds that the answer to this main question is surprisingly positive, even if the discounted resale value of undepreciated houses need not be used as collateral for household debenture loans, or in other words: if household credit constraints do not bind.

We indeed find first empirically not implausible values for sectoral capital production shares, subjective household discount factors, labor productivity growth factors, public debt and government expenditure to GDP ratios such that the financial autarky variant of the OLG model reproduces roughly empirically observed EMU, Asian and US real interest rates, housing stock to GDP ratios and saving rates for the 1990s.

Commencing from the financial autarky solution for internationally differing capital output ratios and associated real interest rates financial integration together with the rapid-growth related increase in Asia's share of world employment induces a larger EMU periphery and US capital output ratio associated with a lower real interest rate and a smaller EMU core and Asian capital output ratio associated with a larger real interest rate. While the intra-EMU interest rate convergence in the shock period is complete, Asian and US' interest rates remain unequal despite of international real interest parity since Asia's rapid growth pushes up the Asian real interest rate and the EMU terms of trade relative to Asia. The rise of EMU periphery's and US capital output ratios deteriorates the respective net foreign asset positions, while the fall of EMU core's and the Asian capital output ratio improves the respective net foreign asset positions albeit at unequal magnitudes due to internationally differing capital production shares, government expenditure and debt ratios, and housing market parameters.

Due to the existence of a non-tradable good, international differences in subjective discount factors and sectoral production shares together with internationally differing fiscal policy parameters induce saving rates diverging substantially internationally and this implies large trade imbalances. This result clearly improves the performance of the model with housing markets compared to that without. Regarding the dynamic development of saving rate levels the present model is also successful in replicating empirical observations. While the model-generated substantial drop in EMU periphery's and the US saving rate as well as the slight decrease in the Asian saving rate can be perfectly aligned with data, only the rather pronounced model-generated increase in the EMU core saving rate in the after-shock period is at odds with data. This also applies to the direction of the dynamic development of housing investment ratios, particularly with respect to EMU periphery and US ratios. The model generates decreasing rates after financial integration while the data show increasing rates.

It remains to be seen in future research whether a better parameter calibration can alleviate these model-data discrepancies or a further extension of the model through the introduction of house price bubbles is needed.

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