



GEP 2015–08

**Pension systems and financial constraints
in a three-country OLG model of
intra-EMU and global trade imbalances**

Karl Farmer, Bogdan Mihaiescu

November 2015

Department of Economics
Department of Public Economics
University of Graz

An electronic version of the paper may be downloaded
from the RePEc website: <http://ideas.repec.org/s/grz/wpaper.html>

Pension systems and financial constraints in a three-country OLG model of intra-EMU and global trade imbalances

Karl Farmer and Bogdan Mihaiescu

Abstract

Farmer and Ban (2014) find in a three-country OLG model (= basic model) that financial integration between both EMU core and periphery and between Asia and USA induce trade surpluses in EMU core and in Asia while in EMU periphery and in USA trade balances become negative when the global economy is dynamically inefficient. While exhibiting the right sign, model-generated steady-state trade balance to GDP ratios turn out, however, being too low compared to empirical counterparts. In order to address this problem we first extend the basic OLG model in line with Eugeni (2015) by introducing pay-as-you-go pension systems in EMU and US but not in Asia. Second, we introduce financial constraints following Coeurdacier et al. (2015) to achieve a better data fit compared to the basic model. Both extensions improve the empirical relevance of the basic model.

JEL: F36

Keywords: Trade Imbalances, European Economic and Monetary Union, Overlapping Generations, Three-Country Model, Global imbalances

Introduction and Motivation

Farmer and Ban (2014) constructed a three-country overlapping generations (OLG) model à la Buiter (1981) (= basic model henceforth) in order to explore the effects of the simultaneous financial integration within the EMU and between Asia and the USA on the widening of current account and trade imbalances between 1999 and 2008. Farmer and Ban (2014) find that financial integration between both EMU core and periphery and between Asia and USA induce current account and trade surpluses in EMU core and in Asia while in EMU periphery and in the USA both balances become negative when the global economy is dynamically inefficient. While exhibiting the right sign, model-generated steady-state trade balance to GDP ratios turn out to be too low compared to the empirical counterparts. In order to address this problem we first extend the basic OLG model in line with Eugeni (2015) by introducing pay-as-you-go pension systems in EMU and US but not in Asia. Second, we introduce internationally differing financial constraints to three periods living younger households in line with Coeurdacier et al. (2015) as these authors claim to be able to explain rising global current account imbalances by internationally differing financial (and growth) constraints.

In order to motivate the set-up of the basic model and its extensions we recall key stylized facts regarding intra-EMU and global trade imbalances between the late 1990s and the onset of the global financial crisis in 2007. First, while euro-related financial integration and convergence expectations (Blanchard and Giavazzi, 2002; Lane, 2006; Japelli and Pagano, 2008; Spiegel, 2009; Kalemli-Ozcan et al., 2010, Schmitz and von Hagen, 2011) contributed without doubt to the evolution of intra-EMU trade imbalances, Chen et al. (2013) emphasize extra-EMU factors as (1) the increasing competitive advantage of Asian exporters vis-à-vis EMU periphery exporters, (2) the rising demand for EMU core capital goods by Asia and oil exporters and (3) the US demand for EMU core financial assets. As a consequence, EMU periphery's current account deficit, while financed mostly by capital inflows from the EMU core, did not increase that much vis-à-vis the core but vis-à-vis Asia and oil exporters. Similarly, EMU core's current account surplus after the Euro launch resulted not from EMU periphery's imports but from rising Asian imports of EMU core (capital) goods.

Financial integration occurred not only in the EMU after Euro inception but also between Asia and the USA – albeit under different institutional ramifications than those existing in the Eurozone. The interest convergence between initially higher US and lower Asian rates is seen as a major driver for the emergence of US current account deficits and Asian current account surpluses (Mendoza et al. 2009; Angeletos and Panousi, 2009). As a consequence of high Asian saving rates due to poor public pension systems (Eugeni, 2015) and due to precautionary saving motives after East Asian currency crises in the late 1990s, Asians accumulated huge amounts of save US government bonds inducing the convergence of nominal and real Asian and US interest rates.

Acknowledging both the strengthened real trade linkages of EMU sub-areas vis-à-vis Asia, and the closer financial linkages among Asia and the USA between 1999 and 2008, it is natural to suggest that the intra-EMU external imbalances are related to the external imbalances among Asia and the USA, and vice versa. Farmer and Ban (2014) find that this suggestion can indeed be verified by use of an intertemporal equilibrium model taking into account both the real and financial trade linkages among EMU's core and periphery, Asia and the USA.

In line with Buiter's (1981) seminal one-good, two-country OLG model the authors depict the pre-Euro and pre-East-Asian currency crisis situation with relatively larger real interest rates in EMU periphery and in the USA as financial autarky and the convergence of (real) interest rates within the Eurozone and between US and Asia after the 1998 currency crisis as financial integration. The higher EMU periphery's and US autarky interest rates are attributed to EMU periphery's and US relatively higher future (utility) discount factors, to EMU periphery's and Asia's relatively higher capital production shares and to EMU periphery's and US relatively larger public debt to GDP's ratios.

Comparing the financial autarky to the financial integration steady state Farmer and Ban (2014, 21-22) show that if the world economy is dynamically inefficient, i.e. the natural growth rate is larger than the common real interest rate, the financial integration net foreign asset to GDP ratio and the current account to GDP ratio of EMU periphery and the USA become negative while the corresponding ratios of EMU core and Asia turn out positive. Thus, commencing with zero external imbalances in financial autarky, the basic three-country OLG model is capable to reproduce qualitatively the emergence of both intra-EMU and global external imbalances.

However, the ultimate test for the explanatory power of an intertemporal equilibrium model resides in its capability to reproduce the empirical facts under scrutiny, i.e. the empirically observed intra-EMU and global external imbalances. To this end a numerical specification of the structural and policy parameters of the extended basic model is in order. The parameters of the intertemporal equilibrium model are specified such that the model is capable to reproduce the empirically observed interest rates and other macro variables in the financial autarky situation. In comparing the model generated external imbalances under financial integration to the empirically observed ones', the prospects and limitations of an extended basic model in explaining intra-EMU and global external imbalances can be evaluated.

The paper has thus two main objectives: First, to extend the basic OLG model by introducing pay-as-you-go pension systems in EMU and in USA and commencing with numerical parameters suitable to financial autarky to compare the model-generated financial-integration solution to the empirically observed trade imbalances both within the EMU and between Asia and USA. Second, since we will see that contrary to Eugeni (2015) the numerical values of the financial-integration steady-state trade imbalance ratios are smaller than the non-steady state counterparts, we secondly

extend the basic model in line with Coeurdacier et al. (2015) by introducing internationally differing credit constraints to younger households in EMU, Asia and USA. To this end the life-time planning horizon of younger households consists of three periods: they borrow in the first period of their life cycle in order to finance their consumption expenditures, they supply labor, repay their loan and save for retirement when in middle age, and when retiring in the third period of their life time they finance their consumption expenditures through the return on their assets accumulated in the second period.

The paper is organized as follows. The next section presents the financial integration variant of the basic model with pay-as-you-go systems in EMU and US, but not in Asia. Then the basic model extended along this way is numerically specified for the financial autarky case, and the numerical solution for the intertemporal-equilibrium trade imbalance ratios in the integration period is compared to main stylized macroeconomic facts, related to EMU and Asia-US financial integration up to the onset of the global financial crisis in 2008. In the following section, credit constraints are introduced into the basic model. Concluding remarks in the final section summarize key results.

First Extension of the Basic Model: Pay-as-you-go pension systems in EMU and US

Consider an infinite-horizon model economy consisting of three areas (“countries”) of the world economy, namely (i) the EMU, comprising two regions, named North (indexed by N) representing EMU’s core, and South (indexed by S), representing EMU’s periphery countries, (ii) the countries characterized by a current account surplus outside the EMU (indexed by A) representing Asia, and (iii) the current-account deficit countries (indexed by U) representing mainly the USA. In each country one commodity, representing the aggregate of thousands of goods and services is produced. This can be used for the purpose of consumption as well as for investment. The EMU specializes completely in the production of good X , Asia in the production of good Y , and US in the production of good Z . Perfectly competitive firms in EMU’s South and North, in Asia and in the USA employ in every period $t = 1, 2, \dots$ labor services N_t^i , $i = S, N, A, U$ and capital services K_t^i , $i = S, N, A, U$, using the Cobb-Douglas (CD) production function $M^i (a_t N_t^i)^{1-\alpha^i} (K_t^i)^{\alpha^i}$, $i = S, N, A, U$, to produce southern (northern) EMU aggregate output X_t^S (X_t^N), Asia’s aggregate output Y_t and US aggregate output Z_t where $M^i > 0$, $i = S, N, A, U$ denote total factor productivity in EMU’s South (North), in Asia and in the USA, respectively. a_t is the common labor productivity and $0 < \alpha^i < 1$, $i = S, N, A, U$ with $\alpha^U \approx \alpha^N < \alpha^S < \alpha^A$ are the capital production shares in EMU South, EMU North, Asia and in the USA.

One-period profit maximization by firms in EMU’s South (North), in Asia and in the USA implies the following FOCs:

$$w_t^i = (1 - \alpha^i) M^i a_t (K_t^i / a_t N_t^i)^{\alpha^i}, i = S, N, A, U, \quad (1)$$

$$q_t^i = \alpha^i M^i (K_t^i / a_t N_t^i)^{\alpha^i - 1}, i = S, N, A, U, \quad (2)$$

whereby $w_t^i, i = S, N, A, U$ denotes the real wage rate in each region and each country. $q_t^i, i = S, N, A, U$ denotes real unit capital user costs in each region and each country $i = S, N, A, U$.

As usual in a Diamond (1965) type OLG framework, two generations of homogeneous individuals overlap in each period t . At date t , a new generation of size L_t^i enters the economy of country (region) $i = S, N, A, U$. For simplicity we assume that the population growth factors of all countries (regions) are identical and are equal to G^L . In view of the empirically rather similar GDP growth rates in southern and northern EMU countries (Fagan and Gaspar 2008) we assume that the respective growth factors of labor productivities G^{a^S} and G^{a^N} are equal in EMU's South and North, an assumption which also applies rather well to the USA, but rather less so to current-account surplus countries like China, India and other Asian countries. However, taking account of the catch-up growth component in emerging countries' GDP growth rates the simplifying assumption $G^{a^S} = G^{a^N} = G^{a^U} = G^{a^A} = G^a$ seems as first pass acceptable. This implies that the natural growth factor $G^n = G^a G^L$ is the same in all countries.

In this section each generation lives for two periods, working during the first when young, and retiring in the second when old. The choice variables of each generation, when young, are denoted by superscript 1, and, when old, they are denoted by superscript 2. For each member of the generation entering the economy in period t , the supply of labor to firms is wage-inelastic since households attribute no value to leisure.

In order to describe the optimization problems of households more specifically the institutional framework regarding international transactions across the three countries and across EMU core and periphery is now addressed. Regarding the three countries, we assume that each country has its own currency and that before the inception of the EMU, the southern and northern EMU member countries had their own currency, too. To mimic the period before the introduction of the common currency in 1999 we follow Gourinchas and Jeanne (2006) as well as Fagan and Gaspar (2008), and assume that before 1999, EMU's South and North were financially autarkic. In contrast to the de-facto financial relationships between subsequent EMU countries, Asia and the USA which existed before euro inception, we also assume financial autarky for Asia and the USA in the pre-euro period. In contrast to financial autarky, we do, however, allow for trade relations between later EMU, Asia and US during the pre-euro period, albeit on a moderate and balanced scale, thus mimicking the fact of mainly Japanese trade linkages vis-à-vis later EMU countries and the US. China and India did not play any important role in international trade during the pre-euro period. Furthermore we assume, in line with Eugeni (2015), that EMU North and South, as well as the US, but not Asia, have adopted a pay-as-you-go pension system that shifts

some of the younger households' income towards the old households living during the same time period.

Complete nominal, and to a lesser extent, real interest convergence across EMU's South and North after the euro launch signifies financial integration across EMU's South and North. This stylized fact is portrayed in our intertemporal equilibrium model in line with Fagan and Gaspar (2008) as an equality of real interest rates of southern and northern EMU countries along the intertemporal equilibrium path. While by no means as complete as that within EMU, there is also some real interest convergence or financial integration across Asia and USA in the early 2000s. We take this stylized fact as support for our rather strong modeling assumption that after the euro launch an uncovered parity condition, in terms of real interest rates, holds across both Asia and the USA. In line with the empirical fact (Chen et al. 2013) that investors from outside EMU invested their wealth in northern EMU financial assets we also assume an uncovered real interest parity condition between USA and EMU. In other words: after euro inception financial integration prevails worldwide but not as strictly as within the EMU.

In order to work out the consequences of intra-EMU, Asian and US financial integration and the trade developments of EMU vis-à-vis non-EMU countries as clearly as possible, the optimization problems of (younger) households and firms as well as the market clearing conditions are now described for intra-EMU, Asian and US financial integration.

Against this empirical background of stylized facts and simplifying modeling assumptions the intertemporal utility maximization problem in later EMU's South after euro inception (= financial integration) reads as follows:

$$\max \rightarrow \zeta^x \ln x_t^{S,1} + \zeta^y \ln y_t^{S,1} + \zeta^z \ln z_t^{S,1} + \beta^S (\zeta^x \ln x_{t+1}^{S,2} + \zeta^y \ln y_{t+1}^{S,2} + \zeta^z \ln z_{t+1}^{S,2})$$

s. t.:

$$x_t^{S,1} + (1/e_t^A) y_t^{S,1} + (1/e_t^U) z_t^{S,1} + s_t^S = w_t^S (1 - \tau_{G,t}^S - \tau_{p,t}^S), \text{ with } s_t^S \equiv K_{t+1}^{S,S} / L_t + B_{t+1}^{S,S} / L_t,$$

$$(ii) x_{t+1}^{S,2} + (1/e_{t+1}^A) y_{t+1}^{S,2} + (1/e_{t+1}^U) z_{t+1}^{S,2} = q_{t+1}^S (K_{t+1}^{S,S} / L_t) + (1 + i_{t+1}^S) (B_{t+1}^{S,S} / L_t) + G^n \tau_{p,t}^S.$$

Here $0 < \beta^S \leq 1$ denotes the time discount factor of EMU's southern younger generation, ζ^k , $k = x, y, z$ with $\zeta^x + \zeta^y + \zeta^z = 1$ represents the utility elasticity of the consumption of good k , $x_t^{S,1}$ is the consumption per capita of the commodity produced in EMU's South acquired at unit relative price, $y_t^{S,1}$ is South's consumption of the Asian good bought at the relative price of $1/e_t^A$, and $z_t^{S,1}$ is southern consumption of the US good acquired at the relative price of $1/e_t^U$. e_t^A denotes the units of the Asian good per unit of EMU good (= EMU terms of trade vis-à-vis Asia), while e_t^U portrays the units of the US good per unit of EMU good (=EMU terms of trade vis-à-vis USA). s_t^S is South's per capita savings, $\tau_{G,t}^S$ denotes region South's flat wage tax rate (which is used to fund the government) and $\tau_{p,t}^S$ its contribution to a pay-as-you-go pension system (where each young household contributes in order to finance the old households living at that moment in time). Since $G^n > 1$ for all countries, this pension fund will always stay solvent, as there will

always be more young households compared to old ones. $G^n \tau_{p,t}^S$ denotes the income gained by old southern households thanks to the pension system in place, whilst $x_{t+1}^{S,2} (y_{t+1}^{S,2}, z_{t+1}^{S,2})$ is old-age consumption per capita of the commodity produced in South (Asia, USA), $x_{t+1}^{S,2} (y_{t+1}^{S,2}, z_{t+1}^{S,2})$ is old-age consumption per capita of the commodity produced in South (Asia, USA), $K_{t+1}^{S,S} / L_t^S$ is real capital produced in EMU South which South's younger household wants to hold at the beginning of the retirement period, $B_{t+1}^{S,S} / L_t^S$ stands for EMU South government bonds which South's younger household wants to hold at the beginning of its retirement period and i_{t+1}^S denotes the real interest rate on southern EMU government bonds. In line with pre-crisis empirical reality, the southern EMU young household invests its savings only in domestic real capital and government bonds. Constraint (i) depicts the working period budget constraint while constraint (ii) represents the retirement period budget constraint.

To mimic the financial integration arising through the set-up of the EMU and the Asian-US financial integration we assume in line with stylized facts (Chen et al. 2013) that northern EMU invests its savings in southern physical capital¹ and government bonds, that Asia buys US government bonds, and that the USA purchases northern EMU government bonds without incurring any transaction costs. However, also in line with empirical data, we assume that the southern EMU young household buys neither northern real capital, nor northern government bonds nor Asian or US assets.

Thus, the intertemporal utility maximization problem of the typical northern EMU household reads as follows:

$$\begin{aligned} \text{Max} \rightarrow & \zeta^x \ln x_t^{N,1} + \zeta^y \ln y_t^{N,1} + \zeta^z \ln z_t^{N,1} + \beta^N (\zeta^x \ln x_{t+1}^{N,2} + \zeta^y \ln y_{t+1}^{N,2} + \zeta^z \ln z_{t+1}^{N,2}) \\ \text{s.t. :} & \\ (i) & x_t^{N,1} + (y_t^{N,1} / e_t^A) + (z_t^{N,1} / e_t^U) + s_t^N = w_t^N (1 - \tau_{G,t}^N - \tau_{p,t}^N), \quad s_t^N \equiv K_{t+1}^{N,N} / L_t^N + K_{t+1}^{S,N} / L_t^N + B_{t+1}^{N,N} / L_t^N + B_{t+1}^{S,N} / L_t^N, \\ (ii) & x_{t+1}^{N,2} + (y_{t+1}^{N,2} / e_{t+1}^A) + (z_{t+1}^{N,2} / e_{t+1}^U) = q_{t+1}^N (K_{t+1}^{N,N} / L_t^N) + q_{t+1}^S (K_{t+1}^{S,N} / L_t^N) + (1 + i_{t+1}^N) (B_{t+1}^{N,N} / L_t^N) + (1 + i_{t+1}^S) (B_{t+1}^{S,N} / L_t^N) \\ & + G^n \tau_{p,t}^N. \end{aligned}$$

Here, $x_t^{N,1} (y_t^{N,1}, z_t^{N,1})$ stands for the purchases of later EMU (Asian, US) goods by EMU North young household, with $s_t^N, \tau_{G,t}^N, \tau_{p,t}^N, K_{t+1}^{N,N}, B_{t+1}^{N,N}$ being interpreted in a similar fashion to that stated for the corresponding variables in EMU South. Moreover, $K_{t+1}^{S,N} / L_t$ and $B_{t+1}^{S,N} / L_t$ denote the respective stocks of southern real capital and government bonds which the northern EMU young household wants to hold at the beginning of period $t + 1$.

The typical Asian young household solves the following optimization problem:

$$\begin{aligned} \text{Max} \rightarrow & \zeta^x \ln x_t^{A,1} + \zeta^y \ln y_t^{A,1} + \zeta^z \ln z_t^{A,1} + \beta^A (\zeta^x \ln x_{t+1}^{A,2} + \zeta^y \ln y_{t+1}^{A,2} + \zeta^z \ln z_{t+1}^{A,2}) \\ \text{s.t. :} & \\ (i) & e_t^A x_t^{A,1} + y_t^{A,1} + (e_t^A / e_t^U) z_t^{A,1} + s_t^A = w_t^A (1 - \tau_{G,t}^A), \quad s_t^A = K_{t+1}^{A,A} / L_t^A + B_{t+1}^{A,A} / L_t^A + (e_t^A / e_t^U) (B_{t+1}^{U,A} / L_t^A), \\ (ii) & e_{t+1}^A x_{t+1}^{A,2} + y_{t+2}^{A,2} + (e_{t+1}^A / e_{t+1}^U) z_{t+1}^{A,2} = q_{t+1}^A (K_{t+1}^{A,A} / L_t^A) + (1 + i_{t+1}^A) (B_{t+1}^{A,A} / L_t^A) + (1 + i_{t+1}^U) (e_{t+1}^A / e_{t+1}^U) (B_{t+1}^{U,A} / L_t^A). \end{aligned}$$

¹ To mimic the facts presented in Figures 5 and 6 below we assume that physical capital is mainly accumulated through housing investment.

Here, $x_t^{A,1}$ stands for the purchases (= consumption) of later EMU goods by the Asian young household at the relative price of e_t^A , while the purchase of the US product by the Asian young household occurs at the relative price e_t^A / e_t^U , i.e. units of the Asian product per unit of the US good. Moreover, $B_{t+1}^{U,A} / L_t^A$ denotes the stock of US government bonds which the Asian young household wants to hold at the beginning of period $t+1$. In line with pre-financial-crisis reality the Asian young household does not hold EMU government bonds. Unlike northern, southern or US households, Asian households neither contribute to a pension system nor receive payments as part of it, meaning they need to finance all of their old-age consumption via the savings in capital and government bonds they accumulate in their first period. All other variables may be interpreted similarly to those in EMU South's young household optimization problem.

Finally, the typical US young household faces the following optimization problem:

$$\begin{aligned} \text{Max} \rightarrow & \zeta^x \ln x_t^{U,1} + \zeta^y \ln y_t^{U,1} + \zeta^z \ln z_t^{U,1} + \beta^U (\zeta^x \ln x_{t+1}^{U,2} + \zeta^y \ln y_{t+1}^{U,2} + \zeta^z \ln z_{t+1}^{U,2}) \\ \text{s.t. :} & \\ (i) & e_t^U x_t^{U,1} + (e_t^U / e_t^A) y_t^{U,1} + z_t^{U,1} + s_t^U = w_t^U (1 - \tau_{G,t}^U - \tau_{p,t}^U), \quad s_t^U = K_{t+1}^{U,U} / L_t^U + B_{t+1}^{U,U} / L_t^U + e_t^U (B_{t+1}^{N,U} / L_t^U), \\ (ii) & e_{t+1}^U x_{t+1}^{U,2} + (e_{t+1}^U / e_{t+1}^A) y_{t+1}^{U,2} + z_{t+1}^{U,2} = q_{t+1}^U (K_{t+1}^{U,U} / L_t^U) + (1 + i_{t+1}^U) (B_{t+1}^{U,U} / L_t^U) + (1 + i_{t+1}^N) e_{t+1}^U (B_{t+1}^{N,U} / L_t^U) \\ & + G^n \tau_{p,t}^N. \end{aligned}$$

Here $x_t^{U,1}$ stands for US young household's purchases of the EMU product while e_t^U / e_t^A now indicates the units of the US product per unit of the Asian product which equals the relative price of US consumption for the Asian good, $y_t^{U,1}$. In line with pre-financial-crisis empirical reality, the US young household does hold only northern EMU government bonds.

The government of each country (region) $i = S, N, A, U$ taxes labor income and uses the proceeds of additional borrowing to finance the interest costs on existing government debt and government expenditures. The government budget constraint of country (region) i reads as follows:

$$B_{t+1}^i - B_t^i + \tau_t^i w_t^i L_t^i = i_t^i B_t^i + \Gamma_t^i, \quad i = S, N, A, U, \quad t = 0, 1, 2, \dots, \quad (3)$$

where Γ_t^i denotes real government expenditures and B_t^i is the level of real government debt in country (region) $i = S, N, A, U$ at the beginning of period t . In line with Diamond (1965), we assume that government expenditures are unproductive.

In addition to the restrictions imposed by household and firm optimization and by the above government budget constraints, markets for labor have to clear in all countries (regions) and in all periods.

$$N_t^i = L_t^i, \quad i = S, N, A, U, \quad t = 0, 1, 2, \dots \quad (4)$$

Since the asset markets are competitive, transaction and adjustment costs do not occur, no risk (aversion) prevails, the following no-arbitrage condition (= national Fisher equation) holds in all countries (regions):

$$1 + i_{t+1}^i = q_{t+1}^i + 1 - \delta, \quad i = S, N, A, U, \quad t = 0, 1, 2, \dots, \quad (5)$$

whereby $0 < \delta \leq 1$ depicts the common fixed depreciation rate of private capital (period by period) in country (region) i .

Since physical capital and government bonds in each EMU region are perfectly substitutable, the following international Fisher equation (=international real interest parity condition) holds in addition to the national Fisher equations (5):

$$1 + i_{t+1}^S = 1 + i_{t+1}^N. \quad (6)$$

In order to ensure arbitrage-free terms of trade, the following international real interest parity conditions in addition to (6) ought to hold:

$$1 + i_{t+1}^A = (e_{t+1}^A/e_t^A)(1 + i_{t+1}^N), \quad \forall t = 0, 1, 2, \dots, \quad (7)$$

$$1 + i_{t+1}^U = (e_{t+1}^U/e_t^U)(1 + i_{t+1}^N), \quad \forall t = 0, 1, 2, \dots. \quad (8)$$

The markets for southern and northern EMU, Asian and US real capital clear according to:

$$K_{t+1}^S = K_{t+1}^{S,S} + K_{t+1}^{S,N}, \quad K_{t+1}^N = K_{t+1}^{N,N}, \quad K_{t+1}^A = K_{t+1}^{A,A}, \quad K_{t+1}^U = K_{t+1}^{U,U}, \quad t = 0, 1, 2, \dots \quad (9)$$

The markets for southern and northern EMU, Asian and US government bonds clear according to:

$$B_{t+1}^S = B_{t+1}^{S,S} + B_{t+1}^{S,N}, \quad B_{t+1}^N = B_{t+1}^{N,N} + B_{t+1}^{N,U}, \quad B_{t+1}^A = B_{t+1}^{A,A}, \quad B_{t+1}^U = B_{t+1}^{U,U} + B_{t+1}^{U,A}, \quad t = 0, 1, 2, \dots \quad (10)$$

The real interest parity condition (6) and the open real interest parity conditions (7) and (8) ensure that the worldwide amount of savings equals the worldwide supply of assets from southern and northern EMU, Asia and the US. Thus:

$$L_t s_t^S + L_t s_t^N + (L_t^A s_t^A)/e_t^A + (L_t^U s_t^U)/e_t^U = K_{t+1}^S + K_{t+1}^N + B_{t+1}^S + B_{t+1}^N + (K_{t+1}^A + B_{t+1}^A)/e_t^A + (K_{t+1}^U + B_{t+1}^U)/e_t^U. \quad (11)$$

Finally, the product markets in EMU, Asia and US clear for all $t = 0, 1, 2, \dots$ according to the following conditions:

$$X_t^S + X_t^N = L_t x_t^{S,1} + L_{t-1} x_t^{S,2} + \Gamma_t^S + K_{t+1}^S + L_t x_t^{N,1} + L_{t-1} x_t^{N,2} + \Gamma_t^N + K_{t+1}^N + L_t^A x_t^{A,1} + L_{t-1}^A x_t^{A,2} + L_t^U x_t^{U,1} + L_{t-1}^U x_t^{U,2}, \quad (12)$$

$$Y_t = L_t^A y_t^{A,1} + L_{t-1}^A y_t^{A,2} + \Gamma_t^A + K_{t+1}^A + L_t y_t^{S,1} + L_{t-1} y_t^{S,2} + L_t y_t^{N,1} + L_{t-1} y_t^{N,2} + L_t^U y_t^{U,1} + L_{t-1}^U y_t^{U,2}, \quad (13)$$

$$Z_t = L_t^U z_t^{U,1} + L_{t-1}^U z_t^{U,2} + \Gamma_t^U + K_{t+1}^U + L_t z_t^{S,1} + L_{t-1} z_t^{S,2} + L_t z_t^{N,1} + L_{t-1} z_t^{N,2} + L_t^A z_t^{A,1} + L_{t-1}^A z_t^{A,2}. \quad (14)$$

Numerical Specification and Model Generated versus Stylized Macro Facts

The numerical specification of the basic model extended by pension systems in EMU and US is used to illustrate both the explanatory power and the limitations of our three-country OLG model. As mentioned above, structural and policy parameters are specified such that the diverging real interest rates between EMU core and periphery and between Asia and USA as well as their approximately balanced current accounts and zero net foreign asset positions in the 1990s (before euro inception and the East-Asian currency crises) are reproduced by the financial autarky version of the basic model.

The structural parameters are specified as follows: $G^n = 2$, $\zeta^x = 0.4$, $\zeta^y = 0.3$, $\zeta^z = 0.3$, $\beta^s = 0.5$, $\beta^N = 0.55$, $\beta^A = 0.65$, $\beta^U = 0.47$, $\alpha^S = 0.17$, $\alpha^N = 0.15$, $\alpha^A = 0.18$, $\alpha^U = 0.16$, $M^S = 1$, $M^N = 1.2$, $M^A = 0.8$, $M^U = 1.4$, $L^S = L^N = 85$, $L^A = 1200$, $L^U = 120$. Before introducing the policy parameter it is worthwhile to mention that all endogenous and policy variables are transformed into national GDP ratios denoted by lower case letters: $\gamma_t^S \equiv \Gamma_t^S / X_t^S$, $\gamma_t^N \equiv \Gamma_t^N / X_t^N$, $\gamma_t^A \equiv \Gamma_t^A / Y_t$, $\gamma_t^U \equiv \Gamma_t^U / Z_t$ and $b_t^S \equiv B_t^S / X_t^S$, $b_t^N \equiv B_t^N / X_t^N$, $b_t^A \equiv B_t^A / Y_t$, $b_t^U \equiv B_t^U / Z_t$. Since we want to focus on the consequences of world-wide financial integration on external balances we assume roughly in line with empirical reality that both the government expenditure and the debt to GDP ratios remain constant over time, i.e. $\gamma_t^i = \gamma_{t+1}^i = \gamma^i$, $\tau_{p,t}^i = \tau_{p,t+1}^i = \tau_p^i$ and $b_t^i = b_{t+1}^i = b^i$, $i = S, N, A, U$. Under these stationarity assumptions we specify the policy parameters as follows: $\gamma^S = 0.16$, $\gamma^N = 0.2$, $\gamma^A = 0.15$, $\gamma^U = 0.16$, $b^S = 0.025$, $b^N = 0.023$, $b^A = 0.015$, $b^U = 0.024$, $\tau_p^S = \tau_p^N = \tau_p^U = 0.005$.

Table 1 reports the steady state values of main endogenous variables in EMU South, EMU North, in Asia and in USA under financial autarky.

Table 1 Main endogenous variables (as ratio to GDP) in EMU South, EMU North, in Asia and in USA (calculated on a yearly basis) under *financial autarky*

	Capital Output ratio	Real interest rate (in %)	EMU terms of trade relative to	Trade imbalance to GDP ratio	Ratio of Net Foreign Assets to GDP
EMU South	2.06	2.94		0	0
EMU North	2.23	2.09		0	0
Asia	2.96	1.69	4.27	0	0
USA	2.02	2.77	1.30	0	0

Source: Own calculations

Note that the yearly real interest rate in EMU South, North, in Asia and in USA exhibited in Table 1 is not too far from the real interest rates in the late 1990s portrayed in Fig. 1 and Fig. 2.²

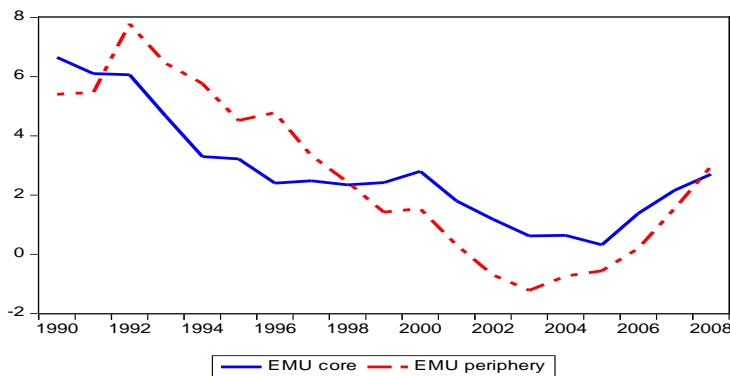


Fig. 1 Real short-term interest rates in EMU core and periphery 1990-2008. Source: AMECO

² Following Fagan and Gaspar (2008, p. 9), the EMU countries are separated into two groups based on the differences in short-term real interest rates in the late 1990s, i.e. before the euro launch. The first group, usually denoted as the “core” countries, comprises the low interest rate countries Austria, Belgium, France, Germany and the Netherlands. The second group, denoted as “periphery” or converging countries, consists of countries which had relatively high interest rates before the introduction of the euro (see Fig. 1).

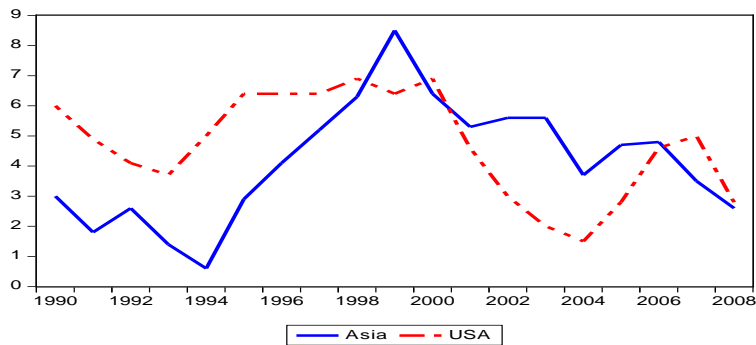


Fig. 2 Real short-term interest rates in USA and Asia 1990-2008. Source: World Bank: World Development Indicators

Financial autarky implies also zero net foreign asset positions. This is true for the net foreign asset position of EMU core but lesser so for EMU periphery's net foreign asset position in 1990s (see Fig. 3).

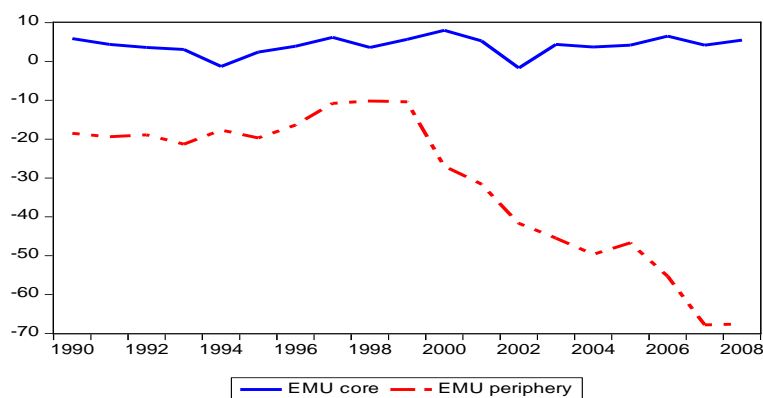


Fig. 3 Net foreign assets (as percent of GDP) in EMU core and periphery 1990-2008. Source: updated and extended version of a dataset constructed by Lane and Milesi-Ferretti (2007)

While the financial autarky assumption of zero net foreign asset position applies well to the US, it is much lesser true with respect to the Asian net foreign asset position in 1990s (see Fig. 4).

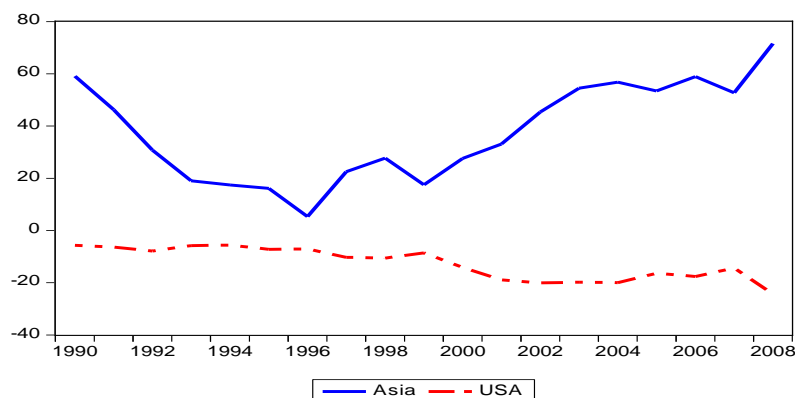


Fig. 4 Net foreign asset positions (in percent to GDP) of Asia and USA 1990-2008. Source: updated and extended version of a dataset constructed by Lane and Milesi-Ferretti (2007)

While in view of the data presented in Figures 3 and 4 the assumption of zero net foreign asset positions in financial autarky is too restrictive with respect to EMU South and Asia it is warranted as a simplifying assumption, in particular, if compared to the evolution of the net foreign asset

positions under financial integration in the 2000s. This brings us to the model generated results with respect to main endogenous variables in EMU, Asia and USA presented in Table 2.

Table 2 Main endogenous variables in EMU South, EMU North, in Asia and in USA (calculated on a yearly basis) under *financial integration* in steady state

	Capital Output ratio	Real interest rate (in %)	Net Foreign Assets to GDP (%)	Trade balance to GDP ratio (%)	Trade balance to GDP ratio (%) (Shock period)	EMU Terms of trade vis-à-vis	Saving ratio (Shock period)	Saving ratio (steady state)
EMU South	2.42	2.27	-34.17	-0.34	-3.51		0.2143	0.2166
EMU North	2.14	2.27	+15.40	+ 0.088	+1.17		0.2248	0.2242
Asia	2.57	2.27	+37.61	+ 0.37	+3.25	4.36	0.2664	0.2654
USA	2.28	2.27	-24.79	-0.24	-2.18	1.29	0.2091	0.2107

Source: Own calculation

Before comparing the results in Table 2 to those in Table 1, the definitions of the net foreign asset to GDP ratio, the current account to GDP ratio and the trade balance to GDP ratio, respectively, need to be introduced. The steady state ratio of the net foreign asset position to GDP in country $i = S, N, A, U$ denoted by ϕ^i is defined as $\phi^i = (\sigma^i / G^n)[1 - \alpha^i - \gamma^i - (\alpha^i b^i) / v^i] - [v^i + b^i(1 - \sigma^i)]$. Using the budget constraints of young and old households plus the zero-profit and market clearing conditions one can show³ that in steady state the trade balance to GDP ratio denoted as $tb^i, i = S, N, A, U$ is related to the respective net foreign asset to GDP ratio as follows: $tb^i = [G^n - (1 + i)]\phi^i$.

The comparison of the results in Table 2 to those in Table 1 reveals both satisfactory and non-satisfactory facts. Regarding the satisfactory results we observe: First, through financial integration (equalization of real interest rates across EMU, Asia and USA) the capital output ratio both in EMU South and US increases while it decreases in EMU North and in Asia in line with empirical observations regarding construction and housing investment (see Figures 5 and 6).

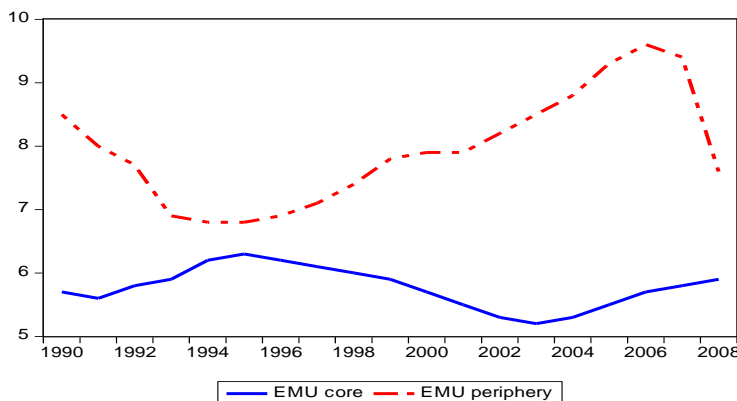


Fig. 5 Housing investment (as percent of GDP) in EMU periphery and core 1990-2008. Source: AMECO

³ Farmer and Ban (2015) derive the relationships between the ratios of the net foreign asset position and the trade balance to GDP in non-steady state intertemporal equilibrium from which the steady state relation in text follows.

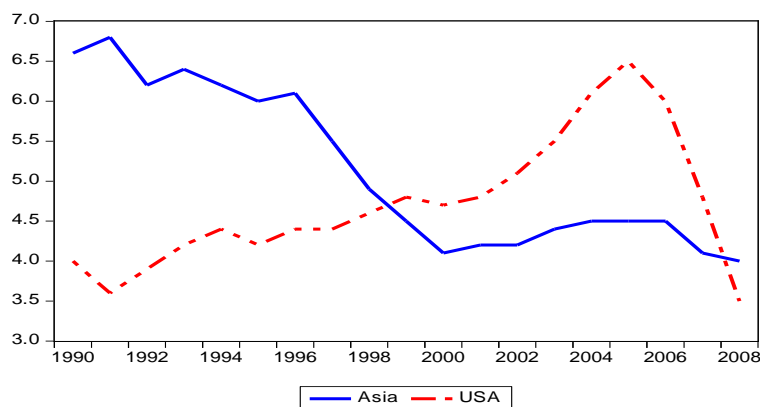


Fig. 6 Housing investment (as percent of GDP) in Asia and in USA 1990-2008. Source: BEA National Economic Accounts

Second, the common real interest rate is significantly lower than the autarky real interest rate in EMU South and in US and higher than the autarky real interest rate in EMU North and Asia, also in line with empirical facts (see Figures 1 and 2 above). Third, the net foreign asset to GDP ratios exhibited in the fourth column of Table 2 feature rather close to those portrayed in Figures 3 and 4. It is also worth mentioning that the huge intra-EMU imbalance with respect to the net foreign asset to GDP ratio can be much better reproduced by our three-country than by Farmer's (2014) two-country model. Fourth, the trade imbalance to GDP ratios for the integration shock period portrayed in the sixth column of Table 2 reproduce rather well the empirically observed trade imbalance to GDP ratios for EMU core and periphery as shown in Figure 7 below and for Asia and US as depicted in Figure 8 below.

Regarding the non-satisfactory results, we observe from Table 2 that the steady state trade imbalance to GDP ratios before 2008 feature too small in comparison to the empirical facts portrayed in figures 7 and 8 below.

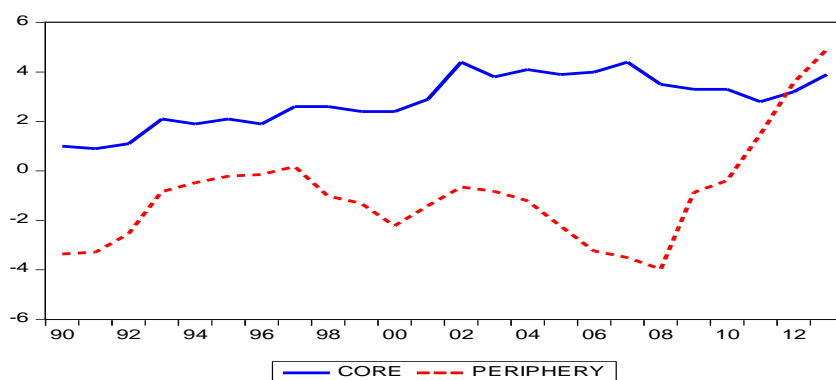


Fig. 7: Trade balance on goods and services (as percent of GDP) in EMU core and periphery 1990-2008. Source: World Bank Indicators.

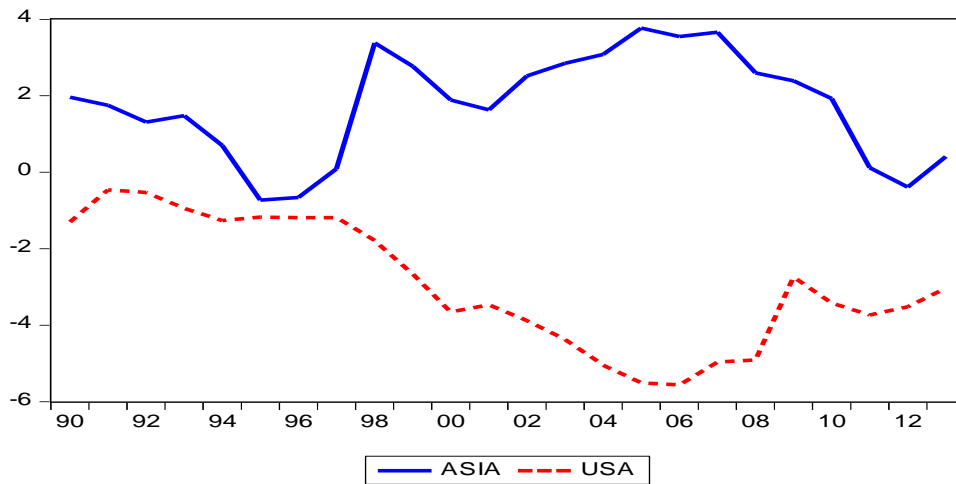


Fig. 8: Trade balance on goods and services (as percent of GDP) in Asia and USA 1990-2008. Source: World Bank Indicators.

However, the reader must not overrate this non-satisfactory result. Cova et al. (2009) obtain in a comparable intertemporal equilibrium model of the world economy similarly small steady-state trade imbalance to GDP ratios, albeit with opposite signs compared to ours. This observation suggests that the performance of our extended three-country OLG to reproduce the empirical data ought not be evaluated by the comparison of the magnitudes of steady state (model generated) trade imbalance to GDP ratios to the empirical counterparts for the financial integration period 1999-2008. It seems to be better to evaluate the data-reproduction performance of the model by comparing the magnitudes of the non-steady state trade imbalance to GDP ratios in the shock period (see the sixth column in Table 2) to the corresponding empirical values portrayed in Figures 7 and 8. This comparison reveals that the model generated trade imbalance to GDP ratios for EMU South and Asia are not far from the corresponding empirical values while significant discrepancies remain for EMU North and the USA. True, one can argue (as Farmer and Ban, 2015 do) that the empirical values for the US trade imbalance to GDP ratios depicted in Figure 8 is by 2 percentage points too large since it includes petroleum imports which are not part of our model. Respecting this fact the discrepancy between model generated and empirical US trade imbalance to GDP ratios gets much smaller.

While the US model-data discrepancy can be explained by adjusting the data to the model, the positive divergence of the observed trade surplus to GDP ratio in EMU North in Figure 7 from the model generated magnitude in Table 2 necessitates the adjustment of the model to the data, and not the other way round. There are several reasons for this claim (see Farmer and Ban, 2015) but more important than this reasoning is a discrepancy between the model-generated evolution of private saving rates and the empirically observed evolution portrayed in Figures 9 and 10 below.

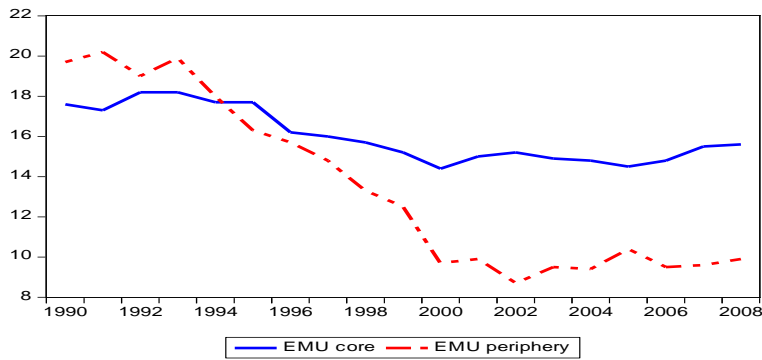


Fig. 9 Personal saving rates in EMU core and periphery 1995-2008. Source: AMECO.

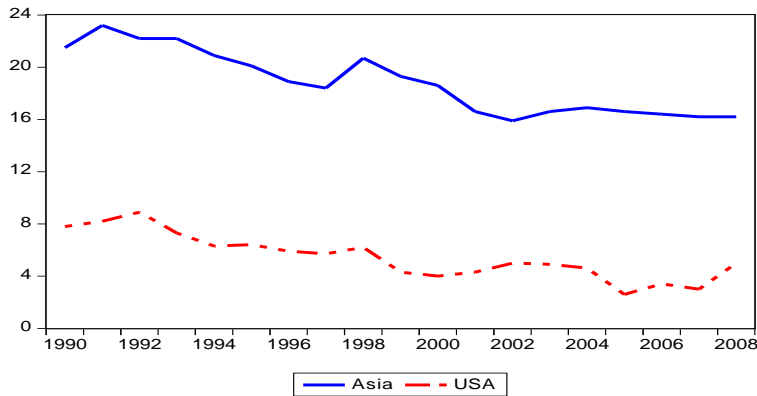


Fig. 10 Asian and US personal saving rates 1990-2008. Source: Source: China Statistical Yearbook Database, CEIC, AMECO, Economic Statistics System, Bank of Korea, FRED.

Figure 9 shows rather similar personal saving in EMU South and EMU North in the 1990s and significantly diverging saving rates in 2000s. In contrast, Figure 10 features significantly higher personal saving rates in Asia than in USA already in the 1990s. Common to both figures is a marked decrease of saving rates in EMU South and USA, and only first a decrease and in the second halve of 2000s even a slight increase of saving rates in EMU North and Asia. Contrary to these observations. the three-country model with pension systems in EMU and US but not in Asia generates rising EMU South and US as well as declining EMU core and Asian personal saving rates. Coeurdacier et al. (2015) forcefully point to this dynamic saving rates divergence (between Asia and US) as one major macro trend of the global economy since the beginning 2000s. These authors attribute the US-Asian divergence of saving rates to the interaction between GDP growth differentials and household financial (credit) constraints – more severe in fast growing Asia than in the relatively slowly growing USA. This suggestion to explain theoretically the dynamic saving rates divergence brings us to the second extension of the basic model.

Second extension: Introducing households' financial constraints and endogenous growth

As mentioned in the Introduction the planning horizon of younger households in all countries consists now of three rather than two periods: they offer private bonds when young (indexed by superscript 1), labor when in middle age (superscript 2), and they consume the returns on their assets accumulated in the second period when old (superscript 3).

In particular, the Southern younger household maximizes the following three-period intertemporal utility function subject to the following three temporal budget constraints:

$$\begin{aligned} \max \rightarrow & \zeta^x \ln x_t^{S,1} + \zeta^y \ln y_t^{S,1} + \zeta^z \ln z_t^{S,1} + \ln f_t^S + \beta^S (\zeta^x \ln x_{t+1}^{S,2} + \zeta^y \ln y_{t+1}^{S,2} + \zeta^z \ln z_{t+1}^{S,2}) \\ & + (\beta^S)^2 (\zeta^x \ln x_{t+2}^{S,3} + \zeta^y \ln y_{t+2}^{S,3} + \zeta^z \ln z_{t+2}^{S,3}) \end{aligned}$$

s. t.:

$$\begin{aligned} (i) \quad & x_t^{S,1} + (1/e^A) y_t^{S,1} + (1/e^U) z_t^{S,1} = b_{t+1}^{S,1}, \\ (ii) \quad & x_{t+1}^{S,2} + \frac{y_{t+1}^{S,2}}{e_{t+1}^A} + \frac{z_{t+1}^{S,2}}{e_{t+1}^U} + s_{t+1}^{S,2} = h_{t+1}^S w_{t+1}^S (1 - \tau_{t+1}^S) - (1 + i_{t+1}^S) b_{t+1}^{S,1}, \end{aligned}$$

$$\text{with } s_{t+1}^{S,2} = (K_{t+2}^{S,S}/L_{t+1}^{S,2}) + (B_{t+2}^{S,S}/L_{t+1}^{S,2}) + b_{t+2}^{SS,2}, b_{t+1}^{S,1} \leq \theta^S \frac{w_{t+1}^S h_{t+1}^S}{1 + i_{t+1}^S}, h_{t+1}^S = h_0^S [(1 - f_t^S) h_t^S]^\mu \left(\frac{\Delta_t^S}{L_t^{S,1}} \right)^{1-\mu},$$

$$(iii) \quad x_{t+2}^{S,3} + \frac{y_{t+2}^{S,3}}{e_{t+2}^A} + \frac{z_{t+2}^{S,3}}{e_{t+2}^U} = \frac{q_{t+2}^S K_{t+2}^{S,S}}{L_{t+1}^{S,2}} + \frac{(1 + i_{t+2}^S) B_{t+2}^{S,S}}{L_{t+1}^{S,2}} + (1 + i_{t+2}^S) b_{t+2}^{SS,2}.$$

In contrast to the two-period intertemporal optimization problem of the Southern younger household in the model with pension systems, first-period consumption expenditures are now covered by the revenues from private bond selling, $b_{t+1}^{S,1}$, which ought to be repaid in the second period including the interest rate on Southern private bonds, i_{t+1}^S (see budget constraint (ii)). In order to address international GDP growth rates differentials, the Southern younger household uses part of her leisure time to invest in human capital accumulation with f_t^S denoting Southern leisure time, h_t^S representing the stock of Southern human capital, $0 < \mu < 1$ indicating the Southern future human capital stock elasticity of non-leisure time allocated to private education and $1 - \mu$ denoting human capital accumulation elasticity with respect to public per-capita human capital investment expenditures $\Delta_t^S / L_t^{S,1}$. Second-period consumption expenditures and second-period savings $s_{t+1}^{S,2}$ are to be equal to effective net wage $h_{t+1}^S w_{t+1}^S (1 - \tau_{t+1}^S)$ minus private credit repayments. Southern younger household's borrowing in the first period is constrained by a fraction θ^S of the present value of her future labor income. In line with Coeurdacier et al. (2015) credit constraints are assumed to be binding an assumption which is satisfied when $\theta^S < 1/[1 + \beta^S + (\beta^S)^2]$. Southern middle-age savings $s_{t+1}^{S,2}$ are used to buy Southern real capital per capita $K_{t+2}^{S,S}/L_{t+1}^{S,2}$, Southern government bonds per capita $B_{t+2}^{S,S}/L_{t+1}^{S,2}$ and Southern private bonds $b_{t+2}^{SS,2}$. Old-age consumption expenditures are financed by the revenues from renting real capital services to Southern firms, $q_{t+2}^S K_{t+2}^{S,S}/L_{t+1}^{S,2}$, and from repaid public and private bonds (inclusive of interest) $(1 + i_{t+2}^S) B_{t+2}^{S,S}/L_{t+1}^{S,2} + (1 + i_{t+2}^S) b_{t+2}^{SS,2}$. In line with pre-financial crisis experience in EMU South the interest rates on public and private bonds are assumed to be equal.

In order to save on space the intertemporal optimization calculi of younger households in EMU North, Asia and the USA are merely formally stated and only the differences with respect to the Southern EMU younger household are described in words.

The intertemporal optimization problem of the typical northern EMU younger household reads as follows:

$$\begin{aligned} \max \rightarrow & \zeta^x \ln x_t^{N,1} + \zeta^y \ln y_t^{N,1} + \zeta^z \ln z_t^{N,1} + \ln f_t^N + \beta^N (\zeta^x \ln x_{t+1}^{N,2} + \zeta^y \ln y_{t+1}^{N,2} + \zeta^z \ln z_{t+1}^{N,2}) \\ & + (\beta^S)^2 (\zeta^x \ln x_{t+2}^{N,3} + \zeta^y \ln y_{t+2}^{N,3} + \zeta^z \ln z_{t+2}^{N,3}) \end{aligned}$$

s. t.:

$$\begin{aligned} (i) \quad & x_t^{N,1} + (1/e_t^A) y_t^{N,1} + (1/e_t^U) z_t^{N,1} = b_{t+1}^{N,1}, \\ (ii) \quad & x_{t+1}^{N,2} + \frac{y_{t+1}^{N,2}}{e_{t+1}^A} + \frac{z_{t+1}^{N,2}}{e_{t+1}^U} + s_{t+1}^N = h_{t+1}^N w_{t+1}^N (1 - \tau_{t+1}^N) - (1 + i_{t+1}^N) b_{t+1}^{N,1}, \\ & \text{with } s_{t+1}^N = K_{t+2}^{N,N} / L_{t+1}^{N,2} + B_{t+2}^{N,N} / L_{t+1}^{N,2} + K_{t+2}^{S,N} / L_{t+1}^{N,2} + B_{t+2}^{S,N} / L_{t+1}^{N,2} + b_{t+2}^{NN,2} + b_{t+2}^{SN,2}, \\ & b_{t+1}^{N,1} \leq \theta^N \frac{w_{t+1}^N h_{t+1}^N}{1 + i_{t+1}^N}, h_{t+1}^N = h_0^N [(1 - f_t^N) h_t^N]^\mu \left(\frac{\Delta_t^N}{L_t^N} \right)^{1-\mu}, \\ (iii) \quad & x_{t+2}^{N,3} + (1/e_{t+2}^A) y_{t+2}^{N,3} + (1/e_{t+2}^U) z_{t+2}^{N,3} = q_{t+2}^N (K_{t+2}^{N,N} / L_{t+1}^{N,2}) + q_{t+2}^S (K_{t+2}^{S,N} / L_{t+1}^{N,2}) + (1 + i_{t+2}^N) (B_{t+2}^{N,N} / L_{t+1}^{N,2}) \\ & + (1 + i_{t+2}^S) (B_{t+2}^{S,N} / L_{t+1}^{N,2}) + (1 + i_{t+2}^N) b_{t+2}^{NN,2} + (1 + i_{t+2}^S) b_{t+2}^{SN,2}. \end{aligned}$$

$b_{t+2}^{SN,2}$ represents the main difference to southern younger household's middle-age budget constraint: the northern younger household buys in middle-age not only private bonds issued in her own young age but also the private bonds issued by the southern young ager.

The typical Asian younger household solves the following optimization problem:

$$\begin{aligned} \text{Max} \rightarrow & \zeta^x \ln x_t^{A,1} + \zeta^y \ln y_t^{A,1} + \zeta^z \ln z_t^{A,1} + \ln f_t^A + \beta^A (\zeta^x \ln x_{t+1}^{A,2} + \zeta^y \ln y_{t+1}^{A,2} + \zeta^z \ln z_{t+1}^{A,2}) \\ & + (\beta^A)^2 (\zeta^x \ln x_{t+2}^{A,3} + \zeta^y \ln y_{t+2}^{A,3} + \zeta^z \ln z_{t+2}^{A,3}) \end{aligned}$$

s.t.:

$$\begin{aligned} (i) \quad & e_t^A x_t^{A,1} + y_t^{A,1} + (e_t^A / e_t^U) z_t^{A,1} = b_{t+1}^{A,1}, \\ (ii) \quad & e_{t+1}^A x_{t+1}^{A,2} + y_{t+1}^{A,2} + (e_{t+1}^A / e_{t+1}^U) z_{t+1}^{A,2} + s_{t+1}^{A,2} = h_{t+1}^A w_{t+1}^A (1 - \tau_{t+1}^A) - (1 + i_{t+1}^A) b_{t+1}^{A,1}, \\ & \text{with } s_{t+1}^A = K_{t+2}^{A,A} / L_{t+1}^{A,2} + B_{t+2}^{A,A} / L_{t+1}^{A,2} + (e_{t+1}^A / e_{t+1}^U) B_{t+2}^{U,A} / L_{t+1}^{A,2} + b_{t+2}^{AA,2} + (e_{t+1}^A / e_{t+1}^U) b_{t+2}^{UA,2}, \\ & b_{t+1}^{A,1} \leq \theta^A \frac{w_{t+1}^A h_{t+1}^A}{1 + i_{t+1}^A}, h_{t+1}^A = h_0^A [(1 - f_t^A) h_t^A]^\mu \left(\frac{\Delta_t^A}{L_t^A} \right)^{1-\mu}, \\ (iii) \quad & e_{t+2}^A x_{t+2}^{A,3} + y_{t+2}^{A,3} + (e_{t+2}^A / e_{t+2}^U) z_{t+2}^{A,3} = q_{t+2}^A (K_{t+2}^{A,A} / L_{t+1}^{A,2}) + (1 + i_{t+2}^A) (B_{t+2}^{A,A} / L_{t+1}^{A,2}) + (1 + i_{t+2}^U) (e_{t+2}^A / e_{t+2}^U) (B_{t+2}^{U,A} / L_{t+1}^{A,2}) \\ & + (1 + i_{t+2}^A) b_{t+2}^{AA,2} + (1 + i_{t+2}^U) (e_{t+2}^A / e_{t+2}^U) b_{t+2}^{UA,2}. \end{aligned}$$

Finally, the typical US younger household faces the following optimization problem:

$$\begin{aligned} \text{Max} \rightarrow & \zeta^x \ln x_t^{U,1} + \zeta^y \ln y_t^{U,1} + \zeta^z \ln z_t^{U,1} + \ln f_t^U + \beta^U (\zeta^x \ln x_{t+1}^{U,2} + \zeta^y \ln y_{t+1}^{U,2} + \zeta^z \ln z_{t+1}^{U,2}) \\ & + (\beta^U)^2 (\zeta^x \ln x_{t+2}^{U,3} + \zeta^y \ln y_{t+2}^{U,3} + \zeta^z \ln z_{t+2}^{U,3}) \end{aligned}$$

s.t.:

$$\begin{aligned} (i) \quad & e_t^U x_t^{U,1} + (e_t^U / e_t^A) y_t^{U,1} + z_t^{U,1} = b_{t+1}^{U,1}, \\ (ii) \quad & e_{t+1}^U x_{t+1}^{U,2} + (e_{t+1}^U / e_{t+1}^A) y_{t+1}^{U,2} + z_{t+1}^{U,2} = h_{t+1}^U w_{t+1}^U (1 - \tau_{t+1}^U) - (1 + i_{t+1}^U) b_{t+1}^{U,1}, \end{aligned}$$

with $s_{t+1}^U = K_{t+2}^{U,U} / L_{t+1}^{U,2} + B_{t+2}^{U,U} / L_{t+1}^{U,2} + e_{t+1}^U B_{t+2}^{N,U} / L_{t+1}^{U,2} + b_{t+2}^{UU,2}, b_{t+1}^U = \theta^U \frac{w_{t+1}^{U,E}}{1+i_{t+1}^U}, h_{t+1}^U = h_0^U [(1-f_t^U)h_t^U]^\mu \left(\frac{\Delta_t^U}{L_t^U}\right)^{1-\mu}$,

(iii) $e_{t+2}^U x_{t+2}^{U,3} + (e_{t+1}^U / e_{t+1}^A) y_{t+2}^{U,3} + z_{t+2}^{A,3} = q_{t+2}^U (K_{t+2}^{U,U} / L_{t+1}^{U,2}) + (1+i_{t+2}^U) B_{t+2}^{U,U} / L_{t+1}^{U,2} + (1+i_{t+2}^U) e_{t+1}^U B_{t+2}^{N,U} / L_{t+1}^{U,2} + (1+i_{t+2}^U) b_{t+2}^{UU,2}$.

The government budget constraints read now as follows:

$$B_{t+1}^i - B_t^i + \tau_i^i w_i^i h_i^i L_t^{i,2} = i_t^i B_t^i + \Gamma_t^i + \Delta_t^i, i = S, N, A, U, t = 0, 1, 2, \dots, \quad (15)$$

where $L_t^{i,2}$ denotes the number of middle-aged households in country i in period t .

While the clearing conditions for the labor markets, the markets for real capital and government bonds remain as in the basic model presented above, the clearing conditions for private bond markets read as follows:

$$L_t^{S,1} b_{t+1}^{S,1} = L_t^{S,2} b_{t+1}^{SS,2} + L_t^{N,2} b_{t+1}^{SN,2}, L_t^{N,1} b_{t+1}^{N,1} = L_t^{N,2} b_{t+1}^{NN,2}, L_t^{A,1} b_{t+1}^{A,1} = L_t^{A,2} b_{t+1}^{AA,2}, L_t^{U,1} b_{t+1}^{U,1} = L_t^{U,2} b_{t+1}^{UU,2} + L_t^{A,2} b_{t+1}^{UA,2}, \quad (16)$$

where $L_t^{i,1}$ denotes the number young households in country i in period t .

The worldwide amount of savings equals the worldwide supply of assets from southern and northern EMU, Asia and the US. Thus:

$$L_t^{S,2} s_t^{S,2} - L_t^{S,1} \theta^S \frac{h_{t+1}^S w_{t+1}^S}{1+i_{t+1}^S} + L_t^{N,2} s_t^{N,2} - L_t^{N,1} \theta^N \frac{h_{t+1}^N w_{t+1}^N}{1+i_{t+1}^N} + (L_t^{A,2} s_t^{A,2} - L_t^{A,1} \theta^A \frac{h_{t+1}^A w_{t+1}^A}{1+i_{t+1}^A}) / e_t^A \quad (17)$$

$$+ (L_t^{U,2} s_t^{U,2} - L_t^{U,1} \theta^U \frac{h_{t+1}^U w_{t+1}^U}{1+i_{t+1}^U}) / e_t^U = K_{t+1}^S + K_{t+1}^N + B_{t+1}^S + B_{t+1}^N + (K_{t+1}^A + B_{t+1}^A) / e_t^A + (K_{t+1}^U + B_{t+1}^U) / e_t^U.$$

Finally, the clearing conditions for the product markets change due to the existence of a third generation as follows:

$$X_t^S + X_t^N = L_t^{S,1} x_t^{S,1} + L_t^{S,2} x_t^{S,2} + L_t^{S,3} x_t^{S,3} + \Gamma_t^S + \Delta_t^S + K_{t+1}^S + L_t^{N,1} x_t^{N,1} + L_t^{N,2} x_t^{N,2} + L_t^{N,3} x_t^{N,3} + \Gamma_t^N + \Delta_t^N \quad (18)$$

$$+ K_{t+1}^N + L_t^{A,1} x_t^{A,1} + L_t^{A,2} x_t^{A,2} + L_t^{A,3} x_t^{A,3} + L_t^{U,1} x_t^{U,1} + L_t^{U,2} x_t^{U,2} + L_t^{U,3} x_t^{U,3},$$

$$Y_t = L_t^{A,1} y_t^{A,1} + L_t^{A,2} y_t^{A,2} + L_t^{A,3} y_t^{A,3} + \Gamma_t^A + \Delta_t^A + K_{t+1}^A + L_t^{S,1} y_t^{S,1} + L_t^{S,2} y_t^{S,2} + L_t^{S,3} y_t^{S,3} \quad (19)$$

$$+ L_t^{N,1} y_t^{N,1} + L_t^{N,2} y_t^{N,2} + L_t^{N,3} y_t^{N,3} + L_t^{U,1} y_t^{U,1} + L_t^{U,2} y_t^{U,2} + L_t^{U,3} y_t^{U,3},$$

$$Z_t = L_t^{U,1} z_t^{U,1} + L_t^{U,2} z_t^{U,2} + L_t^{U,3} z_t^{U,3} + \Gamma_t^U + \Delta_t^U + K_{t+1}^U + L_t^{S,1} z_t^{S,1} + L_t^{S,2} z_t^{S,2} + L_t^{S,3} z_t^{S,3} \quad (20)$$

$$+ L_t^{N,1} z_t^{N,1} + L_t^{N,2} z_t^{N,2} + L_t^{N,3} z_t^{N,3} + L_t^{A,1} z_t^{A,1} + L_t^{A,2} z_t^{A,2} + L_t^{A,3} z_t^{A,3}$$

Numerical Specification of the extended model under exogenous growth

For the numerical specification of the basic model extended by household credit constraints we have to assume that the growth factors of human capital are exogenous and equal across countries. As in the version with pension systems in all countries except Asia, all structural and policy parameters are again specified such that the diverging real interest rates between EMU core and periphery and between Asia and USA as well as their approximately balanced trade account and zero net foreign asset positions in the 1990s (before euro inception and the East-Asian currency crises) are reproduced by the financial autarky version of the extended model.

The structural parameters are largely similar to those specified in the model with pension systems in all countries but in Asia as follows: $G^n = 2$, $\zeta^x = 0.4$, $\zeta^y = 0.3$, $\zeta^z = 0.3$, $\beta^S = 0.55$, $\beta^N = 0.7$, $\beta^A = 0.8$, $\beta^U = 0.55$, $\alpha^S = 0.16$, $\alpha^N = 0.15$, $\alpha^A = 0.18$, $\alpha^U = 0.16$, $M^S = 1$, $M^N = 1.2$,

$M^A = 0.8, M^U = 1.4, L^S = L^N = 85, L^A = 1200, L^U = 120$. Since we want to focus on the consequences of world-wide financial integration on external balances we assume roughly in line with empirical reality that both the government expenditure and the debt to GDP ratios remain constant over time, i.e. $\gamma_t^i = \gamma_{t+1}^i = \gamma^i, \delta_t^i = \delta_{t+1}^i = \delta^i, \tau_t^i = \tau_{t+1}^i = \tau^i$ and $b_t^i = b_{t+1}^i = b^i$. Under these stationarity assumptions we specify the policy parameters as follows: $\delta^S = 0.04, \delta^N = 0.07, \delta^A = 0.03, \delta^U = 0.05, b^S = 0.025, b^N = 0.023, b^A = 0.015, b^U = 0.024, \tau^S = 0.16, \tau^N = 0.18, \tau^A = 0.15, \tau^U = 0.15, \theta^S = 0.03058, \theta^N = 0.045998, \theta^A = 0.02978, \theta^U = 0.03842$.

Given these structural and policy parameters, the steady state and first-period results for main variables read as follows:

Table 3 Main endogenous variables in EMU South, EMU North, in Asia and in USA (calculated on a yearly basis) under *financial integration and households' credit constraints* in steady state

	Capital Output ratio	Real interest rate (in %)	Net Foreign Assets to GDP (%)	Trade balance to GDP ratio (%)	Trade balance to GDP ratio (%) (Shock period)	EMU Terms of trade vis-à-vis	Saving ratios	
							shock period	steady state
EMU South	2.34	2.16	-33.17	-0.39	-2.54		0.2114	0.2111
EMU North	2.20	2.16	+4.17	+ 0.048	+0.49		0.2256	0.2251
Asia	2.64	2.16	+ 36.76	+ 0.43	+2.54	2.045	0.2699	0.2703
USA	2.20	2.16	- 0.21.54	-0.26	-1.54	1.30	0.2270	0.2065

Source: Own calculation

Starting with the same financial autarky solution Table 3 reports the results of financial integration under households' credit constraints and exogenous growth of human capital. The Table shows rather similar (not exactly the same) capital output ratios, real interest rates, net foreign asset positions and trade balance to GDP ratios as compared to the corresponding variables in Table 2. The main difference to the results in Table 2 concerns the evolution pattern of the saving ratios. While in the model with pension systems except in Asia saving ratios increase over time (see Table 2), they now decrease (although rather slightly) in line with the empirical observations in Figures 9 and 10. This can be seen as main advantage of the extension of the basic model through the introduction of household credit constraints.

Conclusion

Farmer and Ban (2014, 2015) investigated in a three-country, three-good OLG model (= basic model) simultaneously the financial integration effects on both intra-EMU and global external imbalances. While the authors were able to show how EMU core and Asian trade surpluses as well as EMU periphery and US trade deficits can be qualitatively attributed to intra-EMU and global financial integration, the empirical fact of significantly higher saving rates in Asia than in USA already before Asian-US financial integration and the divergence of personal saving rates after financial integration, particularly intra EMU, the basic model was unable to address.

This paper asks whether Farmer and Ban's (2015) quantitative results can be improved (1) by addressing the pre-integration Asian-US saving rates difference through the incorporation of pay-as-you-go pension systems in EMU and in US but not in Asia following Eugeni (2015), and (2) by focusing on the divergence of intra-EMU and global personal saving rates after financial

integration through the incorporation of credit constraints to three-period lived younger households in line with Coeurdacier et al. (2015). We find that both extensions of the basic model improve its results.

The introduction of pay-as-you-go pension systems in EMU and US but not in Asia can better reproduce the much higher Asian saving rates relative to their US counterparts both before and after financial integration. However, in contrast to Eugeni's two-country OLG model of global imbalances, the international difference in pension systems (in institutions in general) alone does not explain the post-integration saving rates divergence between EMU South and EMU North as well as to lesser extent between Asia and the USA.

To address this dynamic divergence of saving rates we probed into the second extension, namely the incorporation of credit constraints into the intertemporal optimization calculi of younger households in line with Coeurdacier et al. (2015). We found that even under exogenous and internationally equal productivity growth rates the extended model generates decreasing personal saving rates in EMU South and US, and also in EMU North and Asia, in the latter to a lesser extent. This result clearly improves the performance of the basic model (without financial constraints), since in the latter the personal saving rates in all countries increased over time after financial integration.

However, the change of the numerical values of the saving ratios exhibited in Table 3 while pointing into the right direction is rather modest in view of the marked change in the empirical values portrayed in Figures 9 and 10. In order to address also this model-data discrepancy, endogenous growth with internationally different growth rates need to be introduced into the basic model with internationally differing pension systems and household financial constraints. It remains to be seen in future research whether this suggestion to modify the three-country, three-good OLG model really improve the empirical performance of the intertemporal equilibrium modeling of intra-EMU and global trade imbalances.

References

- Angletos, G.-M. & V. Panousi (2011). Financial integration, entrepreneurial risk and global dynamics. *Journal of Economic Theory* 146, 863-896.
- Blanchard, O. & F. Giavazzi (2002). Current account deficits in the Euro area: The end of the Feldstein-Horioka Puzzle? *Brookings Papers on Economic Activity* 33, 147-210.
- Buiter, W. H. (1981). Time preference and international lending and borrowing in an overlapping-generations model. *Journal of Political Economy* 89: 769-797.
- Chen, R., Milesi-Ferretti, G.-M. & Th. Tressel (2013). External imbalances in the eurozone. *Economic Policy* 28, 101-142..
- Coeurdacier, N., St. Guiband & K. Jin (2015), Credit constraints and growth in a global economy. *American Economic Review* 105(9): 2838-81.
- Cova, P., M. Pisani & A. Rebucci (2009). Global imbalances: The role of emerging Asia. *Review of International Economics* 17 (4), 716-733.
- Diamond, P. A. (1965). National debt in a neoclassical growth model. *American Economic Review* 55: 1135-1150.

- Eugeni, S. (2015). An OLG model of global imbalances. *Journal of International Economics*, 95(1), 83-97.
- Fagan, G. & V. Gaspar (2008). Macroeconomic adjustment to monetary union. *ECB Working Paper Series No 946/October*.
- Farmer, K. (2014). Financial integration and EMU's external imbalances in a two-country OLG model. *International Advances in Economic Research* 20 (1), 1-21.
- Farmer, K. & I. Ban (2015). The Intertemporal approach to Intra-EMU and Global External Imbalances. In: A. Tavidze (Ed.), *Progress in Economic Research* 33, 101-126, New York: Nova Publishers.
- Farmer, K. & I. Ban (2014). Modeling financial integration, intra-EMU and Asian-US external imbalances. *GEP-Graz Economics Papers* 2014-06.
- Farmer, K. (2013). Financial integration and EMU's external imbalances in a two-country OLG model. *GEP-Graz Economics Papers* 2013-07.
- Gourinchas, P. S. & O. Jeanne (2006). The elusive gains from international financial integration. *Review of Economic Studies* 73 (3): 715-741.
- Japelli, T. & M. Pagano (2008), Financial market integration under EMU. *European Economy. Economic Papers* 312.
- Kalemli-Ozcan, S., E. Papaioannou & J.-L. Peydró (2010). What lies beneath the Euro's effect on financial integration? Currency risk, legal harmonization, or trade? *Journal of International Economics* 81 (1), 75-88.
- Lane, P. R. (2006). The real effects of European Monetary Union. *Journal of Economic Perspectives* 20 (4): 47-66.
- Lane, P. R. & G. M. Milesi-Ferretti (2007). The external wealth of nations mark II: Revised and extended estimates of foreign assets and liabilities. *Journal of International Economics* 73 (2), 223-250.
- Mendoza, E. G., V. Quadrini & J.-V. Rios-Rull (2009). Financial integration, financial development, and global imbalances. *Journal of Political Economy* 117 (3), 371-416.
- Schmitz, B. and J. v. Hagen (2011). Current account imbalances and financial integration in the Euro area. *Journal of International Money and Finance* 30 (8), 1676-1695.
- Spiegel, M. M. (2009). Monetary and financial integration in the EMU: Push or pull? *Review of International Economics* 17 (4), 751-776.

Graz Economics Papers

For full list see:

<http://ideas.repec.org/s/grz/wpaper.html>

Address: Department of Economics, University of Graz,
Universitätsstraße 15/F4, A-8010 Graz

- 08–2015 **Karl Farmer, Bogdan Mihaiescu:** [Pension systems and financial constraints in a three-country OLG model of intra-EMU and global trade imbalances](#)
- 07–2015 **Javier López Prol, Karl W. Steininger:** [Photovoltaic self-consumption regulation in Spain: profitability analysis and alternative regulation schemes.](#)
- 06–2015 **Miriam Steurer, Caroline Bayr:** [Quantifying Urban Sprawl using Land Use Data](#)
- 05–2015 **Porfirio Guevara, Robert J. Hill, Michael Scholz:** [Hedonic Indexes for Public and Private Housing in Costa Rica](#)
- 04–2015 **Sofie R. Waltl:** [A Hedonic House Price Index in Continuous Time](#)
- 03–2015 **Ronald Wendner:** [Do Positional Preferences for Wealth and Consumption Cause Inter-Temporal Distortions?](#)
- 02–2015 **Maximilian Gödl, Christoph Zwick:** [Stochastic Stability of Public Debt: The Case of Austria](#)
- 01–2015 **Gabriel Bachner:** [Land Transport Systems under Climate Change: A Macroeconomic Assessment of Adaptation Measures for the Case of Austria](#)
- 10–2014 **Thomas Schinko, Judith Köberl, Birgit Bednar-Friedl, Franz Prettenthaler, Christoph Töglhofer, Georg Heinrich, Andreas Gobiet:** [An Integrated Economic Analysis of Climate Change Impacts on Tourism in Austria Across Tourism Types and Seasons](#)
- 09–2014 **Sugata Ghosh, Ronald Wendner:** [Positional Preferences, Endogenous Growth, and Optimal Income- and Consumption Taxation](#)
- 08–2014 **Stan Metcalfe:** [George Shackle and The Schumpeterian Legacy.](#)
- 07–2014 **Birgit Bednar-Friedl, Karl Farmer:** [Existence and efficiency of stationary states in a renewable resource based OLG model with different harvest costs](#)

- 06–2014 **Karl Farmer, Irina Ban:** [Modeling financial integration, intra-EMU and Asian-US external imbalances](#)
- 05–2014 **Robert J. Hill, Michael Scholz:** [Incorporating Geospatial Data in House Price Indexes: A Hedonic Imputation Approach with Splines](#)
- 04–2014 **Y. Hossein Farzin, Ronald Wendner:** [The Time Path of the Saving Rate: Hyperbolic Discounting and Short-Term Planning](#)
- 03–2014 **Robert J. Hill, Iqbal A. Syed:** [Hedonic Price-Rent Ratios, User Cost, and Departures from Equilibrium in the Housing Market](#)
- 02–2014 **Christian Gehrke:** [Ricardo’s Discovery of Comparative Advantage Revisited](#)
- 01–2014 **Sabine Herrmann, Jörn Kleinert:** [Lucas Paradox and Allocation Puzzle – Is the euro area different?](#)
- 08–2013 **Christoph Zwick:** [Current Account Adjustment in the Euro-Zone: Lessons from a Flexible-Price-Model](#)
- 07–2013 **Karl Farmer:** [Financial Integration and EMUs External Imbalances in a Two-Country OLG Model](#)
- 06–2013 **Caroline Bayr, Miriam Steurer, Rose-Gerd Kobltschnig:** [Scenario Planning for Cities using Cellular Automata Models: A Case Study](#)
- 05–2013 **Y. Hossein Farzin, Ronald Wendner:** [Saving Rate Dynamics in the Neoclassical Growth Model – Hyperbolic Discounting and Observational Equivalence](#)
- 04–2013 **Maximilian Gödl, Jörn Kleinert:** [Interest rate spreads in the Euro area: fundamentals or sentiments?](#)
- 03–2013 **Christian Lininger:** [Consumption-Based Approaches in International Climate Policy: An Analytical Evaluation of the Implications for Cost-Effectiveness, Carbon Leakage, and the International Income Distribution](#)
- 02–2013 **Veronika Kulmer:** [Promoting alternative, environmentally friendly passenger transport technologies: Directed technological change in a bottom-up/top-down CGE model](#)
- 01–2013 **Paul Eckerstorfer, Ronald Wendner:** [Asymmetric and Non-atmospheric Consumption Externalities, and Efficient Consumption Taxation](#)

- 10–2012 **Michael Scholz, Stefan Sperlich, Jens Perch Nielsen:** [Nonparametric prediction of stock returns with generated bond yields](#)
- 09–2012 **Jörn Kleinert, Nico Zorell:** [The export-magnification effect of offshoring](#)
- 08–2012 **Robert J. Hill, Iqbal A. Syed:** [Hedonic Price-Rent Ratios, User Cost, and Departures from Equilibrium in the Housing Market](#)
- 07–2012 **Robert J. Hill, Iqbal A. Syed:** [Accounting for Unrepresentative Products and Urban-Rural Price Differences in International Comparisons of Real Income: An Application to the Asia-Pacific Region](#)
- 06–2012 **Karl Steininger, Christian Lininger, Susanne Droege, Dominic Roser, Luke Tomlinson:** [Towards a Just and Cost-Effective Climate Policy: On the relevance and implications of deciding between a Production versus Consumption Based Approach](#)
- 05–2012 **Miriam Steurer, Robert J. Hill, Markus Zahrnhofer, Christian Hartmann:** [Modelling the Emergence of New Technologies using S-Curve Diffusion Models](#)
- 04–2012 **Christian Groth, Ronald Wendner:** [Embodied learning by investing and speed of convergence](#)
- 03–2012 **Bettina Brüggemann, Jörn Kleinert, Esteban Prieto:** [The Ideal Loan and the Patterns of Cross-Border Bank Lending](#)
- 02–2012 **Michael Scholz, Jens Perch Nielsen, Stefan Sperlich:** [Nonparametric prediction of stock returns guided by prior knowledge](#)
- 01–2012 **Ronald Wendner:** [Ramsey, Pigou, heterogenous agents, and non-atmospheric consumption externalities](#)