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Credit constraints and differential growth in equilibrium modeling of EMU and global trade imbalances

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Abstract

Increases in the private saving rate in emerging economies and its steady decline in advanced economies, huge external surpluses in the former and similar deficits in the latter and a persistent decline in the world long-term interest rate characterize the evolution of the world economy since Euro-related intra-EMU and global financial integration in the beginning 2000s. While the intra-EMU and global trade imbalances and the interest decline can be explained by means of Farmer and Ban’s (2015) three-country OLG model, the saving rate divergence cannot. Coeurdacier et al. (2015) attribute this divergence to the interaction of household credit constraints and international growth differentials in a two-country OLG model. This paper introduces these novel modeling elements into Farmer and Ban’s (2015) three-country OLG model, and finds that country-specific credit constraints and Asian’s rapid growth enlarge the model-generated trade imbalances and let saving rates decline – to some extent in line with empirical evidence.

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Keywords: Trade Imbalances, European Economic and Monetary Union, Overlapping Generations, Three-Country Model, Global imbalances

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Introduction and Motivation

Coeurdacier et al. (2015) forcefully attribute the following macro trends of the global economy to the integration of emerging markets into worldwide capital markets and their rapid growth, particularly in Asia: (1) a large and persistent increase in the household saving rate in emerging Asia in contrast to its steady decline in advanced economies, particularly in the USA; (2) the emergence of global imbalances with external surpluses in emerging markets and external deficits in advanced economies; and (3) a sustained fall in the world long-term interest rate.

Household saving rates diverged and external imbalances popped up not only between emerging and advanced economies, but also between the so-called core and periphery countries of the European Economic and Monetary Union (EMU) after Euro-related financial integration. While in EMU periphery the household saving rate significantly fell and the trade balances turned into huge deficits, the household saving rate in EMU core slightly rose and its trade balance exhibited surpluses. Moreover, due to its financial integration into the world economy, the common EMU (long-term and) short-term interest declined between 1999 (euro launch) and the onset of the global financial crisis in 2007.

Acknowledging the similarity in the pre-crisis development of the household saving rate and the trade imbalance for US and EMU periphery on the one hand, and for Asia and EMU core on the other hand, it seems to be natural to address the issues of global and intra-EMU saving rate divergence and trade imbalances within a unified framework capable to address also the trade and financial interlinkages and spillovers among EMU, Asia and the USA. To the best of our knowledge, as yet only Farmer and Ban (2015) developed a three-country (EMU, Asia, USA), two-region (EMU core, EMU periphery) intertemporal equilibrium model à la Buiter (1981) which accounts explicitly for the trade linkages between EMU sub-areas and extra-EMU trading partners which were forcefully pointed out by Chen et al. (2013). Global and intra-EMU external imbalances emerge in Farmer and Ban’s three-country, two-region OLG due to real interest rate convergence both within the EMU, and also between Asia and the USA. However, while Farmer and Ban (2015) are able to attribute both global and intra-EMU external imbalances and the world-wide interest rate decline to intra-EMU and global financial integration, these authors could not satisfactorily explain the observed divergence of private saving rates between both Asia and the USA and EMU core and EMU periphery.

Fortunately, Coeurdacier et al. (2015) can explain (to some extent) the global saving rate divergence by the interaction between growth differentials and household credit constraints – more severe in fast-growing economies. Thus, it seems to be tempting to introduce internationally differing household credit and growth rate differentials into the three-country, two-region OLG model in order to replicate the empirically observed intra-EMU and global saving rate divergence. However, Coeurdacier et al.’s (2015) attribution of the global saving rate divergence to
differences in growth rates and in the severity of household credit constraints between advanced and emerging countries cannot analogously applied to the explanation of intra-EMU external imbalances. After euro inception and before the onset of the global financial crisis, differences in both household credit constraints and GDP growth rates (Fagan and Gaspar, 2008) were much less pronounced between EMU core and EMU periphery than between emerging countries (Asia) and advanced countries (USA). Moreover, while at the global level financial capital flow upstream from emerging Asia to the advanced USA, financial capital within the EMU flows downstream from EMU North to EMU South – in line with neo-classical international growth theory (Lane and McQuade, 2014). Finally, as an important institutional fact, outside the EMU no common currency exists, and thus significant international growth differentials between Asia and EMU and the USA will affect the terms of trade between these large currency areas. The one-good model of the world economy in Coeurdacier et al. (2015) or Eugeni (2015) cannot address the terms of trade impacts of differential growth and financial integration. In contrast, Farmer and Ban’s (2015) three-good model with intra-temporal trade between EMU, Asia and USA is capable to investigate these terms of trade effects.

In order to be able to replicate in this three-good model the intra-EMU and Asian-US trade imbalances, interest rate convergence and saving rate divergence before and under financial integration we assume in this paper in line with Coeurdacier et al. (2015) three-period instead of two-period lived overlapping generations, subjected to credit constraints on the future discounted wage income, and international differences in exogenous productivity growth rates. This extended three-good, three-country, two-region OLG model will be used to see whether the empirically observed trade imbalances, the decline of the world interest rate and of EMU periphery’s and US saving rates can be better replicated than in the model without credit constraints.

In order to motivate the set-up of the extended model we recall key stylized facts regarding intra-EMU and global trade imbalances between the late 1990s and the onset of the global financial crisis in 2007. First, while euro-related financial integration and convergence expectations (Blanchard and Giavazzi, 2002; Lane, 2006; Japelli and Pagano, 2008; Spiegel, 2009; Kalemli-Ozcan et al., 2010, Schmitz and von Hagen, 2011) contributed without doubt to the evolution of intra-EMU trade imbalances, Chen et al. (2013) emphasize extra-EMU factors as (1) the increasing competitive advantage of Asian exporters vis-à-vis EMU periphery exporters, (2) the rising demand for EMU core capital goods by Asia and oil exporters and (3) the US demand for EMU core financial assets. As a consequence, EMU periphery’s current account deficit, while financed mostly by capital inflows from the EMU core, did not increase that much vis-à-vis the core but vis-à-vis Asia and oil exporters. Similarly, EMU core’s current account surplus after the Euro launch resulted not from EMU periphery’s imports but from rising Asian imports of EMU core (capital) goods.
As mentioned above, financial integration occurred not only in the EMU after Euro inception but also between Asia and the USA – albeit under different institutional ramifications than those existing in the Eurozone. The interest convergence between initially higher US and lower Asian rates is seen as a major driver for the emergence of US current account deficits and Asian current account surpluses (Mendoza et al. 2009; Angeletos and Panousi, 2009). As a consequence of high Asian saving rates due to credit-constrained younger households and high GDP growth rates, Asians accumulated huge amounts of save US government bonds inducing the convergence of nominal and real Asian and US interest rates.

In line with Buiter’s (1981) seminal one-good, two-country OLG model we depict the pre-Euro and pre-East-Asian currency crisis situation with relatively larger real interest rates in EMU periphery and in the USA as financial autarky and the convergence of (real) interest rates within the Eurozone and between US and Asia after the 1998 currency crisis as financial integration. The higher EMU periphery’s and US autarky interest rates are attributed to EMU periphery’s and US relatively higher future (utility) discount factors, to EMU periphery’s and Asia’s relatively higher capital production shares, to EMU periphery’s and US relatively larger public debt to GDP’s ratios and their less stringent household credit constraints.

The paper is organized as follows. The next section presents key stylized international macroeconomic facts characterizing both the financial autarky situation of 1980s and 1990s and the financial integration period between 1999 and 2008. Then, the financial autarky variant of the three-country, two-region OLG model with credit constraints for youngest households in all countries is presented. Thereafter we show the household budget constraints and the market clearing conditions under financial integration, derive the intertemporal equilibrium dynamics and investigate how key international macro indicators as the net foreign asset position, the trade imbalance and the aggregate saving rate are related to the main variables of the intertemporal equilibrium dynamics. In the following section our model economy is numerically specified for the financial autarky case, and the numerical solution for net foreign asset positions, trade imbalances and aggregate saving rates in the integration period is compared to main stylized macroeconomic facts, related to EMU and Asia-US financial integration up to the onset of the global financial crisis in 2007. Concluding remarks in the final section summarize key results.

Stylized Macroeconomic Facts: Financial Autarky versus Financial Integration

In order to illuminate the design of the three-good, three-country and two-region OLG model, some of the relevant stylized facts are now presented in this section. These relate to the macroeconomic performance of the EMU members, Asia and the USA, and the evolution of the trade balance and the net foreign asset positions in the EMU, Asia and the USA before the launch of the euro in 1999 and up to 2008. Following Fagan and Gaspar (2008, p. 9), the EMU countries are separated into two groups based on the differences in short-term real interest rates in the late
1990s, i.e. before the euro launch. The first group, usually denoted as the “core” countries, comprises the low interest rate countries Austria, Belgium, France, Germany and the Netherlands.\textsuperscript{2} The second group, denoted as “periphery” or converging countries, consists of Ireland, Italy, Spain and Portugal which had relatively high interest rates before the introduction of the euro (see Fig. 1).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1}
\caption{Real short-term interest rates in EMU core and periphery 1990-2008. Source: AMECO}
\end{figure}

Figure 1 reveals that in contrast to the pre-EMU situation (before 1999), EMU periphery’s higher real interest rates decreased towards lower rates in EMU core.\textsuperscript{3}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2}
\caption{Real short-term interest rates in USA and Asia 1990-2008. Source: World Bank: World Development Indicators}
\end{figure}

Figure 2 shows a similar convergence of higher US short term real interest rates in the late 1990s towards the lower Asian\textsuperscript{4} rates, following the 1990s East-Asian currency crises (= at the time of euro inception).

\textsuperscript{2} Nowadays Finland is included within core countries. In line with Fagan and Gaspar (2008) we exclude Finland from core countries since in the 1990s the Finnish economy was distorted by special factors after the collapse of the Soviet Union.

\textsuperscript{3} Remaining differences in the real interest rates are due to inflation rate differences across EMU core and periphery.

\textsuperscript{4} The Asian magnitudes result from the unweighted averages of Chinese, Indian, Japanese, Hong Kong and South Korean magnitudes. All remaining Asian macro variables are comprised by these country values.
Fig. 3 Personal saving rates in EMU core and periphery 1990-2008. Source: AMECO.

Figure 3 portrays for the pre-euro period of the 1990s on average rather similar personal saving ratios (rates) (= gross household savings as percent of gross disposable income) for EMU core and EMU periphery, and a significant decline of the saving rate in the EMU periphery after the euro launch. In contrast, Figure 4 reveals that the US personal saving rate is persistently, substantially lower than the Asian rate, both in the 1990s and 2000s.

Fig. 4 Asian and US personal saving rates 1990-2008. Source: Source: China Statistical Yearbook Database, CEIC, AMECO, Economic Statistics System, Bank of Korea, FRED.

Fig. 5 Housing investment (as percent of GDP) in EMU periphery and core 1990-2008. Source: AMECO

Figures 5 and 6 portray housing investment (as percent of GDP) in EMU periphery and core, and in Asia and the USA, respectively. While housing investment rose significantly in the EMU periphery and in the USA, it declined in the EMU core and in Asia.
Fig. 6 Housing investment (as percent of GDP) in Asia and in USA 1990-2008. Source: BEA National Economic Accounts

Through the sharp increase in private domestic expenditures in the periphery, and the muted response of output (Fagan and Gaspar 2008), trade balances in the periphery significantly deteriorated (see Fig. 7).

Fig. 7: Trade balance on goods and services (as percent of GDP) in EMU core and periphery 1990-2012. Source: World Bank Indicators.

Not surprisingly, EMU periphery’s trade balance deficits led to the accumulation of a significant net foreign debtor position. See Figure 8 below.

Fig. 8: Net foreign assets (as percent of GDP) in EMU core and periphery 1990-2008. Source: updated and extended version of a dataset constructed by Lane and Milesi-Ferretti (2007)

The differences with respect to the evolution of trade balance and net foreign asset positions in Asia and the USA are again similar to those in Europe. See figures 9 and 10.
Consider an infinite-horizon model economy consisting of three areas (“countries”) of the world economy, namely (i) the EMU, comprising two regions, named North (indexed by $N$) representing EMU’s core, and South (indexed by $S$), representing EMU’s periphery countries, (ii) the countries characterized by a current account surplus outside the EMU (indexed by $A$) representing Asia, and (iii) the current-account deficit countries (indexed by $U$) representing mainly the USA. In each country one commodity, comprising the aggregate of thousands of goods and services is produced. This can be used for the purpose of consumption as well as for investment. The EMU specializes completely in the production of good $X$, Asia in the production of good $Y$, and US in the production of good $Z$. Perfectly competitive firms in EMU’s South and North, in Asia and in the USA employ in every period $t=1,2,...$ labor services $N_i^t$, $i=S,N,A,U$ and capital services $K_i^t$, $i=S,N,A,U$, using the Cobb-Douglas (CD) production function

\[ M^i(a_i^t N_i^t)^{1-a'} (K_i^t)^{a'}, i = S,N,A,U, \]

to produce southern (northern) EMU aggregate output $X_i^S (X_i^N)$, Asia’s aggregate output $Y_i$ and US aggregate output $Z_i$ where $M^i > 0$, $i = S, N, A, U$ denote total factor productivity in EMU’s South (North), in Asia and in the USA, respectively. $a_i^t$ is the country-specific labor productivity and $0 < a' < 1$, $i = S, N, A, U$ with $\alpha^V = \alpha^N < \alpha^S < \alpha^A$ are the capital production shares in EMU South, EMU North, Asia and in the USA.
One-period profit maximization by firms in EMU’s South (North), in Asia and in the USA implies the following FOCs:

\[ w_t^i = (1 - \alpha^t)M^i a_t^i (K_t^i / a_t^i, N_t^i)^{d_t^i}, i = S, N, A, U, \]

\[ q_t^i = \alpha^t M^i (K_t^i / a_t^i, N_t^i)^{d_t^{i-1}}, i = S, N, A, U, \]

whereby \( w_t^i, i = S, N, A, U \) denotes the real wage rate in each region and each country. \( q_t^i, i = S, N, A, U \) denotes real unit capital user costs in each region and each country \( i = S, N, A, U \).

Following Japelli and Papano (1994), three generations of homogeneous individuals overlap in each period \( t \). At date \( t \), a new generation of size \( L_t^{1,3} \) enters the economy of country (region) \( i = S, N, A, U \). For simplicity we assume that the population growth factors of all countries (regions) are identical and are constant over time equal to \( G^L \). On the other hand, empirical (productivity) GDP growth rates for 1990s and 2000s differ both across intra-EMU regions and between Asia and USA, the former lesser than the latter. Thus, we assume in line with Fagan and Gaspar (2008) that the southern EMU growth factor of labor productivity \( G^S \) is equal to the corresponding northern productivity growth factor \( G^N \). As a consequence, the empirically higher southern EMU GDP growth rate is attributed to catch-up growth. While this modeling assumption also applies to Asia, we assume in addition that the substantially larger Asian GDP growth rates in the data are also due to a higher productivity growth factor \( G_t^A \) than in USA and in EMU.

Each generation lives for three periods, borrowing as young-aged in the first period, working during the second period when middle-aged, and retiring in the third when old. The choice variables of each generation, when young, are denoted by superscript 1, when middle-aged, they are denoted by superscript 2, and when old, they are depicted by superscript 3. For each member of the middle-aged generation, the supply of labor to firms is wage-inelastic since households attribute no value to leisure.

In order to describe the optimization problems of households more specifically the institutional framework regarding international transactions across the three countries and across EMU core and periphery is now addressed. Regarding the three countries, we assume that each country has its own currency and that before the inception of the EMU, the southern and northern EMU member countries had their own currency, too. To mimic the period before the introduction of the common currency in 1999 we follow Gourinchas and Jeanne (2006) as well as Fagan and Gaspar (2008), and assume that before 1999, EMU’s South and North were financially autarkic. In contrast to the de-facto financial relationships between subsequent EMU countries, Asia and the USA which existed before euro inception, we also assume financial autarky for Asia and the USA in the pre-euro period. In difference to financial autarky, we do, however, allow for trade relations between later EMU, Asia and US during the pre-euro period, albeit on a moderate and balanced scale, thus mimicking the fact of mainly Japanese trade linkages vis-à-vis later EMU.
countries and the US. China and India did not play any important role in international trade during the pre-euro period.

Complete nominal, and to a lesser extent, real interest convergence across EMU’s South and North after the euro launch signifies financial integration across EMU’s South and North. This stylized fact is portrayed in our intertemporal equilibrium model in line with Fagan and Gaspar (2008) as an equality of real interest rates of southern and northern EMU countries along the intertemporal equilibrium path. While by no means as complete as that within EMU, there is also some real interest convergence or financial integration across Asia and USA in the early 2000s. We take this stylized fact as support for our rather strong modeling assumption that after the euro launch an uncovered parity condition, in terms of real interest rates, holds across both Asia and the USA. In line with the empirical fact (Chen et al. 2013) that investors from outside EMU invested their wealth in northern EMU financial assets we also assume an uncovered real interest parity condition between USA and EMU. In other words: after euro inception financial integration prevails worldwide but not as strictly as within the EMU.

In order to work out the consequences of intra-EMU, Asian and US financial integration and the trade developments of EMU vis-à-vis non-EMU countries as clearly as possible, the optimization problems of (younger) households and firms as well as the market clearing conditions are first described for financial autarky and then for intra-EMU, Asian and US financial integration.

Pre-Euro and Asian-US Financial Autarky
In order to facilitate the modeling of the pre-euro situation as financial autarky, we first recall that before euro launch southern EMU real interest rates were sizeable larger than the corresponding northern rates (see Figure 1 above). Second, in the 1990s EMU South (with the exception of Portugal) did not run large current account deficits (as ratio of GDP). Hence, our modeling assumes that in the pre-euro period both the trade balance and the net foreign asset position of EMU South and North were zero. In contrast, in the 1990s Asia (including oil exporters) ran current account surpluses (as percent of GDP) roughly equivalent in size to the current account deficit of the USA (Engler 2009, p. 2). However, since at this time the US net foreign asset position was only moderately negative and China and other emerging Asian countries did not contribute much to the imbalance, we assume that the USA and Asia were financially autarkic, just as the later EMU South and North were. Third, in contrast to the tremendous post-crisis accumulation of public debt, particularly in southern EMU, in Japan and in the USA, in the 1990s and 2000s (up to 2008), the average debt to GDP ratios for EMU periphery, EMU core and the USA remained constant over time or even receded slightly. In Asia (with the exception of Japan) public debt to GDP ratios decreased, and remained far below the EMU and US ratios. We also assume that the (un-weighted) average of government debt to GDP ratios in Asia (including
Japan) remains constant over time. Additionally, as Figure 3 above shows, after 1993 the personal saving rate in EMU South was lower than in EMU North. From Figure 4 we know that Asia’s personal saving rate is significantly higher than the corresponding northern EMU rate, while the US personal saving rate is slightly below the southern EMU personal saving rate (see Figures 3 and 4).

In line with Chen et al. (2013) we accept the stylized fact of merchandise trade between EMU’s North and South being relatively modest after euro launch, and we assume that there is no intra-temporal trade between EMU South and North in the model economy before and after euro launch. For the sake of simplicity we assume that the same composite commodity is produced in North and South. Thus, while younger households in EMU South (North) cannot choose between consumption of the domestic and of the northern (southern) commodity they can buy Asian and US goods in addition to the domestically produced good even before the euro launch.

Against this empirical background of stylized facts and simplifying modeling assumptions the intertemporal utility maximization problem in later EMU’s South before euro inception (= financial autarky) reads as follows:

\[
\max_{x, y, z, S, t} \left\{ \left[ \left( \zeta^x \right)^{\frac{1}{\eta}} \left( x_{t+1}^{S,1} \right)^{\eta-1} + \left( \zeta^y \right)^{\frac{1}{\eta}} \left( y_{t+1}^{S,1} \right)^{\eta-1} + \left( \zeta^z \right)^{\frac{1}{\eta}} \left( z_{t+1}^{S,1} \right)^{\eta-1} \right]^{\frac{\sigma}{\eta}} \right\} - 1
\]

\[
+ \frac{\beta^S}{\sigma - 1} \frac{\sigma - 1}{\sigma}
\]

\[
+ \left( \beta^S \right)^2 \frac{\sigma - 1}{\sigma}
\]

s. t.:
\begin{align}
(i) \quad x_{t+1}^{S,1} + (1/e_t^{U}) y_{t+1}^{S,1} + (1/e_t^{U}) z_{t+1}^{S,1} = b_{t+1}^{S,1}, \quad b_{t+1}^{S,1} \leq \theta^S w_{t+1}^S + 1, \\
(ii) \quad x_{t+1}^{S,2} + y_{t+1}^{S,2} + z_{t+1}^{S,2} = w_{t+1}^S (1 - \tau_{t+1}^S) - (1 + \tau_{t+1}^S) b_{t+1}^{S,1}, \\
\text{with} \quad s_{t+1}^{S,2} = [(1 - \delta)(1 - \delta)K_{t+1}^{S,2}/L_{t+1}^{S,2}] + (B_{t+1}^{S,1}/L_{t+1}^{S,1}) + b_{t+2}^{S,2}, \\
(iii) \quad x_{t+2}^{S,2} + y_{t+2}^{S,2} + z_{t+2}^{S,2} = (1 - \delta)(1 - \delta)K_{t+1}^{S,2} + \frac{1 + i_{t+2}^S}{L_{t+1}^{S,2}} + \frac{1 + i_{t+2}^S}{L_{t+1}^{S,1}} + (1 + i_{t+2}^S) b_{t+2}^{S,2}.
\end{align}

Here \( 0 < \beta^S \leq 1 \) denotes the time discount factor of (later) EMU’s southern younger generation, \( \sigma \leq 1 \) is the elasticity of intertemporal substitution in consumption, \( 0 < \eta < 1 \) denotes the elasticity of intra-temporal substitution in consumption of the domestic and foreign goods, \( \zeta_k \), \( k = x, y, z \) with \( \zeta^x + \zeta^y + \zeta^z = 1 \) represents the utility elasticity of the consumption of good \( k \), \( x_{t+1}^{S,1} \) is the consumption per capita of the commodity produced in EMU’s South acquired at unit relative price, \( y_{t+1}^{S,1} \) is South’s consumption of the Asian good bought at the relative price of \( 1/e_t^{U} \), and \( z_{t+1}^{S,1} \) is southern consumption of the US good acquired at the relative price of \( 1/e_t^{U} \). \( e_t^{U} \) denotes the units of the Asian good per unit of EMU good (= EMU terms of trade vis-à-vis Asia), while \( e_t^{U} \) portrays the units of the US good per unit of EMU good (=EMU terms of trade vis-à-vis USA). Since southern younger household do not work when entering the economy, first-period consumption expenditures are covered by the revenues from private bond selling, \( b_{t+1}^{S,1} \), which ought to be repaid in the second period including the interest rate on southern private bonds, \( i_{t+1}^S \) (see budget constraint (ii)). Second-period consumption expenditures and second-period savings \( s_{t+2}^{S,2} \) are to be equal to net wage \( w_{t+1}^S (1 - \tau_{t+1}^S) \) minus private credit repayments where \( \tau_{t+1}^S \) denotes the flat wage tax rate in period \( t \). Southern younger household’s borrowing in the first period is constrained by a fraction \( \theta^S \) of the present value of her future labor income. In line with Coeurdacier et al. (2015) credit constraints are assumed to be binding. Southern middle-age savings \( s_{t+1}^{S,2} \) are used to buy southern real capital per capita \( K_{t+1}^{S,2}/L_{t+1}^{S,2} \), southern government bonds per capita \( B_{t+1}^{S,1}/L_{t+1}^{S,1} \), and southern private bonds \( b_{t+1}^{S,2} \). Old-age consumption expenditures are financed by the revenues from renting real capital services to southern firms, \( q_{t+1}^S K_{t+1}^{S,2}/L_{t+1}^{S,2} \), and from repaid public and private bonds (inclusive of interest) \( (1 + i_{t+1}^S) B_{t+1}^{S,1}/L_{t+1}^{S,1} + (1 + i_{t+1}^S) b_{t+1}^{S,2} \). In line with pre-financial crisis experience in EMU South the interest rates on public and private bonds are assumed to be equal. Constraint (ii) depicts the working period budget constraint while constraint (iii) represents the retirement period budget constraint.

In order to solve the intertemporal utility maximization problem of southern young household, we define southern real consumption per capita in period \( t + \tau, \tau = 0,1,2 \) as
\[
c^S_{t+\tau} \equiv (\zeta^x)^{1/q}(x_{t+\tau}^{S,1})^{(q-1)/q} + (\zeta^y)^{1/q}(y_{t+\tau}^{S,1})^{(q-1)/q} + (\zeta^z)^{1/q}(z_{t+\tau}^{S,1})^{(q-1)/q} w_{t+\tau}^{S,1} \quad \text{and calculate the expenditure-minimizing price indices of these consumption bundles denoted by } \pi_{t+\tau}^S, \tau = 0,1,2 \text{ as follows: } \pi_{t+\tau}^S = (\zeta^x + \zeta^y + \zeta^z)^{1/q} (e_t^{U})^{-1} \quad \text{and } \tau = 0,1,2. \text{ Using these definitions we find the following optimal saving and domestic and foreign consumption relations for the southern younger household:}
\]
\[
s_{t+1}^{S,1} = -b_{t+1}^{S,1} = -\theta^S w_{t+1}^S / (1 + i_{t+1}^S), \quad s_{t+1}^{S,2} = w_{t+1}^S (1 - \tau_{t+1}^S - \theta^S) [1 + (\beta^S)\pi_{t+\tau}^S / \pi_{t+1}^S] \quad \text{and } \tau = 0,1,2. \quad (3)
\]
In order to save on space the intertemporal optimization calculus of northern EMU younger household is not explicitly stated since it is sufficient to substitute the region index $S$ for $N$.

The same holds true mutatis mutandis for the intertemporal utility function of the typical Asian younger household. Since, however, the budget constraints of the Asian younger household differ from the budget constraints of EMU households, it is in order to state them explicitly as follows:

\[
\begin{align*}
(i) \quad & x_{t+1}^A + y_{t+1}^A + (e_t^A/e_t^U)z_{t+1}^A = b_{t+1}, \quad b_{t+1} \leq \theta_t \frac{w_{t+1}^A}{1+i_t^A}, \\
(ii) \quad & x_{t+1}^A + y_{t+1}^A + (e_t^A/e_t^U)z_{t+1}^A + s_{t+1}^A = w_{t+1}(1-\tau_{t+1})-(1+i_t^A)b_{t+1}, \\
& \text{with } s_{t+1}^A = [(I_{t+1} + (1-\delta)K_{t+1})/L_{t+1}] + B_{t+1}/L_{t+2} + d_{t+1}^A, \\
(iii) \quad & x_{t+1}^A + y_{t+1}^A + (e_t^A/e_t^U)z_{t+1}^A = (d_{t+1} + 1-\delta)[(I_{t+1} + (1-\delta)K_{t+1})/L_{t+1}] \\
& + (1+i_t^A)(B_{t+1}/L_{t+2}) + (1+i_t^A)b_{t+1}.
\end{align*}
\]

Here, $x_{t+1}^A$ stands for the purchases (= consumption) of later EMU goods by the Asian young household at the relative price of $e_t^A$, while the purchase of the US product by the Asian young household occurs at the relative price $e_t^U/e_t^A$, i.e. units of the Asian product per unit of the US good. All other variables may be interpreted similarly to those in EMU South’s young household optimization problem.

Analogously to above we define Asian real consumption per capita in period $t+\tau$, $\tau = 0,1,2$ as

\[
\begin{align*}
& c_{t+1}^A = \left[\left(x_{t+1}^A/z_{t+1}^A\right)^{1-q} \left(x_{t+1}^A/z_{t+1}^A\right)^{q-1}\right]^{q/1-q} + \left[\left(y_{t+1}^A/z_{t+1}^A\right)^{1-q} \left(y_{t+1}^A/z_{t+1}^A\right)^{q-1}\right]^{q/1-q} + \left[\left(z_{t+1}^A/z_{t+1}^A\right)^{1-q} \left(z_{t+1}^A/z_{t+1}^A\right)^{q-1}\right]^{q/1-q}, \\
& \text{and calculate the expenditure-minimizing price indices of these consumption bundles denoted by } \pi_{t+\tau}^A, \tau = 0,1,2 \text{ as follows: } \pi_{t+\tau}^A = \left[\frac{\zeta^A(e_t^A/e_t^U)^{1-q} + \zeta^Y(e_t^A/e_t^U)^{1-q}}{\zeta^X(e_t^A/e_t^U)^{1-q}}\right]^{1/(1-q)}, \tau = 0,1,2.
\end{align*}
\]

Using these definitions we find the following optimal saving and domestic and foreign consumption relations for the Asian younger household:

\[
\begin{align*}
& s_{t+1}^A = -b_{t+1} = -\Theta^A w_{t+1}/(1+i_t^A), \quad s_{t+1}^A = w_{t+1}(1-\tau_{t+1}) - \Theta^A[1+(\beta^A)^{1-\sigma}(\pi_{t+1}^A/\pi_{t+2}^A)^{1-\sigma}(1+i_t^A)^{1-\sigma}]^{-1}, \\
x_{t+1}^A = \zeta^A(e_t^A/e_t^U)^q \pi_{t+\tau}^A c_{t+\tau}^A, \quad y_{t+1}^A = \zeta^Y (\pi_{t+\tau}^A)^{q-1} \pi_{t+\tau}^A c_{t+\tau}^A, \quad z_{t+1}^A = \zeta^Z (e_t^A/e_t^U)^q (\pi_{t+\tau}^A)^q \pi_{t+\tau}^A c_{t+\tau}^A, \\
& \text{with } \pi_{t+\tau}^A e_t^A = \Theta^A w_{t+1}/(1+i_t^A), \quad \pi_{t+\tau}^A e_t^A = \frac{w_{t+1}(1-\tau_{t+1}) - \Theta^A(\beta^A)^{1-\sigma}(\pi_{t+1}^A/\pi_{t+2}^A)^{1-\sigma}(1+i_t^A)^{1-\sigma}}{1+(\beta^A)^{1-\sigma}(1+i_t^A)^{1-\sigma}(\pi_{t+1}^A/\pi_{t+2}^A)^{1-\sigma}}.
\end{align*}
\]

Finally, the typical US younger household faces the following budget constraints:
Again, we define US real consumption per capita in period \( t + \tau, \tau = 0,1,2 \) as

\[
e^{U}_{t+\tau} x^{U,1}_{t+\tau} + \left( e^{U}_{t+1}/e^{A}_{t+1} \right) y^{U,1}_{t+\tau} + \zeta^{U,1}_{t+\tau} = b^{U,1}_{t+\tau}, \quad b^{U,1}_{t+\tau} = \theta^{U}_{t+\tau} \frac{w^{U}_{t+\tau}}{1 + i^{U}_{t+\tau}},
\]

\[
(e^{U}_{t+1} x^{U,2}_{t+1} + \left( e^{U}_{t+1}/e^{A}_{t+1} \right) y^{U,2}_{t+1} + \zeta^{U,2}_{t+1} + s^{U,2}_{t+1} = w^{U}_{t+1}(1 - \tau^{U}_{t+1}) - (1 + i^{U}_{t+1})b^{U,1}_{t+1},
\]

with \( s^{U,2}_{t+1} = [(I^{U,1}_{t+1} + (1 - \delta)K^{U,1}_{t+1})/L^{U,2}_{t+1}] + B^{U,2}_{t+1}/L^{U,2}_{t+1} + b^{U,2}_{t+1}, \)

\[
(e^{U}_{t+1} x^{U,3}_{t+2} + \left( e^{U}_{t+2}/e^{A}_{t+2} \right) y^{U,3}_{t+2} + \zeta^{U,3}_{t+2} = (q^{U}_{t+2} + 1 - \delta)[(I^{U,1}_{t+1} + (1 - \delta)K^{U,1}_{t+1})/L^{U,2}_{t+1}]
\]

\[
= (1 + i^{U}_{t+2})B^{U,2}_{t+2}/L^{U,2}_{t+2} + (1 + i^{U}_{t+2})b^{U,2}_{t+2}.
\]

The government of each country (region) \( i = S, N, A, U \) taxes labor income and uses the revenues from additional borrowing to finance the interest costs on existing government debt and government expenditures. The government budget constraint of country (region) \( i \) reads as follows:

\[
\Gamma^{i}_{t} = B^{i}_{t+1} + \tau^{i}_{t}w^{i}_{t}L^{2}_{t} = i^{i}_{t}B^{i}_{t} + \Gamma^{i}_{t}, \quad i = S, N, A, U, \ t = 0,1,2,...,
\]

where \( \Gamma^{i}_{t} \) denotes real government expenditures and \( B^{i}_{t} \) is the level of real government debt in country (region) \( i = S, N, A, U \) at the beginning of period \( t \). In line with Diamond (1965), we assume that government expenditures are unproductive.

In addition to the restrictions imposed by household and firm optimization and by the above government budget constraints, markets for labor have to clear in all countries (regions) and in all periods.

\[
N^{i}_{t} = L^{2}_{t}, \quad i = S, N, A, U, \ t = 0,1,2,...
\]

Since the asset markets are competitive, transaction and adjustment costs do not occur, no risk (aversion) prevails, the following no-arbitrage condition (= national Fisher equation) holds in all countries (regions):

\[
1 + i^{U}_{t+1} = q^{U}_{t+1} + 1 - \delta, \quad i = S, N, A, U, \ t = 0,1,2,...,
\]

whereby \( 0 < \delta \leq 1 \) depicts the common fixed depreciation rate of private capital (period by period) in country (region) \( i \).

The asset market clearing conditions in all countries (regions) read as follows:
Finally, the following clearing conditions for the product markets hold:

\[
X^S_i + X^N_i = L^{i,1}_{x^1} + L^{i,2}_{x^2} + L^{i,3}_{x^3} + \Gamma^S_i (K^S_{i+1} - (1 - \delta) K^S_i) + L^{i,1}_{y^1} + L^{i,2}_{y^2} + L^{i,3}_{y^3} + \Gamma^S_i, \quad t = 1, 2, \ldots
\]  

\[
Y_i = L^{i,1}_{y^1} + L^{i,2}_{y^2} + L^{i,3}_{y^3} + \Gamma^S_i (K^S_{i+1} - (1 - \delta) K^S_i) + L^{i,1}_{z^1} + L^{i,2}_{z^2} + L^{i,3}_{z^3} + \Gamma^S_i, \quad t = 1, 2, \ldots
\]

\[
Z_i = L^{i,1}_{z^1} + L^{i,2}_{z^2} + L^{i,3}_{z^3} + \Gamma^S_i (K^S_{i+1} - (1 - \delta) K^S_i) + L^{i,1}_{w^1} + L^{i,2}_{w^2} + L^{i,3}_{w^3} + \Gamma^S_i, \quad t = 1, 2, \ldots
\]

In order to be able to model the fact of time-stationarity of country (region) \( i \)'s public debt to GDP ratios between 1999 and 2008 we transform total outstanding government debt in country (region) \( i \)'s government budget constraint into debt to GDP ratios. This is achieved by dividing both sides of (3) by \( X_i \), by \( Y_i \), for \( i = A \), by \( Z_i \), for \( i = U \) and by defining the debt to GDP ratios as:

\[
G^X_i b^1_{i+1} = (1 + i^X) b^1_i + \gamma^i - \tau^i (1 - \alpha^X), \quad G^X_i X^i = X^{i+1}_i / X^i, \quad \Gamma^X_i = \Gamma^X_i / X^i, \quad w^i_i L^i_i / X^i = 1 - \alpha^X, \quad i = S, N
\]

\[
G^Y_i b^1_i = (1 + i^Y) b^1_i + \gamma^Y - \tau^Y (1 - \alpha^Y), \quad G^Y_i Y^i = Y^{i+1}_i / Y^i, \quad \Gamma^Y_i = \Gamma^Y_i / Y^i, \quad w^i_i L^i_i / Y^i = 1 - \alpha^Y
\]

\[
G^Z_i b^1_i = (1 + i^Z) b^1_i + \gamma^Z - \tau^Z (1 - \alpha^Z), \quad G^Z_i Z^i = Z^{i+1}_i / Z^i, \quad \Gamma^Z_i = \Gamma^Z_i / Z^i, \quad w^i_i L^i_i / Z^i = 1 - \alpha^Z
\]

Dividing the asset market clearing condition (12) on both sides by \( X^i \), \( i = S, N \), and using the definition of the capital output ratio \( v^i_i \equiv K^i_i / X^i \), \( i = S, N \), (12) can be rewritten as follows:

\[
G^X_i b^1_{i+1} + G^X_i v^i_{i+1} = \frac{-\theta^X (1 - \alpha^X) G^X_i (v^i_{i+1})^{\alpha^X(1-\alpha^X)}}{(1 - \alpha^X + \alpha^X / v^i_{i+1}) (v^i_{i+1})^{\alpha^X(1-\alpha^X)}} (1 - \alpha^X)(1 - \tau^X - \theta^X) + \frac{1 + (\beta^X)^{-\sigma} (\pi^X_i / \pi^X_{i+1})^{\sigma X} (1 - \delta + \alpha^X / v^i_{i+1})^{-\sigma X}}{i = S, N, \sigma X}
\]  

\[
G^Y_i b^1_i + G^Y_i v^i_{i+1} = \frac{-\theta^Y (1 - \alpha^Y) G^Y_i (v^i_{i+1})^{\alpha^Y(1-\alpha^Y)}}{(1 - \alpha^Y + \alpha^Y / v^i_{i+1}) (v^i_{i+1})^{\alpha^Y(1-\alpha^Y)}} (1 - \alpha^Y)(1 - \tau^Y - \theta^Y) + \frac{1 + (\beta^Y)^{-\sigma} (\pi^Y_i / \pi^Y_{i+1})^{\sigma Y} (1 - \delta + \alpha^Y / v^i_{i+1})^{-\sigma Y}}{i = S, N, \sigma Y}
\]
\[ G_t^Z b_{t+1}^U + G_t^Z v_{t+1}^U = (L_t^{U,2} s_{t+1}^{U,2} + L_t^{U,1} s_{t+1}^{U,1})/Z_t = \frac{-\delta^U(1 - \alpha^U) G_t^U (v_{t+1}^U)^{d^U(1-d^U)}}{(1-\delta+\alpha^U/v_{t+1}^U)(v_{t+1}^U)^{d^U(1-d^U)}} \]

\[ + \frac{(1 - \tau^U - \theta^U)}{[1 + (\beta^U) -\sigma (\tau^U / \pi^U)^r(1-\delta + \alpha^U/v_{t+1}^U)^{-\sigma}]} \]. 

\[ (21c) \]

In view of the C-D production function, and noting \[ (a_i^{L,2}/a_i^{L,3})/(K_i^{1,i}/a_i^{L,2})d^i/(K_i^{1,i}/a_i^{L,2})^d, i = S, N ; \]

\[ (a_i^{L,2}/a_i^{L,3})/(K_i^{1,i}/a_i^{L,2})d^i/(K_i^{1,i}/a_i^{L,2})^d, G_t^I = (K_i^{1,i}/a_i^{L,2})d^i/(K_i^{1,i}/a_i^{L,2})^d, \]

\[ (a_i^{L,2}/a_i^{L,3})/(K_i^{1,i}/a_i^{L,2})d^i/(K_i^{1,i}/a_i^{L,2})^d, \]

it turns out that \[ G_t^Y = G_t^I \left( v_{t+1}^I / v_{t+1}^I \right)^{d^I(1-d^I)} \], \[ G_t^Y = (a_i^{L,2}/a_i^{L,3})/(a_i^{L,2}/a_i^{L,3})d^i/(a_i^{L,2}/a_i^{L,3})^d, \]

\[ \alpha^I / v_i^I = q_i = i_i + \delta, i = S, N, A, U. \] 

(23)

The government budget constraints (18-20) together with (22) and (23) yield \( 1 - \tau^i \) as follows:

\[ 1 - \tau^i = \frac{1 - \alpha^I - \gamma^I}{1 - \alpha^I} + \frac{b^I}{1 - \alpha^I} \left[ G_t^{X,i} - \left( 1 + i^I \right) \right], i = S, N, 1 - \tau^I = \frac{1 - \alpha^I - \gamma^I}{1 - \alpha^I} + \frac{b^I}{1 - \alpha^I} \left[ G_t^Y - \left( 1 + i^I \right) \right], \]

\[ 1 - \tau^U = \frac{1 - \alpha^U - \gamma^U}{1 - \alpha^U} + \frac{b^U}{1 - \alpha^U} \left[ G_t^Z - \left( 1 + i^U \right) \right]. \] 

(24)

Using the Cobb-Douglas production function it is easily seen that \[ K_i^{1,i}/K_i^{1,i} = (1/M^i)[K_i^{1,i}/(a_i^N)^{d^i}], i = S, N, K_i^{1,i}/Y_i^{1,i} = (1/M^i)[K_i^{1,i}/(a_i^N)^{d^i}], \]

\[ K_i^{1,i}/Y_i^{1,i} \equiv v_i^U = (1/M^U)[K_i^{1,i}/(a_i^N)^{d^i}]. \]

Thus, the FOC for profit maximizing capital service input (2) can be equivalently written as follows:

\[ \alpha^I / v_i^I = q_i = i_i + \delta, i = S, N, A, U. \] 

(25)

Acknowledging (25) in (24) and considering \( G_t^{Y,i} = G_t^I (v_{t+1}^I / v_i^I)^{d^I(1-d^I)} \), \( i = S, N \) \( (G_t^{Y} = G_t^I (v_{t+1}^I / v_i^I)^{d^I(1-d^I)} \), \( G_t^Z = G_t^U (v_{t+1}^U / v_i^I)^{d^U(1-d^U)} \) yields:

\[ (1 - \tau^I)(1 - \alpha^I) = (1 - \alpha^I - \gamma^I - b^I(1-\delta) + b^I[G_t^I (v_{t+1}^I / v_i^I)^{d^I(1-d^I)} - \alpha^I / v_i^I], i = S, N, A, U. \]

(26)

The intertemporal equilibrium dynamics of the capital-output ratio in all countries (regions) is obtained by inserting (26) into (21):
\[ G^i_t[v^i_{t+1} + b^i] \left( \frac{v^i_{t+1}}{v^i_t} \right)^{a^i/(1-a^i)} + \theta^i (1 - \alpha^i) G^i_t(v^i_{t+1}) \left( \frac{v^i_{t+1}}{v^i_t} \right)^{a^i/(1-a^i)} \]

\[ = \frac{(1 - \alpha^i)(1 - \theta^i) - \gamma^i - b^i(1 - \delta) + b^i [G^i_t(v^i_{t+1}/v^i_t)^{a^i/(1-a^i)} - \alpha^i/v^i_t]}{1 + (\beta^i)\varepsilon^i (\pi^i_t / \pi^i_{t+1}) (1 - \delta + \alpha^i / v^i_t)^{1-\sigma}} \]

\[ \pi^i_t = \pi^i_{t+1} = [\xi^x + \xi^y (e^a_{t+1})^{q-1} + \xi^z (e^U_{t+1})^{q-1}]^{(1-q)}, \tau = 0,1 \]

\[ \pi^A_t = [\xi^x (e^A_{t+1})^{q-1} + \xi^y (e^d_{t+1})^{q-1}]^{(1-q)}, \tau = 0,1 \]

\[ \pi^U_t = [\xi^x (e^U_{t+1})^{q-1} + \xi^y (e^d_{t+1})^{q-1} + \xi^z]^{(1-q)}, \tau = 0,1 \]

\[ \varepsilon^i = \left\{ \frac{\xi^x}{1 - \gamma^i - G^A_t v^d_t \left( \frac{v^d_{t+1}}{v^d_t} \right)^{\alpha^d} + (1 - \delta) v^d_t} \right\} \]

\[ \varepsilon^U = \left\{ \frac{\xi^x}{1 - \gamma^i - G^U_t v^d_t \left( \frac{v^d_{t+1}}{v^d_t} \right)^{\alpha^d} + (1 - \delta) v^d_t} \right\} \]

\[ X^i_t = \left( \frac{M^i}{1-\sigma^i} \right)^{1/(1-\sigma^i)} a^i_t L^2_t \left( \frac{v^i_t}{v^i_{t+1}} \right)^{\alpha^i/(1-\sigma^i)} \]

\[ Z^i_t = \left( \frac{M^i}{1-\sigma^i} \right)^{1/(1-\sigma^i)} a^i_t L^2_t \left( \frac{v^i_t}{v^i_{t+1}} \right)^{\alpha^i/(1-\sigma^i)} \]

\[ a^i_t = G^i_{t-1} a^i_{t-1}, L^2_t = G^i_{t-1} L^2_{t-1}, \quad G^i_t \equiv G^d_t G^U_t, \quad i = S, N, A, U, t = 0,1,2,... \]

Equilibrium equations (31) and (32) are obtained by dividing market clearing condition (16) respective (17) on both sides by market clearing condition (15) and transforming main quantity variables into variable to GDP ratios.

For exogenously given structural and policy parameters \( \beta^i, \alpha^i, \theta^i, b^i, \gamma^i, \)
\( M^i, G^d_t, G^U_t (i = S, N, A, U, t = 0,1,2,...) \delta, \eta, \sigma, \xi^x, \xi^y, \xi^z \) and initial values of the variables \( v^i_0, a^i_0, L^2_0 (i = S, N, A, U), \) the difference equation system (27)-(33) represents a determinate dynamic system.
Before switching to intra-EMU and global financial integration, it is worth noting that in all countries (regions) foreign trade in goods is balanced under financial autarky. To see this we need the following definitions of country- (region-) specific trade balances in period $t$ in country $i$ denoted by $TB^i_t, i = S, N, A, U$ which equals the difference between exports and imports in terms of the country-specific good:

$$TB^i_t = X^i_t - \Gamma^i_t - (K^i_{t+1} - (1 - \delta)K^i_t) - L^i_t \times_i^1 - L^i_{t+1} \times_i^2 - L^i_{t+2} \times_i^3$$

$$- (1 / e^U_t) L^i_t z_{i_t}^1 - (1 / e^U_t) L^i_t z_{i_t}^2 - (1 / e^U_t) L^i_t z_{i_t}^3,$$  

(34)

$$TB^N_t = X^N_t - \Gamma^N_t - (K^N_{t+1} - (1 - \delta)K^N_t) - L^N_t \times_i^1 - L^N_{t+1} \times_i^2 - L^N_{t+2} \times_i^3$$

$$- (1 / e^U_t) L^N_t z_{i_t}^1 - (1 / e^U_t) L^N_t z_{i_t}^2 - (1 / e^U_t) L^N_t z_{i_t}^3,$$  

(35)

$$TB^A_t = Y^i_t - \Gamma^A_t - (K^A_t - (1 - \delta)K^A_t) - L^{A1}_t \times_i^1 - L^{A1}_{t+1} \times_i^2 - L^{A1}_{t+2} \times_i^3$$

$$- e^A_t L^{A1}_t \times_i^1 - e^A_t L^{A2}_t \times_i^2 - e^A_t L^{A3}_t \times_i^3$$

$$- (e^A_t / e^U_t) L^{A1}_t z_{i_t}^1 - (e^A_t / e^U_t) L^{A2}_t z_{i_t}^2 - (e^A_t / e^U_t) L^{A3}_t z_{i_t}^3,$$  

(36)

$$TB^U_t = Z_t - \Gamma^U_t - (K^U_t - (1 - \delta)K^U_t) - L^{U1}_t \times_i^1 - L^{U1}_{t+1} \times_i^2 - L^{U1}_{t+2} \times_i^3$$

$$- e^U_t L^{U1}_t \times_i^1 - e^U_t L^{U2}_t \times_i^2 - e^U_t L^{U3}_t \times_i^3$$

$$- (e^U_t / e^U_t) L^{U1}_t z_{i_t}^1 - (e^U_t / e^U_t) L^{U2}_t z_{i_t}^2 - (e^U_t / e^U_t) L^{U3}_t z_{i_t}^3.$$

(37)

Using the market clearing conditions (12)-(17), the budget constraints for all households in period $t$ and the zero-profit and no-arbitrage conditions, it is not difficult to show that the following equality holds in all periods of the intertemporal equilibrium:

$$TB^i_t = L^{i1}_t \times_i^1 + L^{i2}_t \times_i^2 - K^i_{t+1} - B^i_{t+1}, i = S, N, A, U, t = 0,1,2,...$$  

(38)

Asset market clearing condition (12) immediately implies $TB^i_t = 0, i = S, N, A, U, t = 0,1,2,...$, i.e. balanced trade. It also implies that financial openness represents a necessary condition for trade imbalances. Only if national savings are not only invested in national real capital and national government bonds, trade imbalances can occur. While international capital mobility being a necessary condition for the emergence of trade imbalances, it remains an open question whether international financial integration is sufficient to explain the signs and the sizes of trade imbalances observed empirically between EMU core and periphery since euro-related financial integration, and between Asia and the USA since global financial integration. To provide an answer to this question is the objective of the next section.

**International Equilibrium under intra-EMU and Global Financial Integration**

To mimic the financial integration arising through the set-up of the EMU and the Asian-US financial integration we assume in line with stylized facts (Chen et al. 2013) that northern EMU
invests its savings in southern physical capital\textsuperscript{5} and government bonds, that Asia buys US government bonds, and that the USA purchases northern EMU government bonds without incurring any transaction costs. However, also in line with empirical data, we assume that the southern EMU young household buys neither northern real capital, nor northern government bonds nor Asian or US assets.

Thus, the intertemporal optimization problem of the southern young household under financial integration is the same as under financial autarky. Analogously, the intertemporal utility maximization problem of the typical northern EMU household under financial integration is essentially similar to that under financial autarky with the exception of the investment of northern savings per capita and the old-age budget constraint which read now as follows:

\[
\begin{align*}
S_{t+1}^{N,2} &\equiv \frac{I_{t+1}^{N,N}}{L_{t+1}^{N,2}} + (1 - \delta)K_{t+1}^{N,N} + \frac{I_{t+1}^{S,N}}{L_{t+1}^{S,2}} + (1 - \delta)K_{t+1}^{S,N} + \frac{B_{t+1}^{N,N}}{L_{t+1}^{N,2}} + \frac{B_{t+1}^{S,N}}{L_{t+1}^{S,2}} + b_{t+2}^{N,2} + b_{t+2}^{S,2} \, , \\
(iii) \, x_{t+2}^{N,3} + \frac{y_{t+2}^{N,3}}{e_{t+2}^{A}} + \frac{z_{t+2}^{N,3}}{e_{t+2}^{L}} &= \left[ \frac{I_{t+1}^{N,N}}{L_{t+1}^{N,2}} + (1 - \delta)K_{t+1}^{N,N} \right] + \left[ \frac{I_{t+1}^{S,N}}{L_{t+1}^{S,2}} + (1 - \delta)K_{t+1}^{S,N} \right] + (1 + i_{t+2}^{N}) \left( \frac{B_{t+1}^{N,N}}{L_{t+1}^{N,2}} \right) + (1 + i_{t+2}^{S}) \left( \frac{B_{t+1}^{S,N}}{L_{t+1}^{S,2}} \right) \, .
\end{align*}
\]

Here, \( I_{t+1}^{N,N} \) denotes the per-capita investment in southern real capital in period \( t+1 \), \( B_{t+1}^{N,N} \) is the per-capita stock of southern government bonds which the northern EMU young household wants to hold at the beginning of period \( t+2 \), and \( b_{t+2}^{N,2} \) is the stock of private bonds emitted by southern young households which the northern EMU young household wants to hold at the beginning of period \( t+2 \). The northern younger household buys in middle-age not only private bonds issued by northern EMU young ager but also the private bonds issued by the southern young ager. Since for the time period between euro inception and the onset of the global financial crisis physical capital, government and private bonds in each EMU region can be assumed to be perfectly substitutable, and since within the monetary union these assets can be assumed to be perfectly mobile across South and North, the following international Fisher equation (= real international interest parity condition) holds in addition to the national Fisher equations (11):

\[
1 + i_{t+1}^{N} = 1 + i_{t+1}^{N} .
\]

The typical Asian young household solves under financial integration essentially the same problem as under financial autarky with the exception of the use of per-capita savings and the old-age budget constraint which read now as follows:

\[
\begin{align*}
s_{t+1}^{A,2} &\equiv [I_{t+1}^{A,2} + \frac{K_{t+1}^{A,2}}{L_{t+1}^{A,2}} + B_{t+1}^{A,2} + b_{t+2}^{A,2} + \frac{e_{t+1}^{A}}{e_{t+1}^{U}} + \frac{e_{t+1}^{L}}{e_{t+1}^{L}} \, b_{t+2}^{U,2} \, , \\
(iii) \, e_{t+2}^{A,2} + \frac{y_{t+2}^{A,2}}{e_{t+2}^{A}} + \frac{z_{t+2}^{A,2}}{e_{t+2}^{L}} &= \left[ I_{t+1}^{A,2} + \frac{K_{t+1}^{A,2}}{L_{t+1}^{A,2}} + B_{t+1}^{A,2} + b_{t+2}^{A,2} + \frac{e_{t+1}^{A}}{e_{t+1}^{U}} + \frac{e_{t+1}^{L}}{e_{t+1}^{L}} \, b_{t+2}^{U,2} \, \right] + \left[ I_{t+1}^{A,2} + \frac{K_{t+1}^{A,2}}{L_{t+1}^{A,2}} + B_{t+1}^{A,2} + b_{t+2}^{A,2} + \frac{e_{t+1}^{A}}{e_{t+1}^{U}} + \frac{e_{t+1}^{L}}{e_{t+1}^{L}} \, b_{t+2}^{U,2} \, \right] + (1 + i_{t+2}^{N}) \left( \frac{B_{t+1}^{A,2}}{L_{t+1}^{A,2}} \right) + (1 + i_{t+2}^{N}) \left( \frac{B_{t+1}^{A,2}}{L_{t+1}^{A,2}} \right) \, .
\end{align*}
\]

\textsuperscript{5} To mimic the facts presented in Figures 5 and 6 above we assume that physical capital is mainly accumulated by housing investment.
Here $B_{t+2}^{U,A} / [L_{t+2}^{U}]_t$ $(B_{t+2}^{U,A})$ denotes the stock of US government (private) bonds which the Asian young household wants to hold at the beginning of period $t+2$. In line with pre-crisis reality the Asian young household does not hold EMU government bonds.

Analogously, savings per capita and the old-age budget constraint of the typical US young household are as follows:

$$s_{t+1}^{U,A} = [i^{U,A}_{t+1} + (1 - \delta)K_{t+1}^{U,A}] / L_{t+1}^{U} + b_{t+1}^{U,A} / L_{t+1}^{U} + i^{U,A}_{t+1}B_{t+1}^{N,A} / L_{t+1}^{U} + b_{t+1}^{U,N} + e_{t+1}^{U,A}$$

$$e_{t+2}^{U,A} + e_{t+2}^{U,N} + e_{t+2}^{A} = [d_{t+2}^{U} + (1 - \delta)] [L_{t+1}^{U} + (1 - \delta)K_{t+1}^{U,A}] / L_{t+1}^{U}$$

$$+ (1 + i_{t+1}^{U,A}) B_{t+1}^{U,A} / L_{t+1}^{U} + (1 + i_{t+1}^{U}) B_{t+1}^{U,N} / L_{t+1}^{U} + (1 + i_{t+1}^{N}) e_{t+1}^{U,A} B_{t+1}^{N,A} / L_{t+1}^{U} + (1 + i_{t+1}^{A}) e_{t+1}^{U,A} b_{t+1}^{U,N}.$$

Again in line with pre-crisis empirical reality, the US young household does hold from abroad only northern EMU government bonds.

In order to ensure arbitrage-free terms of trade, the following international real interest parity conditions in addition to (40) ought to hold:

$$1 + i_{t+1}^{A} = e_{t+1}^{A} (1 + i_{t+1}^{N}), \forall t = 0, 1, 2,..., \tag{43}$$

$$1 + i_{t+1}^{U} = e_{t+1}^{U} (1 + i_{t+1}^{N}), \forall t = 0, 1, 2,..., \tag{44}$$

The markets for southern and northern EMU, Asian and US real capital clear according to:

$$K_{t+1}^{S} = K_{t+1}^{S,A} + K_{t+1}^{S,N}, K_{t+1}^{N,A} = K_{t+1}^{N,N}, K_{t+1}^{A,A} = K_{t+1}^{U,A}, K_{t+1}^{U,N} = K_{t+1}^{U,U}, \ t = 0, 1, 2,..., \tag{45}$$

The markets for southern and northern EMU, Asian and US government bonds clear now according to:

$$B_{t+1}^{S} = B_{t+1}^{S,A} + B_{t+1}^{S,N}, B_{t+1}^{N,A} = B_{t+1}^{N,N}, B_{t+1}^{A,A} = B_{t+1}^{A,A}, B_{t+1}^{U} = B_{t+1}^{U,U} + B_{t+1}^{U,A}, \ t = 0, 1, 2,..., \tag{46}$$

While the clearing conditions for the labor markets and good markets remain as under financial autarky, the clearing conditions for the following private bond markets read now as follows:

$$L_{t+1}^{A} b_{t+1}^{A,A} = L_{t+1}^{A,A} b_{t+1}^{A,A} + L_{t+1}^{A,N} b_{t+1}^{A,N} = L_{t+1}^{A,U} b_{t+1}^{A,U} + L_{t+1}^{A,U} b_{t+1}^{A,U} + L_{t+1}^{A,A} = L_{t+1}^{A,A} + L_{t+1}^{A,A} + L_{t+1}^{A,A} + L_{t+1}^{A,A}. \tag{47}$$

Financial integration affects mostly the asset market clearing condition (12). Now, the worldwide amount of savings ought to be equal to the worldwide supply of assets from southern and northern EMU, Asia and the US:

$$K_{t+1}^{S} = K_{t+1}^{N} + B_{t+1}^{N,A} + B_{t+1}^{N,U} + (K_{t+1}^{A,A} + K_{t+1}^{U,U}) / e_{t+1}^{A} + (K_{t+1}^{U} + B_{t+1}^{U,A}) / e_{t+1}^{U}. \tag{48}$$

Having described the optimization problems of households and firms as well as the market clearing conditions, the intertemporal equilibrium dynamics can now be derived.

From (25) and from the international Fisher equations (40), (43) and (44) the following relationships between southern EMU, Asian, US and northern EMU capital output ratios and the terms of trade result:
\begin{equation}
\varepsilon^{S}_{t+1} = \alpha^{S} \varepsilon^{N}_{t+1} N_{t+1}, \quad t = 0, 1, 2, ..., \quad (49)
\end{equation}

\begin{equation}
\varepsilon^{A}_{t+1} = \alpha^{A} \varepsilon^{N}_{t+1} N_{t+1}, \quad t = 0, 1, 2, ..., \quad (50)
\end{equation}

\begin{equation}
\varepsilon^{A}_{t+1} = \alpha^{U} \varepsilon^{N}_{t+1} U_{t+1}, \quad t = 0, 1, 2, ..., \quad (51)
\end{equation}

Dividing (48) on both sides by the world efficiency weighted labor defined as 
\[ AL_{t}^{w} \equiv a_{t}^{S} L_{t}^{S, 2} + a_{t}^{N} L_{t}^{N, 2} + a_{t}^{A} L_{t}^{A, 2} + a_{t}^{U} L_{t}^{U, 2} \]
and introducing the definitions of the capital output ratios as well as the debt output ratios, the world-wide asset market clearing condition (48) can be rewritten as follows:

\begin{equation}
\begin{aligned}
&\frac{L_{t}^{S, 1} S_{t}^{1} + L_{t}^{S, 2} S_{t}^{2} \alpha_{t}^{S} L_{t}^{S, 2} + a_{t}^{N} L_{t}^{N, 2} + L_{t}^{A, 1} S_{t}^{1} + L_{t}^{A, 2} S_{t}^{2} \alpha_{t}^{A} L_{t}^{A, 2} + L_{t}^{U, 1} S_{t}^{1} + L_{t}^{U, 2} S_{t}^{2} \alpha_{t}^{U} L_{t}^{U, 2}}{a_{t}^{S} L_{t}^{S, 2} + a_{t}^{N} L_{t}^{N, 2} + a_{t}^{A} L_{t}^{A, 2} + a_{t}^{U} L_{t}^{U, 2}} \\
&= (v_{t+1}^{S} + b_{t+1}^{S})(M_{t}^{S})^{(1-\sigma)} (v_{t+1}^{N} + b_{t+1}^{N})(M_{t}^{N})^{(1-\sigma)} (v_{t+1}^{A} + b_{t+1}^{A})(M_{t}^{A})^{(1-\sigma)} (v_{t+1}^{U} + b_{t+1}^{U})(M_{t}^{U})^{(1-\sigma)} \quad (52)
\end{aligned}
\end{equation}

\begin{equation}
\begin{aligned}
&+ (v_{t+1}^{S} + b_{t+1}^{S})(M_{t}^{S})^{(1-\sigma)} (v_{t+1}^{N} + b_{t+1}^{N})(M_{t}^{N})^{(1-\sigma)} (v_{t+1}^{A} + b_{t+1}^{A})(M_{t}^{A})^{(1-\sigma)} (v_{t+1}^{U} + b_{t+1}^{U})(M_{t}^{U})^{(1-\sigma)} \quad t = 0, 1, 2, ...
\end{aligned}
\end{equation}

Acknowledging the optimal savings functions resulting from household’s utility maximization problems and (26) in (52) yield:

\begin{equation}
\begin{cases}
- \theta^{S}(1-\alpha^{S}) \frac{\alpha^{S}}{(v_{t+1}^{S})^{(1-\sigma)}} + \frac{(v_{t+1}^{S})^{(1-\sigma)}}{(1-\delta+\frac{\alpha^{S}}{v_{t+1}^{S}})} (1-\alpha^{S})(1-\theta^{S}) - \gamma^{S} - b^{S}(1-\delta) + b^{S}[G_{t}^{S} \left( \frac{v_{t+1}^{S}}{v_{t+1}^{S}} \right)^{(1-\sigma)} - \frac{\alpha^{S}}{v_{t+1}^{S}}] \\
(1-\delta+\frac{\alpha^{S}}{v_{t+1}^{S}}) \left[ (1-\alpha^{S})(1-\theta^{S}) - \gamma^{S} - b^{S}(1-\delta) + b^{S}[G_{t}^{S} \left( \frac{v_{t+1}^{S}}{v_{t+1}^{S}} \right)^{(1-\sigma)} - \frac{\alpha^{S}}{v_{t+1}^{S}}] \right]
\end{cases}
\end{equation}

\begin{equation}
\begin{cases}
- \theta^{N}(1-\alpha^{N}) \frac{\alpha^{N}}{(v_{t+1}^{N})^{(1-\sigma)}} + \frac{(v_{t+1}^{N})^{(1-\sigma)}}{(1-\delta+\frac{\alpha^{N}}{v_{t+1}^{N}})} (1-\alpha^{N})(1-\theta^{N}) - \gamma^{N} - b^{N}(1-\delta) + b^{N}[G_{t}^{N} \left( \frac{v_{t+1}^{N}}{v_{t+1}^{N}} \right)^{(1-\sigma)} - \frac{\alpha^{N}}{v_{t+1}^{N}}] \\
(1-\delta+\frac{\alpha^{N}}{v_{t+1}^{N}}) \left[ (1-\alpha^{N})(1-\theta^{N}) - \gamma^{N} - b^{N}(1-\delta) + b^{N}[G_{t}^{N} \left( \frac{v_{t+1}^{N}}{v_{t+1}^{N}} \right)^{(1-\sigma)} - \frac{\alpha^{N}}{v_{t+1}^{N}}] \right]
\end{cases}
\end{equation}
Equation (53) together with equations (28)-(30) represents the fourth equation of the intertemporal equilibrium dynamics of the capital-output ratios, \( v^S, v^A, v^U \), and the terms of trade \( e^S \) and \( e^U \) of the three-good, three-country OLG model under financial integration.

Equations (31) and (32) together with (33) represent the two remaining dynamic equations of the intertemporal international equilibrium under financial integration.

It turns out that the intertemporal equilibrium dynamics consists of six non-linear first-order difference equations which can be used to determine the following six dynamic variables: \( v^S, v^A, v^U, e^S, e^A, e^U \).

In line with Coeurdacier et al. (2015) we assume that in a steady state defined as \( v^S_{t+1} = v^S, v^A_{t+1} = v^A, v^U_{t+1} = v^U, e^S_{t+1} = e^S, e^A_{t+1} = e^A \) and \( e^U_{t+1} = e^U \) the labor productivity growth factors are international equal: \( G^i = G, i = S, N, A, U \). Then, the system of first-order difference equations collapses on the following system of steady state equations:

\[
v^S = \left( \frac{\alpha^S}{\alpha^N} \right) v^N, \quad (54)
\]
\[
v^A = \left( \frac{\alpha^A}{\alpha^N} \right) v^N, \quad (55)
\]
\[
v^U = \left( \frac{\alpha^U}{\alpha^N} \right) v^N, \quad (56)
\]
National Net Foreign Asset Position, Trade Imbalance and Aggregate Savings

Since national savings of young and middle-aged households need no longer be equal to the sum of national real capital and real government bonds, trade imbalances are now feasible along an intertemporal equilibrium path. Since these trade imbalances represent the other side of the divergence between national savings and national asset accumulation it is usual in the international macroeconomics literature (e.g. Zee 1987, p. 609) to introduce an appropriate variable denoted by $\Phi_{i,t} = L_{t}^{\delta_{1,i}}s_{t}^{1,i} + L_{t}^{\delta_{2,i}}s_{t}^{2,i} - K_{t+1}^{i} - B_{t+1}^{i}, i = S, N, A, U, t = 0, 1, 2, ...$

Acknowledging the use of aggregate savings within the period-$t$ budget constraints of young, middle-aged and old-aged households and the market clearing conditions for real capital and government bonds, it turns out that $\Phi_{i,t}^{i}$ equals the net foreign asset position of country $i = S, N, A, U$. In particular, it follows:

$$\Phi_{t+1}^{S} = -K_{t+1}^{S,N} - B_{t+1}^{S,N} - L_{t}^{N,2}s_{t+1}^{SN}, t = 0, 1, 2, ...$$

$$\Phi_{t+1}^{N} = K_{t+1}^{S,N} + B_{t+1}^{S,N} + L_{t}^{N,2}s_{t+1}^{SN} - B_{t+1}^{N,U}, t = 0, 1, 2, ...$$

$$\Phi_{t+1}^{A} = (e_{t}^{U}/e_{t}^{U})B_{t+1}^{U,A} + (e_{t}^{U}/e_{t}^{U})L_{t}^{U,2}s_{t+1}^{UA}, t = 0, 1, 2, ...$$

$$\Phi_{t+1}^{U} = e_{t}^{U}B_{t+1}^{N,U} + e_{t}^{U}L_{t}^{N,2}s_{t+1}^{NU} - L_{t}^{N,2}s_{t}^{UA} - B_{t+1}^{U,A}, t = 0, 1, 2, ...$$
Using again the budget constraints of young, middle-aged and old households plus the zero-profit and no-arbitrage conditions as well as the market clearing conditions one can show that the trade balance of country \(i = S, N, A, U\) is related to the respective net foreign asset position as follows:

\[
TB_i^S = -K_{i+1}^{S,N} - B_{i+1}^{S,N} - \tau_{i}^{N,2}b_{i+1}^{S,N,2} - (1 + i)^S_t(-K_{i}^{S,N} - B_{i}^{S,N} - L_{i}^{S,3}b_{i}^{S,N,2}) = \Phi_{i+1}^{S} - (1 + i)^S_t\Phi_{i}^{S},
\]

(64)

\[
TB_i^N = K_{i+1}^{S,N} + B_{i+1}^{S,N} + L_{i}^{N,2}b_{i+1}^{S,N,2} - B_{i}^{N,U} - (1 + i)^N_t(K_{i}^{S,N} + B_{i}^{S,N} + L_{i+1}^{N,2}b_{i}^{S,N,2} - B_{i}^{N,U}) = \Phi_{i+1}^{N} - (1 + i)^N_t\Phi_{i}^{N},
\]

(65)

\[
TB_i^A = (e_i^U / e_i^J)(B_i^{U,A} + L_i^{A,2}b_{U,i}^{A,2}) - (1 + i^A_t)(e_i^U / e_i^J)(B_i^{U,A} + L_i^{A,2}b_{U,i}^{A,2}) = \Phi_{i+1}^{A} - (1 + i^A_t)\Phi_{i}^{A},
\]

(66)

\[
TB_i^U = e_i^U B_i^{N,U} + e_i^U L_i^{U,2}b_{U,i}^{N,2} - L_i^{U,2}b_{i}^{N,2} - B_{i+1}^{U,A} - (1 + i^U_t)[e_i^U(B_i^{N,U} + L_i^{U,2}b_{U,i}^{N,2}) - B_i^{U,A} - L_i^{U,2}b_{U,i}^{A,2}] = \Phi_{i+1}^{U} - (1 + i^U_t)\Phi_{i}^{U}.
\]

(67)

Dividing the \(TB_i^t\)'s in equation (64)-(67) by the respective GDPs, we obtain the trade balance to GDP ratios denoted by \(tb_i^t\) and defined as:

\[
tb_i^S = TB_i^S / X_i^S, tb_i^N = TB_i^N / X_i^N, tb_i^A = TB_i^A / Y_i, tb_i^U = TB_i^U / Z_i.
\]

The relation of the trade balance to GDP ratios to the corresponding net foreign asset to GDP ratios defined as \(\phi_i^S = TB_i^S / X_i^S, \phi_i^N = TB_i^N / X_i^N, \phi_i^A = TB_i^A / Y_i, \phi_i^U = TB_i^U / Z_i\) follows immediately from equations (64)-(67):

\[
tb_i^t = \phi_i^t G_t\left(\frac{v_{i+1}^t}{v_t^t}\right)^{1-d} - (1 + i_t^t)\phi_i^t, i = S, N, A, U,
\]

(68)

where \(v_i^S = K_i^S / X_i^S, v_i^N = K_i^N / X_i^N, v_i^A = K_i^A / Y_i, v_i^U = K_i^U / Z_i\) denote the country-specific capital output ratios.

Finally, we define aggregate savings of country (region) \(i = S, N, A, U\) as gross national product less public and private consumption as follows:

\[
S_i^S = X_i^S + i_\phi_i^S - \Gamma_i^S - L_i^{S,1}x_{i}^{S,1} - L_i^{S,2}x_{i}^{S,2} - L_i^{S,3}x_{i}^{S,3} - (1 / e_i^S) L_i^{S,1}y_i^{S,1} - (1 / e_i^A) L_i^{S,2}y_i^{S,2} - (1 / e_i^U) L_i^{S,3}y_i^{S,3} - (1 / e_i^J) L_i^{S,1}x_i^{S,1} - (1 / e_i^J) L_i^{S,2}x_i^{S,2} - (1 / e_i^J) L_i^{S,3}x_i^{S,3}
\]

(69)

\[
S_i^N = X_i^N + i_\phi_i^N - \Gamma_i^N - L_i^{N,1}x_{i}^{N,1} - L_i^{N,2}x_{i}^{N,2} - L_i^{N,3}x_{i}^{N,3} - (1 / e_i^A) L_i^{N,1}y_i^{N,1} - (1 / e_i^A) L_i^{N,2}y_i^{N,2} - (1 / e_i^U) L_i^{N,3}y_i^{N,3} - (1 / e_i^J) L_i^{N,1}x_i^{N,1} - (1 / e_i^J) L_i^{N,2}x_i^{N,2} - (1 / e_i^J) L_i^{N,3}x_i^{N,3}
\]

(70)

\[
S_i^A = Y_i + i_\phi_i^A - \Gamma_i^A - e_i^A L_i^{A,1}x_{i}^{A,1} - e_i^A L_i^{A,2}x_{i}^{A,2} - e_i^A L_i^{A,3}x_{i}^{A,3} - L_i^{A,1}y_{i}^{A,1} - L_i^{A,2}y_{i}^{A,2} - L_i^{A,3}y_{i}^{A,3} - (e_i^A / e_i^J) L_i^{A,1}x_i^{A,1} - (e_i^A / e_i^J) L_i^{A,2}x_i^{A,2} - (e_i^A / e_i^J) L_i^{A,3}x_i^{A,3}
\]

(71)

\[
S_i^U = Z_i + i_\phi_i^U - \Gamma_i^U - e_i^U L_i^{U,1}x_{i}^{U,1} - e_i^U L_i^{U,2}x_{i}^{U,2} - e_i^U L_i^{U,3}x_{i}^{U,3} - (e_i^U / e_i^J) L_i^{U,1}y_{i}^{U,1} - (e_i^U / e_i^J) L_i^{U,2}y_{i}^{U,2} - (e_i^U / e_i^J) L_i^{U,3}y_{i}^{U,3} - (e_i^U / e_i^J) L_i^{U,1}x_i^{U,1} - (e_i^U / e_i^J) L_i^{U,2}x_i^{U,2} - (e_i^U / e_i^J) L_i^{U,3}x_i^{U,3}
\]

(72)

Upon using again the zero-profit property, the no-arbitrage conditions and the budget constraints of all households it can be shown that \(S_i^t, i = S, N, A, U\) can be equivalently written as follows:

\[
S_i^t = L_i^{1,1}s_{i}^{1,1} + L_i^{2,1}s_{i}^{2,1} - i_t B_i^t - \Gamma_i^t + L_i^{2,2}w_t e_i^t - L_i^{3,1}s_{i-1}^{1,1} - L_i^{2,3}s_{i-1}^{2,2} + \delta K_i^t, i = S, N, A, U, t = 0, 1, 2, \ldots \ (73)
\]
By dividing $S_t^i$ through the respective GDP we obtain the national saving ratios denoted by $sr_t^i$ and defined as: $sr_t^S = S_t^S / X_t^S$, $sr_t^N = S_t^N / X_t^N$, $sr_t^A = S_t^A / Y_t$, $sr_t^U = S_t^U / Z_t$. Inserting the optimal individual saving functions (3), (5) and (7) we obtain:

$$sr_t^i = -\frac{(1-\alpha')(\theta')G_t^i(v_{t+1}^i/v_t^i)}{(1-\delta+\alpha'/v_t^i)} \left[ 1 + \left( \beta'^{-\sigma} \frac{\gamma^i}{\gamma^i} \right) \right] - b' \left( 1-\alpha' \right) \frac{\theta' (1-\alpha')}{(1-\delta+\alpha'/v_t^i)} (1-\delta+\alpha'/v_t^i)^{-\sigma} + \frac{\theta' (1-\alpha')}{(1-\delta+\alpha'/v_t^i)} (1-\delta+\alpha'/v_t^i)^{-\sigma} \right] + \frac{\theta' (1-\alpha')}{(1-\delta+\alpha'/v_t^i)} (1-\delta+\alpha'/v_t^i)^{-\sigma}.$$

In a steady state with $G_t^i = G$, $i = S, N, A, U$, $\forall t$, (78) simplifies as follows:

$$sr_t^i = -\frac{(1-\alpha')(\theta')(G-1)}{1+i} + \frac{\{ (1-\alpha)(1-\theta')\} (G-1)}{[1 + (\beta')^{-\sigma} (1+i)^{-\sigma}]G} - b'(G-1) + \delta v^i, \ i = S, N, A, U.$$

In order to point out the relevance of the interaction between productivity growth and credit constraints for the international divergence of saving rates we set provisionally all fiscal parameters equal to zero, and moreover assume that $\alpha' = \alpha$ and $\beta' = \beta, i = S, N, A, U$. Then, (75) further simplifies to:

$$sr_t^i = -\frac{(1-\alpha')(\theta')(G-1)}{1+i} + \frac{\{ (1-\alpha)(1-\theta')\} (G-1)}{[1 + (\beta')^{-\sigma} (1+i)^{-\sigma}]G} + \delta v^i, \ i = S, N, A, U.$$

As Coeurdacier et al. (2015, p. 2850) aptly remark, the above equation illustrates three important insights: First, the international divergence of saving rates must be attributed to the interaction of productivity growth and credit constraints. Without growth ($G = 1$) saving rates are equal across countries. Second, with growth and under financial integration the levels of saving rates differ: the saving rate is higher the more constrained the economy is. The reason is that the positive influence of the middle-aged ($= second term in equation (76)$) weights more than the negative influence of the young-aged if $\theta'$ is small. Third, the response of the saving rate vis-à-vis a fall in the interest factor is larger in the more constrained economy. “These slope differences, combined with differences in levels, imply that a fall in $R [= 1+i]$ induces a divergence (italics in original) in saving rates across countries.” (Coeurdacier et al. 2015, ibid)

One reason for the decline of the common real interest rate is the expansion of the share of the most constrained economy in the world GDP due to higher GDP growth rates than the other countries (see Proposition 2 in Coeurdacier et al. 2015, p. 2849).
Aggregate savings (74) are composed of the savings of young-aged \( (L_{t}^{1} s_{t}^{1}) \), of middle-aged \( (L_{t}^{2} s_{t}^{2} - L_{t-1}^{2} s_{t-1}^{2}) \) and of old-aged \( (-L_{t}^{3} s_{t}^{3} + \delta K_{t}^{2}) \) assuming provisionally a zero public deficit. Then, it holds:

\[
S_{t}^{1,j} = \frac{(1 - \alpha') \theta' G_{i}^{2}(v_{t+1}^{j})^{1-\alpha'}}{(1 + i_{t+1})(v_{t}^{j})^{1-\alpha'}},
\]  

(77)

\[
S_{t}^{2,j} = \frac{(1 - \alpha')(1 - \theta')}{[1 + (\beta')^{-\sigma}(\pi_{t}^{j} / \pi_{t-1}^{j})^{1-\sigma}(1 + i_{t})^{1-\sigma}]} + \frac{(1 - \alpha') \theta'}{(1 + i_{t})},
\]  

(78)

\[
S_{t}^{3,j} = \frac{(1 - \alpha')(1 - \theta')(v_{t+1}^{j})^{1-\alpha'}}{[1 + (\beta')^{-\sigma}(\pi_{t}^{j} / \pi_{t-1}^{j})^{1-\sigma}(1 + i_{t})^{1-\sigma}]} + \delta v_{t}^{j}.
\]  

(79)

Looking at the respective young-aged and middle-aged saving rates in (77) and (78), it is not difficult to see that in response to a decrease in the interest factor, \( 1 + i_{t+1} \), the young save less and the middle-aged save more provided that \( \sigma < 1 \). The decrease in the young-aged saving rate is larger in less constrained economies, while the increase in savings of the middle-aged is larger in more constrained economies.

**Numerical Specification and Model Generated versus Stylized Macro Facts**

The numerical specification of our three-country, three-good OLG model with internationally differing household credit constraints is used to investigate whether a small number of shocks at the start of the new millennium – intra-EMU and Asian-US capital market integration and the rapid growth of Emerging Asia can generate the decline in the common world interest rate, the emergence of intra-EMU and Asian-US trade imbalances and the divergence of saving rates observed in the data between 2000 and the onset of the global financial crisis. Structural and policy parameters are specified such that the differences in real interest rates between EMU core and periphery and between Asia and USA as well as their approximately balanced current accounts and zero net foreign asset positions in the 1990s (before euro inception and the East-Asian currency crises) are reproduced by the financial autarky version of the model.

In order to highlight the significance of the interaction between capital market integration and productivity growth we do start from financial autarky steady states but assume internationally unequal productivity growth rates in line with Coeurdacier et al. (2015). As these authors we assume that one period (= 20 calendar years) before financial integration Asia exhibits a significantly higher labor productivity growth rate than the USA, and continues higher productivity two periods (= 40 calendar years) after financial integration. Regarding the EMU, we assume in line with empirical reality of 1990s (Fagan and Gaspar, 2008), that the productivity growth rate of EMU periphery is (approximately) equal to that of EMU core - a scenario which continues under EMU financial integration. We assume the following numerical values for the
respective national growth factors (yearly growth rates): \( G^S = G^N = 1.6 \) (2.4% yearly), \( G^0_i = 2.1 \), \( G^t_i = G^4_i = 2.5 (4.7\% \text{ yearly}) \), \( G^{t}_i = 1.6, t = 3, ..., G^{U}_i = 1.7, t = 1, 2 (2.7\% \text{ yearly}) , G^{U}_i = 1.6, t = 3, ... \)

Also in line with the empirical reality of the 1990s and pre-crisis 2000s we assume that both the government expenditure and the debt to GDP ratios remain constant over time, i.e. \( \gamma'_i = \gamma'_{i+1} = \gamma' \) and \( b'_i = b'_{i+1} = b'_i, i = S, N, A, U \). Under these stationarity assumptions we specify the policy parameters roughly in line with 1990s averages as follows: \( \gamma^S = 0.22, \gamma^N = 0.24, \gamma^A = 0.15, \gamma^U = 0.23, b^S = 0.025, b^N = 0.023, b^A = 0.015, b^U = 0.024 \). Given the above specified productivity growth factors, the just mentioned policy parameters, the consumption expenditure shares \( \zeta^S = 0.4, \zeta^N = 0.3, \zeta^A = 0.3 \), the total factor productivity numbers \( M^S = 5, M^N = 6, M^A = 4, M^U = 7 \), the depreciation rate \( \delta = 1 \), and the following subjective utility discount factors taken from the literature \( \beta^S = 0.62, \beta^N = 0.9, \beta^A = 0.82, \beta^U = 0.7 \), we calibrate the capital production shares \( \alpha'_i \) and the credit constraint parameters \( \theta'_i \) such that the country-specific net foreign asset positions are zero, the associated real interest rates correspond roughly to the 1990s country averages and, most importantly, that the aggregate saving rates accord to 1990s data: \( \alpha^S = 0.1625, \alpha^N = 0.15, \alpha^A = 0.1829, \alpha^U = 0.1412, \theta^S = 0.06224, \theta^N = 0.08037, \theta^A = 0.0216, \theta^U = 0.1136 \). For the calibration exercise we assume the following values for the inter- and intra-temporal elasticity in consumption, respectively: \( \sigma = 0.97, \eta = 0.97 \). The intertemporal elasticity in consumption is within the range of empirically estimated values but much higher than in Coeurdacier et al. (2015).

Table 1 reports the steady state values of main endogenous variables in EMU South, EMU North, in Asia and in USA under financial autarky.

| Table 1 Main endogenous variables (as ratio to GDP) in EMU South, EMU North, in Asia and in USA (calculated on a yearly basis) under financial autarky |
|-----------------|-----------------|----------------|-----------------|-----------------|----------------|
| Capital output ratio | Real interest rate (in %) | EMU terms of trade relative to | Ratio of net foreign assets to GDP | Saving rates (in %) |
|-----------------|-----------------|----------------|-----------------|-----------------|----------------|
| EMU South       | 1.63            | 3.53           | 0               | 0               | 13             |
| EMU North       | 1.88            | 2.38           | 0               | 0               | 15             |
| Asia            | 2.29            | 2.38           | 0.48            | 0               | 24             |
| USA             | 1.18            | 4.47           | 1.72            | 0               | 10             |

Source: Own calculations

Note that the yearly real interest rate in EMU South, North, in Asia and in USA exhibited in Table 1 is not too far from the real interest rates in the late 1990s portrayed in Fig. 1 and Fig. 2.

Figure 3 shows rather similar personal saving rates in EMU South and EMU North in the 1990s, while significantly diverging saving rates in 2000s. In contrast, Figure 4 features significantly higher personal saving rates in Asia than in USA already in the 1990s. Both the similarity of pre-euro EMU saving rates and the huge difference between the Asian and the US saving rates levels are rather well reproduced by our financial autarky solution presented in Table 1.

Common to both figures is a marked decrease of saving rates in EMU South and USA, and only first a decrease and in the second halve of 2000s even a slight increase of saving rates in
EMU North and Asia. Following the analytical insights regarding the response of the saving rates to declining real interest rates as a consequence of the interaction between internationally differing credit constraints and high Asian GDP growth rates, the model should generate markedly declining EMU South and US saving rates.

Given the structural and policy parameters calibrated to the financial autarky situation, the first-period results for main variables under financial integration read as follows:

### Table 2
Main endogenous variables in EMU South, EMU North, in Asia and in USA (calculated on a yearly basis) under financial integration and households’ credit constraints in shock period.

<table>
<thead>
<tr>
<th></th>
<th>Capital output ratio</th>
<th>Real interest rate (in %)</th>
<th>Net foreign assets to GDP (%)</th>
<th>Trade balance to GDP ratio (%) (Shock period)</th>
<th>EMU Terms of trade vis-à-vis</th>
<th>Saving rates (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>EMU South</strong></td>
<td>1.95</td>
<td>2.58</td>
<td>-51.35</td>
<td>-4.25</td>
<td>15.02</td>
<td>14.55</td>
</tr>
<tr>
<td><strong>EMU North</strong></td>
<td>1.80</td>
<td>2.58</td>
<td>+0.053</td>
<td>+0.04</td>
<td>15.04</td>
<td>14.77</td>
</tr>
<tr>
<td><strong>Asia</strong></td>
<td>1.76</td>
<td>3.73</td>
<td>+46.55</td>
<td>+5.10</td>
<td>0.59</td>
<td>21.50</td>
</tr>
<tr>
<td><strong>USA</strong></td>
<td>1.54</td>
<td>3.05</td>
<td>-67.88</td>
<td>-5.95</td>
<td>1.89</td>
<td>11.57</td>
</tr>
</tbody>
</table>

Source: Own calculation

Starting from the financial autarky solution Table 2 reports the results of financial integration under households’ credit constraints and exogenous productivity growth. Comparing the results in Table 1 to those in Table 2 we see that the capital output ratio in EMU South and in the USA increases while it decreases in EMU North and in Asia which replicates the development in housing investment in EMU, Asia and USA as presented in Figure 5 and 6, respectively. The comparison of the interest rate column in Table 1 and Table 2, respectively, reveals that the interest rate in EMU South decreases and in EMU North increases featuring the approximate convergence of real interest rates within EMU reported in Figure 1 above. As Figure 2 shows the Asian real interest rate increases sharply around 2000 and then declines but remains above the autarky level a development which is only to some extent replicated by our model. The model reproduces better the decline of the US real interest rate as the comparison of the respective columns in Table 1 and Table 2 shows. The rise of the real interest rate of Asia as a consequence of the much higher productivity growth in Asia than in EMU North is accompanied by an increase in the EMU terms of trade relative to Asia in line with the real interest parity condition (43) and shown in the terms of trade column in Table 1 and 2, respectively. Regarding the development of the net foreign asset position we see in Table 2 a massive worsening of EMU periphery’s position and a slight improvement of EMU core’s net foreign asset position in line with the development of net foreign asset to GDP ratios reported in Figure 8. Table 2 shows also the pronounced accumulation of Asian net foreign assets, and the deterioration of the US position. Most

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6 While the convergence of nominal (short-term) interest rates within EMU is perfect (see chart 4 in Fagan and Gaspar, 2008, p. 34), real interest rates did not converge due to higher inflation rates in EMU periphery than in EMU core as mentioned above.

7 Table 2 reports a larger negative US net foreign asset position than shown in Figure 10. Gourinchas and Rey (2014) estimate that approximately 20% of the US net foreign asset position is due to capital gains on its net foreign assets. Correcting the US net foreign asset position reported in Figure 10 by these percentage points we see that the model generated US net foreign asset position diverges from the empirically observed value not that much.
important for the research results reported in this paper are the model generated signs and sizes of the trade imbalance to GDP ratios presented in the fifth column of Table 2 as compared to the empirical values shown in Figure 7 for the EMU and in Figure 9 for Asia and the US. While the right signs of the trade imbalance to GDP ratios can also be replicated in Farmer and Ban’s (2014, 2015) three-good, three-country OLG model without household credit constraints, the model generated sizes are generally too small compared to the empirical values. Here household credit constraints definitively help to improve the empirical fit of the model. However, Table 2 also shows that the model under the present parameter calibration generates too small EMU core trade surpluses. Further research should show whether a better parameter calibration can help to remedy this problem.

Besides the inadequate sizes of model generated trade imbalances the empirically inadequate levels of aggregate saving ratios and their development in the integration period were another driver for the present research. Contrary to our first expectations we are not able to report overall satisfactory results. On the positive side we can state that internationally diverging household credit constraints enabled us to calibrate the parameters of the financial autarky variant of the model such that the huge difference in saving rate levels between Asia and US can be replicated. Moreover, this level difference remains during the financial integration period as the respective column in Table 2 shows. Without over-interpreting the empirical fit of the model, we see in Figure 4 a sizeable drop of the Asian saving rate and an increase in the US and EMU core saving rate immediately after 2000 developments which are corroborated by the model generated results presented in the shock period column of Table 2. However, clearly on the negative side, the model in its present form is neither able to replicate the massive decline of EMU periphery’s saving rate nor the less massive decline in the US rate between 2000 and 2007, the slight decline in saving rates in the after integration period reported in the last column of Table 2 not contradicting.

**Conclusion**

Farmer and Ban (2014, 2015) investigated in a three-country, three-good OLG model without household credit constraints simultaneously the financial integration effects on both intra-EMU and global external imbalances. While the authors were able to show how EMU core and Asian trade surpluses as well as EMU periphery and US trade deficits can be qualitatively attributed to intra-EMU and global financial integration, the large sizes of the observed trade imbalance to GDP ratios and the empirical fact of significantly higher saving rates in Asia than in USA already before Asian-US financial integration, the model without household credit constraints was unable to address.

This paper investigates whether Farmer and Ban’s (2015) quantitative results can be improved by the incorporation of credit constraints to three-period lived younger households in line with Coeurdacier et al. (2015) in order to be able to replicate the huge difference in saving rate levels between Asia and the US already before financial integration. We indeed find empirically not implausible values for capital production shares and households credit constraint parameters (albeit for rather low values) such that the financial autarky variant of the OLG model reproduces
roughly empirically observed EMU, Asian and US real interest rates and the corresponding saving rates for given values of observed country-specific productivity growth rates for the 1980s and 1990s.

Commencing from the financial autarky solution for internationally differing capital output ratios and associated real interest rates financial integration together with the rapid-growth related increase in Asia’s share of world employment induces a larger EMU periphery and US capital output ratio associated with a lower real interest rate and a smaller EMU core and Asian capital output ratio associated with a larger real interest rate. While the intra-EMU interest rate convergence in the shock period is complete, Asian and US’ interest rates remain unequal despite of international real interest parity since Asia’s rapid growth pushes up the Asian real interest rate and the EMU terms of trade relative to Asia. The rise of EMU periphery’s and US capital output ratios (housing investment) deteriorates the respective net foreign asset positions, while the fall of EMU core’s and the Asian capital output ratio improves the respective net foreign asset positions albeit at unequal magnitudes due to internationally differing capital production shares, government expenditure and debt ratios, and credit constraint parameters.

The structural and policy parameters calibrated to the financial autarky situation are such that under financial integration the world economy is dynamically inefficient meaning that the productivity growth rates are larger than the real interest rates. Compared to productivity growth rather low real interest rates are needed in order to replicate through the model the empirically observed signs of trade imbalances. Due to the household credit constraints saving rates differ substantially internationally and this implies larger trade imbalances. This result clearly improves the performance of the model with credit constraints compared to that without. Regarding the dynamic development of saving rate levels the present model is less successful in replicating empirical observations. While the drop in the Asian saving rate and the increase in the EMU and US rates can be aligned with data, the only slight decrease of Asian and EMU rates (and even more the slight increase of the US saving rate) in the after-shock period is at odds with data, in particular with respect to EMU periphery and US data. As the numerical experimentation with different values for the intertemporal and intra-temporal elasticity of substitution in consumption shows there is a trade-off between the interest rate elasticity of middle-aged savings and the sizes of trade imbalances. High values for both the intertemporal and intra-temporal elasticity of substitution in consumption induce high trade imbalances but a weak interest rate response of middle-aged savers which is needed for rising saving rates in more constrained and falling saving rates in less constrained economies. It remains to be seen in future research whether a better parameter calibration can alleviate the mentioned trade-off or a further extension of the model through the introduction of asset price bubbles is needed.

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