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ABSTRACT

Digital land use data, generally derived by remote sensing and organized in grid form, have become widely and cheaply available for even the most remote areas of the globe. Here we investigate how to use land use data to measure three of the most characteristic aspects of urban sprawl: low density, low continuity of land use type (scatteredness), and low compactness of the shape of the city. For each of these categories we present possible urban sprawl indicators. Some of these indicators have been used in the literature before, others we developed ourselves. We illustrate how simple changes to common density indices can improve their meaningfulness. With respect to scatteredness we show that entropy indices might not be the most suitable type of index as their interpretation is ambiguous. A variant on Moran's I index performs this task better. When it comes to measuring compactness the grid structure of land use data can inflate the boundary of the measured area. We introduce compactness indices that correct for this problem. To illustrate the discussed indices we apply them to the city of Graz, the second largest town in Austria, using data from the CORINE Land Cover (CLC) Project (2006).

1 INTRODUCTION

Measuring urban sprawl is important because it is associated with a variety of major ecological, social, and health effects and developments. The impacts associated with urban sprawl range from the lack of scale economies (Frenkel and Ashkenazi, 2008), ecological problems such as air pollution and congestion due to increased car use associated with urban sprawl (Brueckner, 2000, Nechyba and Walsh, 2004), fragmentation of the eco-system, loss of agricultural land, social problems of increased segregation (e.g. Glaeser and Kahn, 2003) or increased isolation (Frumkin, 2002), health problems such as obesity (e.g. Ewing et al., 2003, Bray et al., 2005), to increased public spending by local governments (Carruthers 2008).

Urban sprawl is however a complex phenomenon and it is therefore not surprising that many different definitions of urban sprawl exist in the literature. Frenkel and Ashkenazi (2008) summed up the situation very well when they wrote: "we know that sprawl is significant, but we are not yet sure what it is exactly or how to measure it." The attributes that are used to describe urban sprawl range from: low density housing (especially at the urban fringe), low diversity, sprawling housing developments, poor accessibility, to aesthetic criteria such as boring architecture. However, the three categories that are most often used to describe urban form are density, diversity, and spatial-structural pattern (Tsai, 2005). In this paper we focus on how to measure these three characteristics of urban sprawl using land use data.

Land use data are typically available in digital code that refers to individual grid cells. Each cell is allocated one special land use type that indicates the dominant use of this plot of land. Typically the data are presented in digital form and entries refer to individual plots of uniform size (e.g. 100m x 100m). Our land use data comes from the Corine Land Cover Project (European Environment Agency, 2010). This data set is freely available for all European countries (and also Turkey, Iceland, Madeira, and the Canary Islands). The grid size in the Corine Land Cover Project is 100 by 100 meters and there are 42 land use classes present in the CLC Project. Population is mainly found in CLC class 1 (continuous urban fabric) and 2 (discontinuous urban fabric).

The most prominent indicator used in the literature to describe urban sprawl is density. In section 2 we illustrate how traditional density measures can be improved by making use of the classifications of land use data. Rather than just using population density per area, land use data can be used to fine-tune the area definition by "cutting out" areas that are (for legal or other reasons) not used for housing. The resulting density measure is a much more meaningful indicator of urban sprawl and also allows for comparisons across cities.

Section 3 discusses diversity-based urban sprawl indices. Entropy indices are the main proponent of this type, but other indices like the Moran's I index provide alternatives to capture diversity or continuity across space. Entropy indices are increasingly being used in the literature to measure the diversity of population and land use patterns across an area (see e.g. Yeh and Li 2001, Sudhira et al. 2004, Li and Yeh 2004, Sun et al. 2007, Jat et al. 2008, Bhatta 2009, Yue et al. 2013). We explore two different variants of Shannon's entropy index and find them difficult to interpret. The difficulty arises because entropy indices measure two components within one index: On the one hand the prevalence of one cell type compared to another, on the other hand how scattered the cell types are within the chosen area. To get around this problem we therefore turn to an alternative measure of diversity, the (global) Moran's I index. The Moran's I index is a spatial autocorrelation index and measures the similarity of neighboring cells. We argue that it is better suited to capture the discontinuity aspect of urban sprawl than entropy measures.

In section 4 we investigate which spatial-structural indices are well suited to measure the compactness of an urban settlement. We introduce two new indices into the urban sprawl literature that take account of the grid structure of land use data. Using conventional geometrical compactness measures with digital land use data would lead to overestimation of circumference measures and thus to an overestimation of sprawl. Our versions of Milne's (1991) shape index and Bribiesca's (1997) compactness index correct for this problem.

In section 5 we illustrate the workings of these indices by applying each of them to the city of Graz, the second biggest city in Austria using data from the Corine Landcover Project (European Environment Agency, 2010). The paper concludes with a discussion of the relative advantages and disadvantages of each urban sprawl indicator.

2   DENSITY MEASURES

2.1   Density Measure 1: number of people divided by total area of district/municipality

The most popular methods to measure urban sprawl are population density measurements. Low population density numbers are suggestive of high degrees of urban sprawl. We can see large differences in the population density for major international cities. The 6,299km^2 city area of Los Angeles has a population of 14,900,000, which corresponds to a population density of 2,400 people per square kilometer according to this method, for the 2,163km^2 area of Seoul-Incheon in South Korea with a population of 22,547,000 people, the population density is 10,400 people per square kilometer. (New Geography, 2013). The standard version of the density measure is to divide the number of people by the total area of a district/municipality (D_1), where the size of the district is generally given by the political boundaries of the settlement.

$$D_1 = \frac{\sum_i \sum_j pop_{ij}}{\sum_i \sum_j x_{ij}}$$

where i indexes the cells $i = 1, \dots, n$. pop_{ij} is the population of cell i in district j. There are 12762 cells within the official city limits of Graz divided up into 259 voting districts (thus there are on average about 49 cells per voting district). $\sum_i \sum_j x_{ij}$ is the total area of the city (sum of all cells within town boundaries).

For Austrian municipalities the population densities per square kilometer are moderate. The population density within the official city limits of Graz is 1,944 people per km^2 while for the 57 surrounding communities within the "Greater Graz Area" it is only 123 people per km^2.

2.2   Density Measure 2: number of people divided by built-up cells in district/municipality

One problem with D_1 is that it is very sensitive to the area included in the measurement. Most often this measurement is performed using the political (administrative) boundary of a city. However, this is a man-made concept and can differ substantially from one town to another. One time the outer suburbs and rural areas are included within a political city boundary another time they are not. Consistency of what constitutes an urban area is needed. Sadly, lack of consistency in where urban boundaries are drawn is the norm – especially in Europe. The consequence is over- or underestimation of urban land use and a lack of comparibility across or within nations (Cohen, 2004, Schneider and Woodcock 2008). Remote sensing data allows for a more uniform definition of functional urban areas instead of arbitrary political boundaries.

Another related problem with the D_1 measure is that it does not take account of different land uses within a city area. For example, a city that consists of dense urban spaces and a large park might have the same density measure as one that consists of a sprawling urban landscape without any large green spaces. To counteract these shortcomings we propose a second density measure here which considers the population density in built-up areas only (D_2).

We use land use data from the Corine Land Cover Project (2006) to classify individual 100m x 100m grid cells according to their dominant usage into 42 possible land use classes. Our first task is to decide statistically which of these land use classes constitute a built-up area.[1] We found that in the city area of Graz significant housing is only found in cells classified as CLC1 and CLC2. We therefore assigned the district level population onto the cells of these classifications.

Population density according to our density measure D_2 is calculated as follows: For each of the 259 sub-districts within the city limits of Graz we divide its population by the number of cells of CLC class 1 and 2 within this district. Population density for each of the 57 municipalities outside of Graz is calculated in the same way. Doing this we get a population density measure per populated area for each of the voting districts. For a city-wide density measure we divide the number of CLC class 1 and 2 cells by the city population. We also do the same for the greater Graz region. The results can be found in Table 1.

The formula for this density measure reads as follows:

$$D_2 = \frac{\sum_i \sum_j pop_{ij}}{\sum_i \sum_j \sum_k x_{ijk}}$$

where i indexes the cells, j the district, and k indexes the land use types that are counted as residential.

3 DIVERSITY MEASURES

### 3.1 Entropy measures to indicate urban sprawl

Entropy measures are an example of inequality measures and are increasingly being used in the urban sprawl literature. The concept of entropy comes from the field of thermodynamics and is generally considered to be a measure of disorder. Entropy indices can be used to measure "disorder" in various areas – apart from thermodynamics; it is used in inequality measurement, diversity measurement, and also for measuring urban sprawl. Entropy is highest when disorder in a system is greatest. As such it seems ideal for the concept of urban sprawl. No one would say that either the Gobi desert or down-town Manhattan is affected by urban sprawl. So urban sprawl is something that happens in between these extremes. Figure 1 graphically illustrates this point. Because they are steadily increasing as more people move into an area, density measures cannot deal with this aspect but entropy measures can.

Entropy measures of urban sprawl attempt to capture the expansion of urban areas into the countryside. In contrast to density measures, entropy measures focus on differences in density across districts rather than

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[1] Because we are interested in urban sprawl, which is a phenomenon primarily linked to housing sprawl, we define a cell as built-up if there is residential housing in that cell.

the density level itself. In this way entropy is an indicator of the continuity of land use type, or if there is lack of continuity, the "scatterdness" of land use types throughout the area.
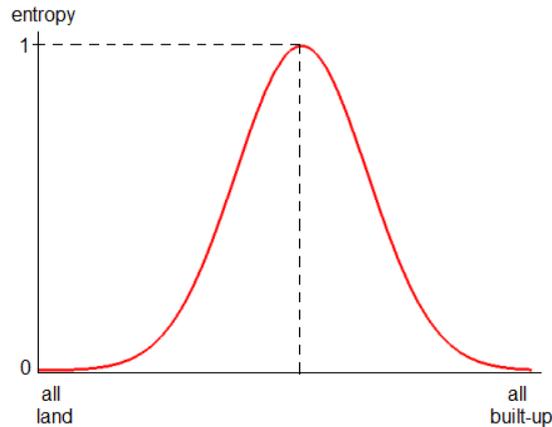
Fig. 1: The curve of the entropy measure.

According to Galster et al. (2001) continuity refers to the degree to which developable land has been built upon at urban densities in an unbroken fashion. Here we use Shannon's measure of entropy to measure the degree of continuity (or discontinuity) of urban development. A high entropy score indicates that population density varies a lot throughout the area which in turn is interpreted as high level of urban sprawl. A low entropy score conversely implies low variability in population density across districts and a low level of urban sprawl. Mathematically it is possible to further decompose entropy scores (Baumgärtner (2004), Ritsema van Eck and Koomen (2004)). The total entropy index could for example be de-composed into an entropy sub-index based on the mix of urban and rural land uses, the sub-index based on the mixture of urban land uses, and the sub-index based on the mixture of rural land uses (Ritsema van Eck and Koomen, 2008).

Entropy measures have been used previously to measure urban sprawl by Yeh and Li (2001) and Li and Yeh (2004) for areas in China, Sudhira, Ramachandra, and Jagadish (2004) as well as Jat et al. (2008) and Bhatta (2009) for India, Sun et al. (2007) for Canada. Cabral et al. (2013) provide a good overview of the use of entropy measures in the urban sprawl literature.

One of the main advantages of entropy measures in the urban sprawl context are their simplicity and the ease of integration with GIS based data (Yeh and Li, 2001). These advantages have made it "the most widely used technique to measure the extent of urban sprawl with the integration of remote sensing and GIS" (Bhatta et al. 2010). Another advantage of the entropy measure is that it is invariant to the size, shape, or number of regions of the area under consideration and it is therefore a more robust spatial statistic than most (Yeh and Lie 2001, Bhatta et al. 2010).

Most entropy applications to urban sprawl use Shannon's entropy measure (or the Theil index, which is closely related to it). However, any variation of the general entropy index could be used to measure urban sprawl. We also believe that other indices of this type – for example the Atkinson index or the Gini

5

coefficient – could be used just as well for this purpose. In this paper we consider two variations of Shannon's entropy measure – one that includes population density measures, and one that does not.

3.2   Measuring entropy at the district/municipality level with population data

Urban sprawl according to Shannon's entropy measure is calculated as follows:

$$E = -\sum_j p_j \cdot \log(p_j),$$

where p_j is the weight for district j (j = 1,… n) and $\sum_j p_j = 1$.

Using the two different concepts of density introduced in the chapter above (D_1 and D_2), we can derive two different measures for p_j. Using the density concept of D_1 leads to weights equal to:

$$p_j = \frac{\sum_i pop_{ij} / \sum_j x_{ij}}{\sum_j \left(\sum_i pop_{ij} / \sum_j x_{ij}\right)}$$

While defining density in the same way as D_2 leads to weights equal to:

$$p_j = \frac{\sum_i pop_{ij} / \sum_j \sum_k x_{ijk}}{\sum_j \left(\sum_i pop_{ij} / \sum_j \sum_k x_{ijk}\right)}$$

Again, the subscript I indexes cells, j indexes the districts, and k indexes land use types. Thus x_{ijk} denotes cell i of type k in district j, while pop_{ij} denotes population of cell i in district j.

The upper bound for E is $\log(n)$, where n is the number of districts. E takes the value $\log(n)$ when the population density is the same in all districts. Dividing E by $\log(n)$ provides us with an index that lies between zero and one. This normalized entropy measure makes it easier to compare entropy results between regions.

Using the two different weights for p_j above gives us two different measures of Shannon's entropy (E_1 and E_2). We calculate these entropy measures for the city of Graz, the surrounding area of Graz, and greater Graz (the latter being the sum of the city and surrounding area).

$$E_1 = \frac{-\sum_j p_j \cdot \log(p_j)}{\log(n)} \qquad \text{with } p_j = \frac{\sum_i pop_{ij} / \sum_j x_{ij}}{\sum_j (\sum_i pop_{ij} / \sum_j x_{ij})}$$

$$E_2 = \frac{-\sum_j p_j \cdot \log(p_j)}{\log(n)} \qquad \text{with } p_j = \frac{\sum_i pop_{ij} / \sum_j \sum_k x_{ijk}}{\sum_j (\sum_i pop_{ij} / \sum_j \sum_k x_{ijk})}$$

3.3 Entropy measures that do not use population data

The Shannon entropy measure can also be applied to density measures that are not based on population data. An advantage of applying a non- population data based approach is that it is readily applicable in areas for which no reliable population data exist. Gathering detailed population data is cost intensive, in contrast gathering land use data is generally very cost effective. Also, as land use data can be gathered remotely they are available for even the remotest parts of the world and areas for which no reliable population data exist.

One example of such a density measure for a district is the number of built-up cells divided by the total number of cells in the district. Applying this definition to the above index we get an entropy measure of the following form:

$$E_3 = -\sum_j p_j \cdot \log(p_j)/\log(n),$$

where

$$p_j = \frac{\sum_i \sum_k x_{ijk}/\sum_i x_{ij}}{\sum_j \left(\sum_i \sum_k x_{ijk}/\sum_i x_{ij} \right)}$$

are the weights summing up to one.

$\sum_i \sum_k x_{ijk}$ denotes the number of all built up cells in district j with land use type k ($k = 1, 2$). On the other hand, $\sum_i x_{ij}$ is the number of all cells in district j. For E_3 urban sprawls depends only on whether land is used or not, not on how many people live on it.

3.4 Critique on the use of entropy indices

Even though entropy indices are very popular in the urban sprawl literature, we find that their interpretation is not straightforward. This difficulty comes from the fact that entropy indices measures two components within one index: on the one hand the prevalence of one cell type compared to another, on the other hand how scattered these cell types are within the chosen area. This means that the same level of entropy can exist in areas that are very different in their properties. For example an area with a high prevalence of on particular cell type and a high degree of scatteredness will get a similar entropy score to another area that is characterized by a low prevalence of this cell type and a low degree of scatteredness. This makes entropy comparisons between different areas unsatisfactory. For this reason we tried to find an alternative measurement of urban continuity (scatteredness) which does not suffer from this problem. The (global) Moran I index, a measure of spatial autocorrelation, does just that.

3.5 Spatial autocorrelation as an alternative for entropy

In this section we introduce the Moran I index, which is a measure of spatial autocorrelation, measuring the degree of clustering within an area. The Moran I index has been introduced to the urban sprawl literature by Tsai (2005) as a measure to distinguish compactness from sprawl and also in order to characterize metropolitan form. Similar to entropy measures, Moran's I also captures unequal distribution in an area. However, while entropy refers to the distribution and composition of cells over the entire area, spatial autocorrelation measures the similarity of neighbouring cells. Thus a gradual decline in intensity of settlement from the inner-city outwards would give us high neighbourhood similarity values while a city with multiple urban centers and agricultural areas in between these centers would give us low

neighbourhood similarity values. Entropy also measures cell similarity, but it captures the cell similarity average over the entire area and those two scenarios would get similar entropy values. For this reason we believe that the Moran I index is better suited to measure the scatteredness aspect of urban sprawl than entropy indices. Low autocorrelation values will generally mean a high degree of scatteredness and thus a high degree of urban sprawl, however high autocorrelation numbers are more difficult to interpret. Whether the Moran I index is high because the cells of the chosen area show similarly high population numbers or show similarly low population numbers will remain context specific.
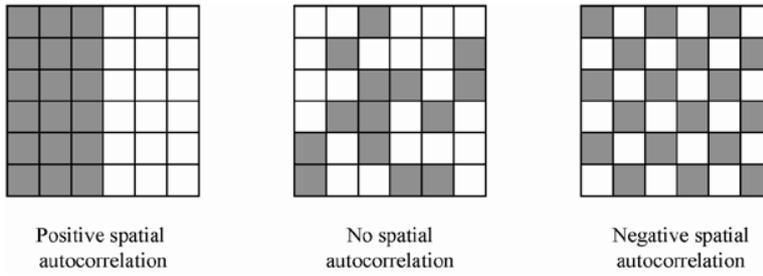

Positive spatial   No spatial   Negative spatial
autocorrelation   autocorrelation   autocorrelation

Figure 2: llustration of spatial autocorrelation

The Moran I index is defined as follows:

$$Moran = \frac{n}{w} \cdot \frac{[\sum_i \sum_j w_{ij} \, (pop_{ij} - \overline{pop})] \cdot [pop_{ij} - \overline{pop}]}{\sum_i \sum_j (pop_{ij} - \overline{pop})^2}$$

where pop_{ij} denotes population of cell i in district j, $\overline{pop}$ denotes average cell population (over the entire area), $w = \sum_i \sum_j w_{ij}$ with the distance weight w_{ij} of cell i in district j and n is the number of cells.

The Moran index has a range from -1 to +1. To be consistent with the entropy measures discussed above – where a value close to 1 indicates a high degree of urban sprawl – we will change the direction of the Moran's I index so that a higher value implies more urban sprawl. Also, we rescale it to lie between 0 and 1. Our modified version of the Moran index is calculated as follows:

$$M_1 = \frac{1 - Moran}{2}$$

The M_1 index will be below 0.5 whenever the values in the dataset tend to cluster spatially – high values cluster near other high values and/or low values cluster near other low values. Thus low values of M_1 mean that neighbouring areas are similar to each other in terms of development intensity. When the value of M_1 becomes close to 0.5 the cell characteristics show no spatial autocorrelation. Values above 0.5 indicate negative spatial autocorrelation – something that we do not expect to find in the empirical urban sprawl analysis. The more positive the M_1 index, the more scattered the housing development over the area and thus the higher urban sprawl.

There is one problem with the interpretation of M_1 however: without knowing the closer context we cannot say whether a low urban sprawl result (low M_1) comes from an environment in which there is very little urban development (and low sprawl because most of the area is uninhabitated), or because of very high and dense urban development, or even if this result is based on low density housing throughout the

area (often considered one of the main characteristics of urban sprawl). Tsai (2005) introduced the Moran i index into the urban sprawl literature in order to characterize metropolitan form, and he sees it as the ideal candidate to replace a mix of other urban sprawl indices (like density indices, inequality indices, and metropolitan size measures). Because of the difficulty we find in interpreting the Moran i index results without looking at the underlying data structure we do not completely agree with Tsai (2005) on this point. We believe that the Moran i index (and our M_1 variant on it) is a valuable addition to the tool kit for measuring urban sprawl but not suitable to replace all other measures on this topic.

We measure M_1 with respect to population per cell for the area within the city limit of Graz and for the wider-Graz area. We find that the M_1 values for of these areas are less than 0.5, which indicates a positive spatial auto-correlation. The index performs well for the test area: our results show that the city of Graz showed a higher degree of spatial autocorrelation (lower M_1) and thus less urban sprawl than when the outer areas where included as well.

4 SPATIAL COMPACTNESS MEASURES

With each of the measures considered so far we moved a step further away from the notion of urban sprawl as a population density concept and more towards urban sprawl as a land use issue. In section 4.2 we already introduced an urban sprawl measure that is independent of population data. Now we take this concept a step further and consider a class of methods for measuring urban sprawl that do not require any population data. We have already mentioned above that such an approach is useful in situations in which no reliable population data exist. Obtaining population density measures can also be problematic when the political boundary of a town does not coincide with its functional boundary and no cell level population data exist. The class of urban sprawl measures in this section focus on the shape and relative size of a town's boundary. The Corine Land Cover Project (European Environment Agency, 2010) provides an ideal framework for illustrating these types of methods. The landscape is divided into 100 by 100 meter cells, thus allowing the boundary to be identified at this fine level. The question of „where does a town end?" has various possible answers. For the compactness measures we define the "end of town" as the end of cells with Corine Land Cover classification 1 or 2 (continuous or discontinuous urban fabric) and use the shape of the urban structure that evolves when we take all urban cells that are less than 200m apart. This follows the Urban Morphological Zone (UMZ) definition by the European Environmental Agency (2006b).

One of the often mentioned aspects of urban sprawl is "strip development" along public roads and highways. This is the result of an externality problem. "Poor land use regulation may allow developers to strip-develop along state-funded roads, rather than absorb the full cost of construction of local road systems deep into residential developments. The social benefits of higher densities at the fringe, such as enhanced social interactions or preservation of green spaces, may not be incorporated into land use and development actions" (Henderson, 2009). Compactness measures will capture this aspect of sprawl which cannot be addressed with the other indices considered so far.

The most compact boundary is a circle. Arranging all populated cells within a circle would minimize the urban sprawl according to this type of measure. The more the shape departs from a circle the higher is the resulting measure of urban sprawl.

A simple compactness measure consists of the scalar-adjusted ratio between area and circumference of a settlement. This measure is dimensionless and the simplest measure of shape complexity.

$$C_0 = \frac{p}{\sum_i x_i} \frac{2}{r} - 1,$$

where p denotes the perimeter of the town, $\sum_i x_i$ denotes the area as the number of cells, and r is the average radius of the settlement. A circle is the most compact two dimensional form, and for a circular settlement C_0 reduces to zero. C_0 can therefore be used to compare the shape of an actual town and compare it with the ideal of a circle.

However, there are some problems with this simple compactness measure C_0. First, it is not clear how to measure the radius of an irregular shape. Second, when using data based on a grid, the grid structure of the data produces "noisy contours" and consequently larger perimeters (Bribiesca 1997) which potentially inflates the compactness measure (C_0 or any other traditional compactness measure). This problem is similar to Mandelbrot's classic example of how the length of the coastline of Britain can be increased by reducing the size of the measuring stick with which it is measured (Mandelbrot, 1967).

To avoid the problem of overstating contours, we now turn to compactness measures that are defined on grids in the first place. Compactness index C_1 treats a square as the most compact shape, while compactness measure C_2 treats a grid approximation to a circle as the most compact shape.

4.1 Compactness Index 1

C_1 is based on an index introduced by Milne (1991) and asks how much a particular area differs from a square. Milne's index is a shape index that calculates patch perimeter (given in number of cell surfaces) divided by the minimum perimeter (given in number of cell surfaces) possible for a maximally compact patch (in a square raster format) of the corresponding patch area (see Department of Environmental Conservation UMass, 2013). We change the index slightly by letting our C_1 index take the form of one minus the reciprocal of Milne's original index. We do this for two reasons: first we want all our parameters be bounded between 0 and one (which Milne's original index is not), second we want all sprawl measurements to be pointing in the same direction with higher numbers indicating a higher level of urban sprawl.

$$C_1 = 1 - \frac{min\ p}{p}$$

Where:
p = perimeter of the town in terms of number of cell surfaces.
$min\ p$ = minimum perimeter of the town in terms of number of cell surfaces. $min\ p$ is calculated as follows:

> $min\ p = 4s$, when $m = 0$,
> $min\ p = 4s + 2$, if $s^2 < a \le s(1+s)$, or
> $min\ p = 4s + 4$, if $a > s(1+s)$

where $s = \sqrt{a}$, $m = a - s^2$, a denotes the area of the shape under consideration (in terms of number of cells), and s is the side of the largest integer square (i.e., a square formed from the individual cells) smaller than a.

C_1 takes on the value of 0 when the settlement area is maximally compact (i.e., square or almost square) and increases towards 1 as patch shape becomes more irregular. Given that GIS based land use data is presented in square grid form, this index is easy to apply in this context. A drawback of this index is however, that a square is not a natural form for settlements to take. Cities that grow over time generally are more likely to resemble circles than squares. Our next compactness measure is able to provide a measure that compares settlements with the ideal compact form of a circle while at the same time correcting for grid-cell data problems.

4.2   Compactness Index 2 – Bribiesca's Contact Perimeter approach

Here we introduce an approach for measuring the compactness of objects composed of a finite number of cells that was developed by Bribiesca (1997). Bribiesca's paper appeared in the Mathematics and Computer Science literature and has so far not been applied in an urban context. Bribiesca's index basically allows us to do with grid data what the C_0 measure can do in continuous space – it can compare the shape of an area with the ideal of a circle while adjusting for the grid structure. Plus, it also is size invariant. Size invariance is important to be able to compare the compactness of settlements of different sizes, but also to compare measurements of the same settlement that are using different size grid structures. The Bribiesca Contact Perimeter approach is a discrete measure of compactness using the individual grid cell as its measurement instrument. Using the individual cell as the unit of measurement has the advantage that the accuracy of the shape contour as well as the shape area are measured using the same element – the cell (Bribiesca 1997).

To understand the method, it is first necessary to introduce the concept of the contact perimeter. The perimeter of a shape that is composed of cells consists of the sum of the lengths of the sides of the closed shape. In contrast the "contact perimeter" P_C of a shape that is composed of cells corresponds to the sum of the lengths of segments which are common to the two cells. The notion of a contact perimeter and how it differs from the "normal" perimeter are illustrated in Figure 3 below.
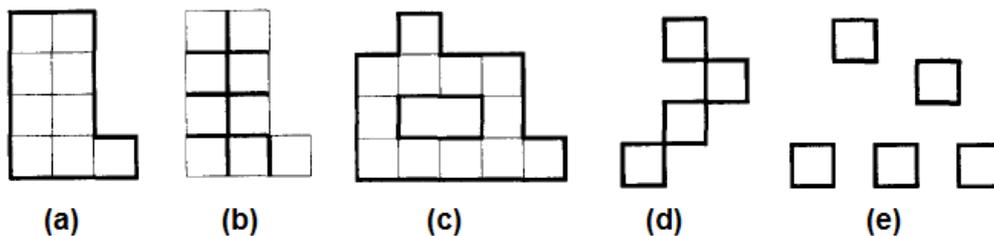
Fig. 3: The perimeters and their relations: (a) the perimeter of a shape using the rectangular tessellation; (b) the contact perimeter; (c) a shape with a hole; (d) a shape without contact perimeter;) a shape without contact perimeter; source Bribiesca (1997).

The measure of normalized discrete compactness according to Bribiesca (1997) is defined as follows:

$$Bribiesca's\ index = \frac{C_D - C_{D_{min}}}{C_{D_{max}} - C_{D_{min}}}$$
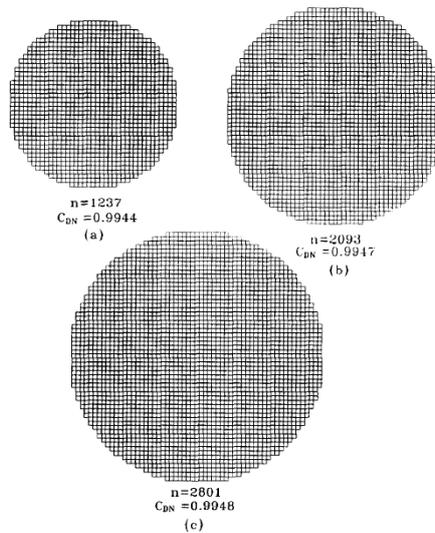
where

$C_D = P_C$ (Contact perimeter)
$C_{D_{min}} = n - 1$
$C_{D_{max}} = 2(n - \sqrt{n}\,)$
n = number of cells in shape

The values of this index varies continuously from 0 (minimum compactness) to 1 (maximum compactness) (Bribiesca, 1997). Figure 4 illustrates digital circles and their values for this index (which are basically equal to 1).

Fig. 4: Examples of three different levels of resolution for a digital circle: (a) the digital circle composed of 1237 pixels; (b) the digital circle composed of 2093 pixels; (c) the digital circle composed of 2801 pixels; source Bribiesca (1997).

This measure of discrete compactness is invariant under translation, rotation, and scaling (Bribiesca 1997). Especially its invariance under scaling is important for our purposes: it allows the index to remain constant when the same shape is presented using different scale grid representations (and therefore a different number of pixels). Thus a settlement's compactness value will remain (more or less) constant if grid size changes from 100m x 100m to 25m x 25m.

Again we will change the index slightly to make it comparable with the other urban sprawl indices we consider. Our C_2 index, which is based on Bribiesca's (1997) index, is defined as follows:

$$C_2 = \frac{C_{D_{max}} - C_D}{C_{D_{max}} - C_{D_{min}}}$$

C_2 will be bounded between 0 and 1, with larger numbers indicating a less compact form and thus a higher degree of urban sprawl. In our trials this index has performed well (sell also results below) and we believe that it is a useful addition to the measurement of urban sprawl.

5 RESULTS

Sprawl Measures	Density Index D_1	Density Index D_2	Entropy Index E_1	Entropy Index E_2	Entropy Index E_3	Moran I Index M_1	Compact sprawl Index 2 C_1	Compact sprawl Index 3 C_2
Graz	1944/km²	3635/km²	0.93	0.94	0.99	0.12	0.88	0.19
Graz including surround-ing area [2]	311/km²	2546/km²	0.92	0.83	0.92	0.19	0.95	0.31

Table 1: Results of Sprawl Indices.

For all but the density measures, results lie between zero and one and higher numbers indicate a higher degree of urban sprawl.

Each family of indices measures a different aspect of urban sprawl. These aspects are related but not identical – therefore we cannot say that one type of sprawl measure is categorically better than another. However within each family we can generalize somewhat:

With respect to density, the measure D_2 is more meaningful than D_1. Excluding those areas where no-one can live (for example rivers, lakes, high mountains) or where no-one is allowed to live (for example forests, parks, or other areas excluded by zoning restrictions) before calculating density numbers makes sense and vastly improves the meaningfulness and comparability of the urban density measurements.

Entropy is often used in the literature to illustrate the scatteredness aspect (or lack of clustering) of urban sprawl. We find that all versions of the entropy index provide somewhat confusing results. First of all, the values are very high (close to one) indicating a high degree of urban sprawl within the city limits and also for the greater Graz area. Knowing that only 12% cells in the area outside the city limits was classified as "populated", this result was difficult to understand. The confusion arises because entropy measures attempt to measure two components within one index: the degree of clustering as well as the relative distribution of cell types. If one of these characteristics is higher within the city area and the other is higher outside the city area the comparison between the corresponding entropy values becomes difficult. We conclude that entropy measures might therefore not be the best candidates for comparing scatteredness between different settlements.[3]

The M_1 index which is based on the (global) Moran's i index also attempts to measure the scatteredness of an area. But while entropy measures the relative prevalence of cell types AND the overall distribution (scatteredness) of relative cell types, M_1 focuses on the distribution of cell types only. This is a positive feature of the M_1 index and gets rid of the interpretation difficulties of the entropy indices. This is probably the reason why Tsai (2005) sees the Moran's I index as the ideal candidate to replace a mix of other urban sprawl indices (like density indices, inequality indices, and metropolitan size measures). While we like the use of spatial autocorrelation as a measurement of urban sprawl, we find that our M_1 index (and thus the related Moran's i index as well) has other difficulties with respect to its interpretation. Low scatteredness numbers (low numbers of M_1) mean low variability between cells. This can come from

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[2] The weights for the assignment are according to their relative area (relative number of cells).

[3] It can still be useful to measure the scatteredness of a particular settlement over time.

extremely dense settlement throughout the area (think about Dhakar in Bangladesh), particularly low settlement throughout the entire area (e.g. Gobi dessert), OR because of a vast area of low density housing development. All three situations are characterized by low variability. The first two we would not want to classify as urban sprawl, but the third situation is one of the key characteristics of urban sprawl. To know which of these situations is the right interpretation one does need to know the underlying data well. Additionally, the interpretation of a single out-of context M_1 number is difficult.

With respect to the compactness indices we can say that C_0 is not a good index for the measurment of urban sprawl. It is not invarient to patch size – comparing similar shaped cities of different size or the same city using a different size grid structure would alter the index significantly. Also it over-estimates the perimeter of grid-based data which would lead to lower compactness scores the smaller the grid size and/or the larger the settlement.

C_1 corrects the size bias and the over-estimation of perimeter that is present in C_0. Here lower values mean a more compact shape (and thus less urban sprawl). The results for C_1 indicate that the city of Graz shows lower urban sprawl than the greater Graz region – which coincides with our regional knowledge. However, the C_1 index is not ideal as its base shape is a square, which is not a typical shape for organically grown cities. C_2 on the other hand performs well and is in our opinion the best index from the third category. Being invariant to size changes, a meaningful comparison between the two explored regional areas can be made. Our results of C_2 show that the city of Graz has a quite low shape-based sprawl value (0.12) which rises as we include the surrounding area of Graz. It performs exactly as it should.

Summing up we can say the following: While we believe that urban sprawl has many different components – each of which needs a different way of approaching it – we find that within each group some indicators are better suited for this tasks than others. For the density indices we favor D_2 as it can correct for areas that cannot (or should not) contain population. Contrary to its recent popularity in the literature we find entropy indices not the ideal tool to capture the scatteredness of urban sprawl. Instead we find that the Moran's I index (and our variant of it M_1) performs better. Compactness indices attempt to formalize the "intuition" we get when looking at a map and decide whether a town has a compact form or not. The grid structure of the data imposes some restrictions that were not considered in the literature so far. We find the C_2 index is well suited to deal with any problems arising from the grid structure and is able to give meaningful, size-invariant results on the compactness of settled areas.
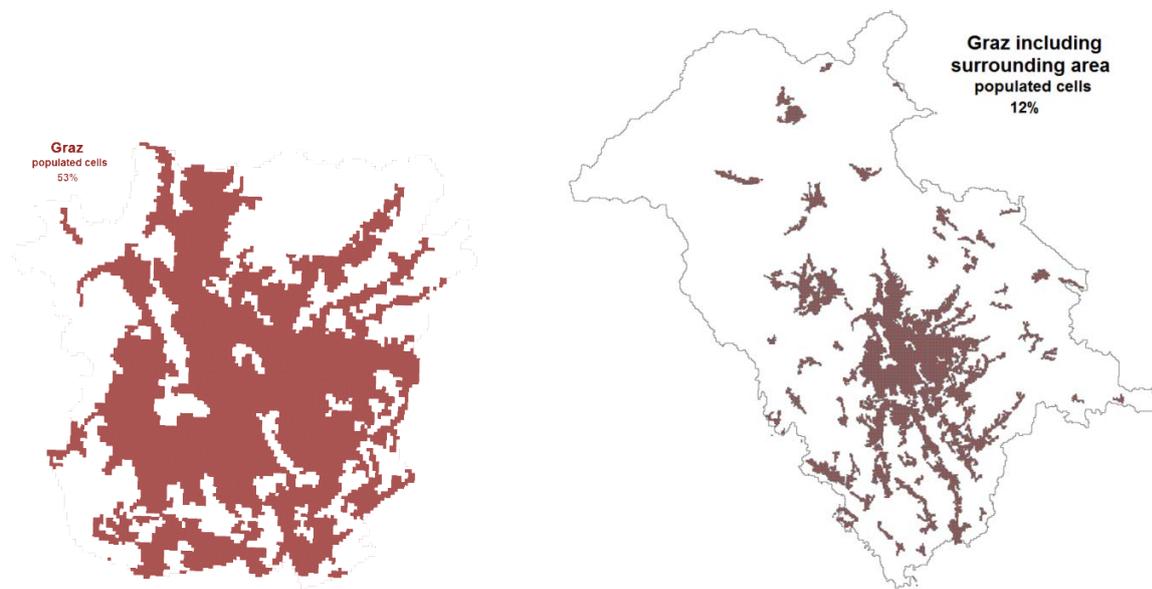
Figure 5: CLC class 1 and 2 cells within city of Graz and for greater Graz region

6 CONCLUSION

Urban sprawl has many interpretations and different measurement concepts capture different aspects of this complex issue. We have concentrated here on three main characteristics of urban sprawl: density, scatteredness, and shape of the urban development. We have developed and applied density, entropy, spatial auto-correlation, and compactness indices that measure these three components of urban sprawl. As traditional methods of measurement are not always appropriate to use on digital data, we presented indices that are optimally suited to measure the aspects of urban sprawl using digital land use data. Each of these indices captures an individual aspect of urban sprawl. However, urban sprawl is a complex phenomenon and it is therefore useful to apply a variety of measurements to capture what is going on. Focusing on a single measurement to represent the complexity of urban sprawl can be misleading (Li and Wu (2004), Frenkel and Ashkenazi (2008), Ritsema and Koomen (2008)). All the presented indices can be easily applied to digital land use data.

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