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Abstract

Harvest costs can reduce the incentive to overexploit a renewable resource stock, particularly when costs are stock dependent. This paper compares different types of harvest costs in a renewable resource based overlapping generations (OLG) model in which resource harvest competes with commodity production for labor. We analyze under which conditions a stationary state market equilibrium exists and whether this equilibrium is intergenerationally efficient. We find that stock dependent harvest costs favor the existence of an equilibrium and that a positive own rate of return on the resource stock is no longer necessary for intergenerational efficiency. Whether constant or inversely stock dependent, harvest cost in general equilibrium necessitate the inquiry of a positive resource stock price to ensure the existence of a stationary state market equilibrium.

Keywords: natural resources, harvest costs, overlapping generations, existence, efficiency

JEL Codes: Q20; D90; C62;

1 Introduction

There is no such thing as free lunch—or in the context of natural resources, harvesting involves effort which translates into costs in terms of time and/or money. For any renewable resource, harvest costs can be constant, depend on the harvest volume or on the resource stock (Bjørndal et al., 1993; Grafton et al., 2007). Constant harvest cost characterize a resource which is difficult to access but once access is achieved harvesting leads to no additional costs.[1] In real world situations, constant harvest cost are hardly found for renewable resources, but it is a common (implicit) modeling assumption because it is merely a generalization of the case without harvest cost (Smith, 1968). Harvest cost which depend on the harvest volume are typical for many renewable resources which are available in abundance, such as aquaculture or wood (Smith, 1968; Heaps and Neher, 1979). Finally, harvesting effort and hence costs can also depend on the resource stock following the general wisdom 'the larger the stock, the easier to catch' (Clark and Munro, 1975).

The importance of harvest costs is fully acknowledged in partial equilibrium models of resource dynamics (Clark and Munro, 1975; Levhari et al., 1981; Olson and Roy, 1996). The seminal finding in these models is that stock dependence of harvest cost reduces the incentive to overexploit the resource stock (Scott, 1955), an effect which does not occur for harvest costs which depend only on the harvest volume.[2] Despite this significant influence of harvest costs on the optimal size of the resource stock, costs are hardly found in dynamic general equilibrium models (exemptions being Krutilla and Reuveny, 2004; Elíasson and Turnovsky, 2004; Bednar-Friedl and Farmer, 2013). The goal of this paper is hence to isolate the effects of different specifications of resource harvest costs for the existence and intergenerational efficiency of stationary state solutions in a renewable resource based overlapping generations (OLG) economy.

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[1] Yet, fixed costs can occur in addition to operating costs (see e.g. Smith, 1968). An example for fixed costs are investment costs for harvesting equipment, such as for the fishing fleet in fisheries or harvesting machines and access roads in forestry.

[2] At least not when harvest costs are linear in the harvest level. For a model with quadratic costs, see e.g. Heaps and Neher (1979).

The advantage of an OLG model as compared to Ramsey–type growth models with infinitely lived agents (ILA) or a benevolent social planner is that the OLG framework is better capable to capture the finite lifetime of households and the consequences for resource harvest and conservation when the resource stock serves as store of value across adjacent generations (e.g. Howarth and Norgaard, 1990; Mourmouras, 1991; Olson and Knapp, 1997; Krautkraemer and Batina, 1999; Koskela et al., 2002; Valente, 2008; Bréchet and Lambrecht, 2011; Bednar-Friedl and Farmer, 2013; Karp and Rezai, 2013). Due to the demographic structure of finitely lived individuals with an old generation selling the resource stock and a young generation buying, an endogenous resource stock price dynamics results in OLG models but not in models of the ILA type.

We employ a standard Diamond (1965)–type OLG model with production where the renewable resource stock plays the role of capital and regenerates according to a concave growth function and resource harvest is used as input in commodity production (as in Koskela et al., 2002). Into this model we introduce resource harvest cost by assuming that harvesting competes with commodity production for labor and that the costs of harvesting are borne by the younger generation as the resource owner (as in Bednar-Friedl and Farmer, 2013). This assumption is very similar to the one used by Elíasson and Turnovsky (2004) in an endogenous growth model of the ILA type in which they assume that labor is allocated between a resource extraction and a commodity sector.

The incorporation of harvest costs, and in particular whether harvest costs depend only on the harvest level or also inversely on the resource stock, has profound implications for the existence and efficiency of stationary state solutions in the OLG framework. Regarding existence, depending on the magnitude of harvest costs, some biologically feasible resource stock values are economically infeasible because the stock price of the renewable resource in general equilibrium would become negative (Bednar-Friedl and Farmer, 2013). This result emerges because of the general equilibrium setting in which the economy's wage rate is endogenous and would become larger than the sales price of the resource harvest. As consequence, the economically admissible range of the stationary state resource stock is only a subset of all resource stock values which are consistent with the resource regeneration function. In this paper, we show that the economic infeasibility cannot emerge in the standard model without harvest cost and that it is more pronounced with linear harvest cost than when the harvest cost depend additionally on the resource stock.

Second, as a consequence of harvest cost a non–trivial stationary state may not exist or may not be unique. This has already been shown in the ILA context: While the Inada conditions are necessary and sufficient for the existence of a unique non–trivial steady state in a standard Ramsey growth model with a renewable resource, incorporating resource harvest cost in such a model can lead to multiple steady states or a steady state may not exist (Krutilla and Reuveny, 2004).[3] In OLG models, the existence is further complicated by the finite life of household which acts as an incentive to save too little of the resource stock for the retirement period and hence a nontrivial solution may not exist, as shown by Galor and Ryder (1989) in a model with productive capital and by Koskela et al. (2002) and Farmer (2000) in a resource based economy without harvest cost. This paper proves that a non–trivial stationary state equilibrium exists only under specific values for the harvest cost parameter, the time preference rate, the resource regeneration rate, and the input elasticities of the production function. However, due to log–linear utility and Cobb–Douglas technology and in particular labor–using resource harvest, we will show that the solution is unique despite harvest cost. This is an important difference to the model of Krutilla and Reuveny (2004) in which harvest costs impact the net regeneration of the resource stock and hence multiple solutions are possible.

Third, in OLG models a non–trivial stationary state may exist but this competitive market equilibrium can be intergenerationally inefficient because of the double infinity of goods and agents (Shell, 1971). Redistribution of savings and hence consumption between the young and old generation could lead to a Pareto improvement (de la Croix and Michel, 2002). In case of a renewable resource based economy, an inefficient stationary state market equilibrium corresponds to case where the opportunity costs of holding the resource stock are negative—a situation which would imply a negative rate of return on assets (Koskela et al., 2002). We therefore investigate whether such a situation is more likely to emerge (in terms of stringency of efficiency conditions) with some types of harvest cost functions than with others.

The remainder of the paper is structured as follows. Section 2 provides the description of the model, including a characterization of the different types of harvest costs. Section 3 derives the conditions for the existence, uniqueness and asymptotic stability of a nontrivial stationary state

[3] The reason for this result is that due to harvest costs one of the isoclines of the model becomes non–concave, despite concave resource harvest and growth functions and well–behaved utility and production functions.

for each type of harvest costs. The intergenerational efficiency of these solutions is analyzed and compared in section 4. Section 5 discusses our results and concludes.

2 Model description

In this chapter, we start with the description of the general modeling framework. This framework consists of an OLG model with a renewable resource stock with concave regeneration and a general harvest cost function. The renewable resource is used as input in commodity production. Moreover, the resource stock serves as asset to transfer income from working to the retirement period. Households split their working time between employment in the production sector and harvesting.

After having outlined the general modeling framework, we proceed to the analysis of intertemporal equilibrium dynamics by distinguishing three cases for the harvest cost function: no harvest cost, constant unit harvest cost, and unit harvest costs that depend inversely on the resource stock.[4]

To be able to analytically elaborate the consequences of different types of harvest cost, we assume log–linear utility and Cobb–Douglas technology and a logistic resource growth. As shown by Lloyd-Braga et al. (2007), more general utility functions generate multiple steady state solutions which we want to avoid in order to be able to focus on the existence and intergenerational efficiency implications of different types of harvest cost. Our model without harvest costs is closely related to the parametric example of log–linear utility and Cobb-Douglas technology in Koskela et al. (2002). Regarding harvest cost, we build on the renewable resource based OLG model with stock dependent harvest cost in Bednar-Friedl and Farmer (2013) but for the purpose of comparison we also consider the cases of no harvest cost and linear harvest cost.

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[4] We focus here on the three most widely used specifications in the partial equilibrium literature. Other specifications would include fixed costs or harvest costs which are convex in the harvest volume. We also do not consider inefficiencies which may arise due to overcapitalization of the resource sector, which may give rise to crowding effects.

2.1 Household and firm optimization

The representative consumer's intertemporal utility depends on consumption during the working period, C_t^1, and consumption during the retirement period, C_{t+1}^2, $0 < \beta < 1$ denoting the time discount factor. The representative young household's preferences are represented by a log–linear intertemporal utility function:

$$u = u(C_t^1, C_{t+1}^2) = \ ln\, C_t^1 + \beta\, ln\, C_{t+1}^2. \tag{1}$$

The young household faces a budget constraint in each period of life. In the first period, the household splits total working time (normalized to one) on employment in the commodity production sector and on resource harvesting effort h. As a consequence of exclusive private property rights (as for e.g. a fish pond or a fishing ground), the younger household acquires the resource stock from the older household in the competitive resource stock market at the beginning of the period and can also appropriate the revenues from resource harvest in the current market period.[5] Moreover, because of the concavity of the regeneration function, rents on the resource stock occur which are appropriated by the resource–owning young generation.

Thus, in addition to wage income, the young household gains income from selling of the resource harvest X_t. This income is spent on consumption C_t^1. For transferring income to their retirement period, young households save in terms of the natural resource stock R_t^d. The budget constraint in the working period is thus:

$$p_t\, R_t^d + C_t^1 = w_t(1 - \mathrm{h}(R_t^d)X_t) + q_t X_t, \tag{2}$$

where w_t denotes real wage, q_t the price of resource harvest, p_t the price of the resource stock demanded, and the price of the consumption good in period t serves as the numeraire.

The unit harvest cost function $\mathrm{h}(R_t^d)$ is assumed to have the following properties: $\mathrm{h}'(R_t^d) \leq 0$. Thus, unit harvest effort is either constant or decreasing in the resource stock.[6] Throughout the paper, different versions of unit harvest cost functions will be used representing the idea

[5] In contrast to this beginning–of–period market equilibrium notion, Koskela et al. (2002) use the end–of–period asset market equilibrium concept.

[6] In line with basic economic reasoning, we could also assume that total harvest costs are not linear but quadratic in the harvest level. While the analysis turns out much more complicated, the qualitative results are similar as in the case of linear harvest cost.

that resource harvest requires labor (or effort) as input (see e.g. Krutilla and Reuveny, 2004; Elíasson and Turnovsky, 2004). In case of constant unit harvest cost, we have:

$$\mathrm{h}(R_t^d) = \lambda, \quad \lambda > 0. \tag{3}$$

Second, we will assume that unit harvest cost is inversely related to the resource stock:[7]

$$h(R_t^d) = \frac{\lambda}{R_t^d}, \tag{4}$$

yielding a total cost function which is linear in the harvest volume and inversely stock dependent: $(\mathrm{h}(R_t^d)X_t = \lambda X_t / R_t^d)$. As a benchmark to which we compare the two types of harvest costs, we will also analyze the case of no harvest costs: $\mathrm{h}(R_t^d) = 0$.

The resource stock serves as a store of value for the household between working and retirement period. From saving the resource stock, the household gains revenues from selling the resource stock in the retirement period, which is spent on consumption C_{t+1}^2:

$$C_{t+1}^2 = p_{t+1} R_{t+1}. \tag{5}$$

The young household owns the resource stock and therefore the dynamics of the resource stock form the third constraint:

$$R_{t+1} = R_t^d + \mathrm{g}(R_t^d) - X_t, \tag{6}$$

where $\mathrm{g}(R_t^d)$ denotes the concave resource regeneration function which is specified as logistic:[8] $\mathrm{g}(R_t^d) = r\left[R_t^d - (R_t^d)^2/R_{\max}\right]$, where $r > 0$ denotes the regeneration rate and $R_{\max}$ the carrying capacity.

The representative household thus chooses C_t^1, C_{t+1}^2, R_t^d, and X_t to maximize (1) with respect to (2) (taking account of (4)), (5), and (6).

This yields the following first order condition for intertemporal optimality in consumption:

$$\frac{C_{t+1}^2}{\beta\, C_t^1} = \frac{\left[1 + \mathrm{g}'(R_t^d)\right] p_{t+1}}{p_t + w_t \mathrm{h}_R(R_t^d) X_t}, \tag{7}$$

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[7] Multiplying the right hand side of (4) by X_t gives total harvest costs. Solving this expression for X_t yields the well–known Schaefer (1954) harvest function, a functional specification popular in mostly partial equilibrium fishery models (see, e.g. Brown, 2000; Conrad, 1999; Clark, 1990).

[8] This is the standard assumption in OLG models with a renewable resource, (see e.g. Krautkraemer and Batina, 1999; Farmer, 2000; Koskela et al., 2002; Bednar-Friedl and Farmer, 2013)

which requires that the intertemporal marginal rate of substitution between consumption when young and consumption when old equals the net return factor on the resource stock.

The second condition equates the price of the resource stock to the net return on resource harvest (i.e. the revenue on selling an incremental additional unit, net of the marginal harvest cost):

$$p_t = \left[q_t - w_t \mathrm{h}(R_t^d)\right] \left[1 + \mathrm{g}'(R_t^d)\right] - w_t \mathrm{h}'(R_t^d) X_t. \tag{8}$$

In (8), assume first that $\mathrm{h}(R_t^d) = 0$. In that case, the price of the resource stock has to be equal to the price of the harvest taking account of the growth of the stock occurring in the respective period. In the more general case when harvest cost depend on harvest volume X_t, the revenue from selling the resource harvest is reduced by the time and hence costs involved in harvesting. If, moreover, harvest cost depend on the resource stock, then the revenues on harvesting are increased (because of $\mathrm{h}'(R_t^d) < 0$) by keeping an additional unit unharvested because the larger the resource stock, the lower is harvesting effort.

The firm is assumed to behave competitively and to maximize profits given output and input prices. Output Y_t is produced according to a constant–returns–to–scale Cobb–Douglas production function $Y_t = (X_t^d)^\alpha (N_t^d)^{(1-\alpha)}$, with labor N_t^d and resource harvest X_t^d as inputs. The firm's first order conditions read as follows:

$$q_t X_t^d = \alpha Y_t, \qquad w_t N_t^d = (1-\alpha) Y_t. \tag{9}$$

All markets are assumed to clear every period, i.e. the markets for the resource stock ($R_t^d = R_t, \forall t$), for resource harvest ($X_t^d = X_t, \forall t$), and for labor ($N_t^d = 1 - \mathrm{h}(R_t) X_t, \forall t$). Finally, commodity market clearing coincides with Walras' Law and is therefore redundant:

$$(X_t)^\alpha (1 - \mathrm{h}(R_t) X_t)^{(1-\alpha)} = C_t^1 + C_t^2. \tag{10}$$

2.2 The intertemporal equilibrium dynamics and the stationary state

As in Koskela et al. (2002), the intertemporal equilibrium dynamics in the model with log–linear utility function and Cobb–Douglas technology can be reduced to a one–dimensional system in R_t. By using household's and firm's first order conditions (2)–(9) in the goods market clearing condition (10) we get:

$$\frac{X_t R_t \left[1 - \gamma(1-\alpha)\right] - {X_t}^2 \left[\mathrm{h}(R_t) R_t - (1-\gamma)(1-\alpha) \mathrm{h}'(R_t) {R_t}^2\right]}{{R_t}^2 \left[\alpha - \mathrm{h}(R_t) X_t\right]} = \Psi(R_t), \tag{11}$$

where $\gamma \equiv 1/(1+\beta)$, $\Psi(R_t) \equiv \{\gamma\Phi(R_t) + [1+\mathrm{g}'(R_t)]\} R_t$, and $\Phi(R_t) \equiv \mathrm{g}(R_t)/R_t - \mathrm{g}'(R_t)$. The intertemporal equilibrium dynamics for the resource stock results after solving (11) with respect to X_t and substituting the result in the net resource growth function (6).

Setting $X_{t+1} = X_t = X, \forall t$ and $R_{t+1} = R_t = R, \forall t$ in (11) and acknowledging that according to (6) $X = \mathrm{g}(R)$, gives the following equation which characterizes the stationary state:

$$\Psi(R) = \frac{\mathrm{g}(R)\left\{[1-\gamma(1-\alpha)]\, R - \left[\mathrm{h}(R)R + (1-\gamma)(1-\alpha)\mathrm{h}'(R)R^2\right]\mathrm{g}(R)\right\}}{R\left[\alpha - \mathrm{h}(R)\mathrm{g}(R)\right]}. \tag{12}$$

Inspecting the denominator of (12) reveals that this function can exhibit a pole when $[\alpha - \mathrm{h}(R)\mathrm{g}(R)] = 0$. The sign of this expression is economically relevant insofar as it determines the sign of the resource stock price. Substituting for the firm's first order conditions in (8) and evaluating the resulting expression at the stationary state yields for the stationary state resource stock price:

$$p = \frac{\left\{[\alpha - \mathrm{h}(R)\mathrm{g}(R)]\,[1+\mathrm{g}'(R)] - (1-\alpha)\mathrm{h}'(R)\mathrm{g}(R)^2\right\}\mathrm{g}(R)^{\alpha}\,[1-\mathrm{h}(R)\mathrm{g}(R)]^{(1-\alpha)}}{\mathrm{g}(R)\,[1-\mathrm{h}(R)\mathrm{g}(R)]}. \tag{13}$$

Inspecting the numerator of (13) reveals that all expressions are clearly positive except for $[\alpha - \mathrm{h}(R)\mathrm{g}(R)]$. Thus, a necassary and sufficient condition for a positive stationary state resource stock price, and hence an economically feasible resource stock, is that $[\alpha - \mathrm{h}(R)\mathrm{g}(R)] > 0$.

Focusing first on the case of linear harvest cost, i.e. $\mathrm{h}(R_t) = \lambda$, in that case, this feasibility requirement reduces to $[\alpha - \lambda\mathrm{g}(R)]$ and implies that the right hand side of (12) exhibits two poles between which the resource stock price would become negative. With inversely stock dependent harvest cost, the economic feasibility requirement reduces to $[\alpha R - \lambda\mathrm{g}(R)]$ and thus one pole results for (12), and left of this pole the resource stock price would become negative. Finally, without harvest cost the feasibility requirement is equal to $\alpha > 0$ and thus no pole emerges. Hence, without harvest cost the resource stock price is positive for the whole range of biologically feasible resource stock values ($R \in (0, R_{max})$). Proposition 1 summarizes this result.

Proposition 1 (Economic feasibility)**.** *Let* $\mathrm{g}(R) = r\left[R - R^2/R_{\max}\right]$ *and* $\mathrm{h}'(R) \leq 0$. *A resource stock is economically feasible in the stationary state when* $[\alpha - \mathrm{h}(R)\mathrm{g}(R)] > 0$. *Without harvest cost, all resource stocks* $R \in (0, R_{max})$ *are economically feasible. With linear harvest costs, when* $\lambda r R_{max} - 4\alpha < 0$ *all resource stocks* $R \in (0, R_{max})$ *are economically* *feasible. If on the other hand* $\lambda r R_{max} - 4\alpha > 0$*, all resource stocks* $R \in (0, \hat{R}_1) \cup (\hat{R}_2, R_{max})$ *are economically feasible and if* $\lambda r R_{max} - 4\alpha = 0$*, all resource stocks* $R \in (\hat{R}_2, R_{max})$ *are economically feasible. With inversely stock dependent harvest cost, for* $\lambda \leq \alpha/r$ *all resource stocks* $R \in (0, R_{max})$ *are economically feasible while for* $\lambda > \alpha/r$ *all resource stocks* $R \in (\hat{R}_2, R_{max})$ *are economically feasible.*

Proof 1. *For* $\mathrm{h}(R) = \lambda$*, denote the left hand side of* (12) *by* $\mathrm{LHSL}(R)$ *and the right hand side by* $\mathrm{RHSL}(R)$ *and the denominator of the latter by* $\mathrm{BL}(R) \equiv \alpha - \lambda \mathrm{g}(R)$*. For logistic regeneration, the poles of* $\mathrm{BL}(R) = 0$ *can be calculated as* $\hat{R}_{1,2} = R_{max}/2 \pm \sqrt{\lambda r R_{max} (\lambda r R_{max} - 4\alpha)}$*. Both solutions are real if* $\lambda r R_{max} - 4\alpha \geq 0 \iff \lambda \geq 4\alpha/(r R_{max})$*. If on the other hand* $\lambda r R_{max} - 4\alpha < 0$*, no pole emerges for* $\mathrm{RHSL}(R)$*.*

For $\mathrm{h}(R) = \lambda/R$*, denote again the left and right hand side of* (12) *by* $\mathrm{LHSR}(R) \equiv \Psi(R)$ *and by* $\mathrm{RHSR}(R) \equiv \mathrm{g}(R) \left\{ [1 - \gamma(1-\alpha)] R - [1 + (1-\gamma)(1-\alpha)] \mathrm{g}(R) \lambda \right\} / \mathrm{BR}(R)$*, where* $\mathrm{BR}(R) \equiv [\alpha R - \lambda \mathrm{g}(R)]$*. By setting* $\mathrm{BR}(R) = 0$ *we find two poles of* $\mathrm{RHSR}(R)$*:* $\hat{R}_1 = 0$ *and* $\hat{R}_2 = [(\lambda r - \alpha) R_{max}] / (\lambda r)$*. For* $\lambda r = \alpha$*,* $\hat{R}_1 = \hat{R}_2 = 0$*, while for* $\lambda > \alpha/r$ *only the second pole* $\hat{R}_2$ *exists.* □

3 Existence of stationary state market equilibrium

3.1 Reference model without harvest cost

Having established that the whole range of biologically feasible resource stock values, i.e. $R \in (0, R_{max})$, is also economically feasible in the model without harvest costs, we now investigate the existence, uniqueness and asymptotic stability of the stationary state resource stock for that case. For a nontrivial stationary state to exist, it is sufficient that the left hand side of (12), denoted by $\mathrm{LHS0}(R)$, cuts the right hand side, $\mathrm{RHS0}(R)$, from below at the intersection point. As summarized in Prop. 2, this is essentially the case if the slope of the left hand side at the origin is flatter than the slope of the right hand side.

Proposition 2 (Existence, uniqueness, and stability without harvest cost)**.** *Let* $\mathrm{g}(R) = r \left[R - R^2/R_{\max} \right]$ *and* $\mathrm{h}(R) = 0$*. Then a unique and asymptotically stable nontrivial stationary state solution* $R \in (0, R_{max})$ *with* $p > 0$ *exists if* $1 - (1-\gamma) r > 0$ *and* $\lim_{R \to 0^+} \mathrm{LHS0}'(R) \equiv 1 + r < \lim_{R \to 0^+} \mathrm{RHS0}'(R) \equiv [1 - \gamma(1-\alpha)] r/\alpha$*.*

Proof 2. *See Appendix A.1.*

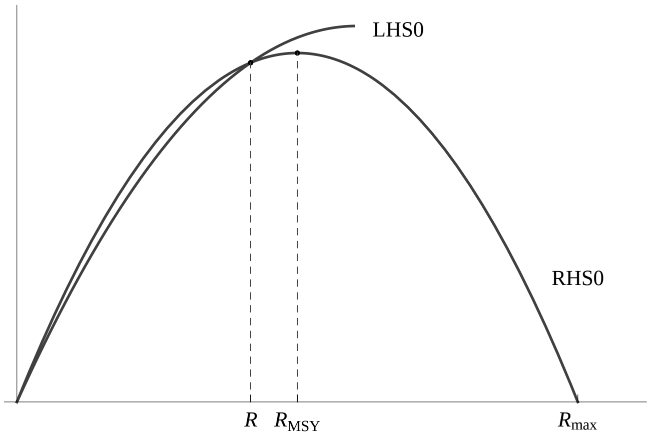

Figure 1: A unique and asymptotically stable stationary state $R < R_{\text{MSY}}$ in model without harvest cost

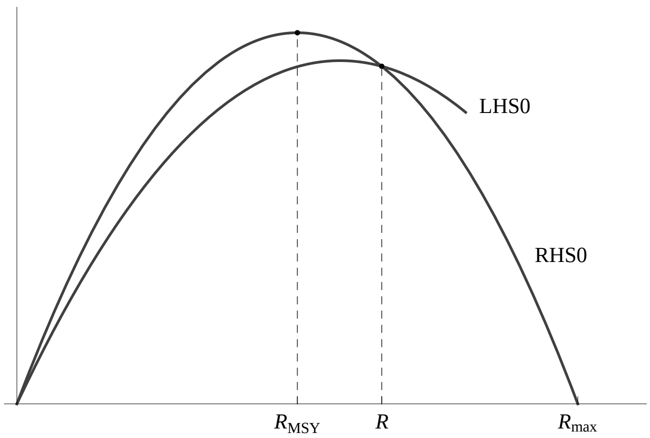

Figure 2: A unique and asymptotically stable stationary state $R > R_{\text{MSY}}$ in model without harvest cost

The economic intuition behind Prop. 2 is that two cases have to be distinguished. In the first case, the model parameters are such that the stationary state lies between the origin and the maximum sustainable yield resource stock, $R_{MSY} = R_{max}/2$. This case is illustrated in Fig. 1.[9] Here, for a resource stock close to the origin, the growth potential of the resource stock is sufficiently large as compared to the productivity of resource use in commodity production. Continuity of functions on the left and right hand side ensures that this also holds for larger levels. As the regeneration function $\mathrm{g}(R)$ is monotonically increasing (but at a decreasing rate)

[9] The figures are drawn for illustrative purposes based on the following parameter set: $\alpha = 0.3, r = 1.4, R_{max} = 10, \lambda = 0$. In Fig. 1, $\beta = 0.6$, while $\beta = 0.9$ in Fig. 2.

until R_{MSY} and hence also $\text{RHS0}(R)$ is, and moreover the slope of LHS0 at the maximum sustainable yield level is higher than the slope of RHS0, there is a unique intersection point which is the nontrivial stationary state. As therefore also the slope of LHS0 at the stationary state is larger than that of RHS0, the stationary state is asymptotically stable.

The second case is depicted in Fig. 2 where the stationary state is larger than the maximum sustainable yield level but smaller than the carrying capacity. In that case, the slope of $\text{RHS0}(R)$ is negative but the slope of $\text{LHS0}(R)$ is certainly less steep leading also to a unique, asymptotically stable intersection point.

3.2 Model with constant unit harvest cost

For constant unit harvest cost, economic feasibility may be violated over some ranges of biologically feasible resource stock values. Thus, we need to distinguish three cases for the existence of a stationary state resource stock.

Proposition 3 (Existence, uniqueness, and stability with constant unit harvest cost)**.** *Let* $\text{g}(R) = r\left[R - R^2/R_{\max}\right]$ *and* $\text{h}(R) = \lambda$. *If* $1-(1-\gamma)r > 0$, $\lambda < 4\alpha/(rR_{max})$ *and moreover* $1 + r < [1 - \gamma(1-\alpha)]\, r/\alpha$, *or if* $\lambda \geq 4\alpha/(rR_{max})$, *then a unique and asymptotically stable nontrivial stationary state solution with* $p > 0$ *exists.*

Proof 3. *See Appendix A.2.*

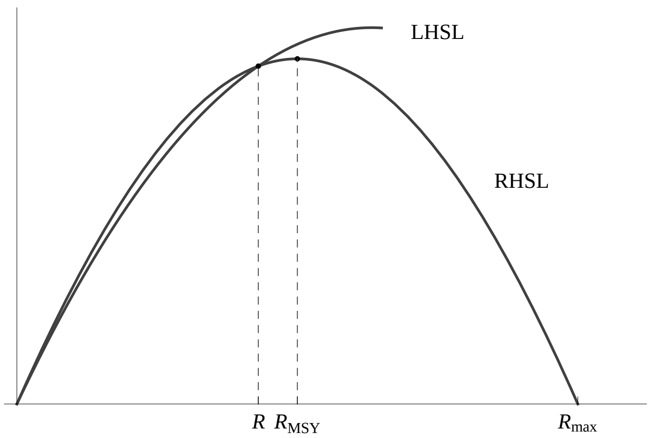

Figure 3: A unique and asymptotically stable stationary state $R < R_{\text{MSY}}$ in model with constant unit harvest cost

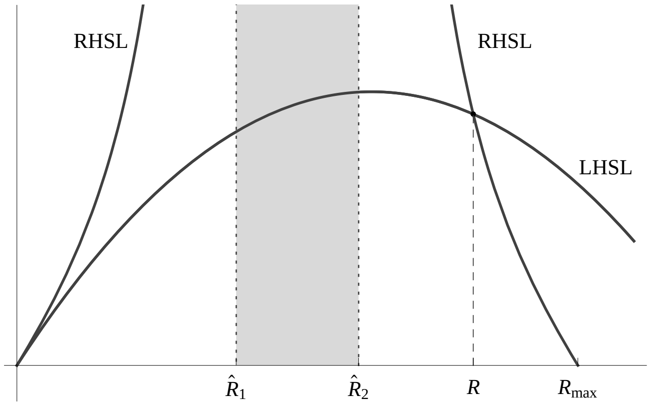

Figure 4: A unique and asymptotically stable stationary state $R > R_{\mathrm{MSY}}$ in model with constant unit harvest cost; the gray shaded area indicates where the resource stock price would be negative

By comparing Prop. 3 to Prop. 2 it can be seen that the slope condition in the model without harvest cost translates to the similar condition in the model with constant unit harvest costs (see Fig. 3).[10]

But in contrast to the model without harvest cost, the positivity of the resource stock price p, which is equivalent to $\mathrm{BL}(R) > 0$, is no longer fulfilled for all biologically feasible resource stocks as Fig. 4 illustrates. Between $\hat{R}_1$ and $\hat{R}_2$ function $\mathrm{BL}(R) < 0$ (see gray shaded area in Fig. 4) and hence the resource stock price p would be negative precluding an economically feasible stationary state solution. While both to the left of $\hat{R}_1$ and to the right of $\hat{R}_2$ the resource stock price would be positive because of $\mathrm{BL}(R) > 0$, the unique, and asymptotically stable stationary state market equilibrium is found at the right arm of RHSL. This stationary state is characterized by a relatively high harvest cost parameter, i.e. $\lambda > 4\alpha/(rR_{max})$, which enables a high stationary state resource stock and a low harvest level.

The economic intuition of this result is that when harvest costs are large, decision makers have a higher incentive to keep the resource stock large. In contrast, when harvest costs are small, there is the danger that the resource stock is overexploited. It is therefore necessary that the natural growth rate r is large compared to the productivity of resource inputs in commodity production. If this latter condition is violated, a nontrivial steady state solution does not exist.

[10] Fig. 3 is drawn for $\lambda = 0.014$, and Fig. 4 for $\lambda = 0.09$. For both figures, $\beta = 0.55$. All other model parameters are set as for Fig. 1.

Since Krutilla and Reuveny (2004) find multiple solutions in a one–sector ILA model due to harvest cost which impact on the resource stock regeneration while Elíasson and Turnovsky (2004) do not in a two–sector ILA model with labor using harvest cost, it remains to be discussed whether harvest costs can lead to multiple solutions in a resource based OLG framework. In our model setting with log–linear utility and Cobb-Douglas technology, the answer is no—on the one hand due to labor using harvest costs and on the other due to log–linear utility and Cobb-Douglas production technology (as in OLG models with endogenous labor supply, see Lloyd-Braga et al., 2007). We will show in the next section, that this result carries also over to the case of inversely stock dependent harvest cost.

3.3 Model with inversely stock dependent harvest cost

As for constant unit harvest cost, economic feasibility may be violated over some ranges of biologically feasible resource stock values. Thus, we need to distinguish again three cases for the existence of a stationary state resource stock.

Proposition 4 (Existence, uniqueness, and stability with inversely stock dependent harvest cost)**.** *Let* $\mathrm{g}(R) = r\left[R - R^2/R_{\max}\right]$ *and* $\mathrm{h}(R) = \lambda/R$*. If* $\lambda < \alpha/r$*,* $1-(1-\gamma)r > 0$ *and moreover* $1 + r < \{(1-\gamma)\left[1-(1-\alpha)\lambda r\right] - \gamma(1-\alpha) - \lambda r\}\, r/(\alpha - \lambda r)$*, or if* $\lambda \geq \alpha/r$*, then a unique and asymptotically stable nontrivial stationary state solution with* $p > 0$ *exists.*

Proof 4. *See Appendix A.3.*

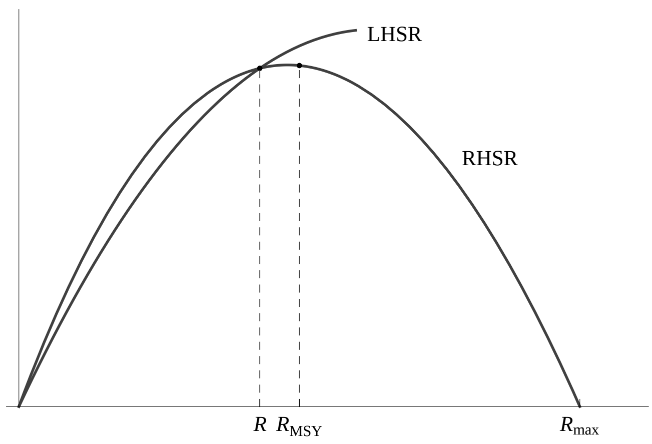

Figure 5: A unique and asymptotically stable stationary state $R < R_{\mathrm{MSY}}$ in model with stock dependent harvest cost

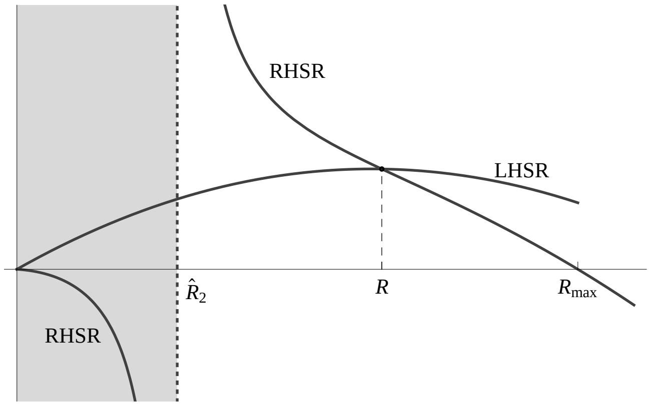

Figure 6: A unique and asymptotically stable stationary state $R > R_{\mathrm{MSY}}$ in model with stock dependent harvest cost; the gray shaded area indicates where the resource stock price would be negative

Again, the first case ($\lambda < \alpha/r$) of Prop. 4 is a generalization of the slope condition in Prop. 2 (model without harvest cost). As a consequence of stock dependent harvest cost, the first case (small harvest cost parameter) is valid for a larger range of R values as compared to the case with constant unit harvest cost (Fig. 5). This is the case as inverse stock dependent harvest cost imply for a small resource stock that unit harvest costs are high while they decrease with a larger resource stock.

For the second case (large harvest cost parameter), illustrated in Fig. 6, again no slope restriction is necessary. Yet, relative to the case with constant unit harvest cost, the harvest cost parameter needs to be larger when harvest costs depend inversely on the resource stock. The reason is again that for a large resource stock, unit harvest costs are driven downwards by the resource stock—an effect which cannot emerge when harvest costs only depend on harvest volume but not the resource stock.

4 Intergenerational efficiency of stationary state market equilibrium

Knowing that a unique and asymptotically stable stationary state exists, we investigate when the stationary state market solution is intergenerationally efficient. This is particularly relevant given the fact that the nontrivial stationary state solutions may or may not be efficient in an OLG model with a renewable resource even without harvest costs (Koskela et al., 2002).

To derive the conditions for intergenerational efficiency of the stationary state solution, we set up the problem of a social planner who maximizes utility of each individual living in the stationary state and require that the utility of the oldest generation alive in the initial period (denoted by subscript 0) achieves a predefined level:[11]

$$\max \mathrm{u}(C^1, C^2) = \ln\ C^1 + \beta\ \ln\ C^2$$

subject to

$$\begin{aligned}
(i) &\quad \ln C_0^2 = \ln \left(C_0^2\right)^{\circ}, \\
(ii) &\quad C^1 + C_0^2 = X^{\alpha}(1 - \mathrm{h}(R_0)X)^{1-\alpha}, \\
(iii) &\quad C^1 + C^2 = X^{\alpha}(1 - \mathrm{h}(R)X)^{1-\alpha}, \\
(iv) &\quad X = \mathrm{g}(R), \\
(v) &\quad R + X = R_0 + \mathrm{g}(R_0), \\
(vi) &\quad (C_0^2, C^1, C^2) \geq 0,\ (R, R_0) \geq 0,\ X \geq 0,
\end{aligned}$$

where R_0 is the resource stock owned by the initially old generation.

To see whether individual utility and profit maximization in perfectly competitive markets leads to intergenerational efficiency, we compare household and firm first order conditions (2)–(9) as well as market clearing conditions in the stationary state market equilibrium to (A.3). We start again with the reference case without harvest cost before proceeding to linear and inversely stock dependent harvest cost.

[11] Koskela et al. (2002) alternatively put the utility function of the initially old generation with a positive weight into the welfare function of the social planner.

4.1 Reference case without harvest cost

Proposition 5 states when stationary market equilibria without harvest cost are intergenerationally efficient, with Diamond's (1965) 'Golden Age', in which the utility of the initially old generation is disregarded as constraint for utility maximization of the young generation in the stationary state, as a special case.

Proposition 5 (Intergenerational efficiency without harvest cost)**.** *Let* $\mathrm{g}(R) = r\left[R - R^2/R_{\max}\right]$ *and* $\mathrm{h}(R) = 0$*. The stationary market equilibrium* R *from* (12) *is intergenerationally efficient (Golden Age) if* $\mathrm{g}'(R) > 0$ *(*$\mu_{-1}^C = 0$ *and* $\mathrm{g}'(R) = 0$*). Otherwise, the stationary state market equilibrium is intergenerationally inefficient.*

Proof 5. *See Appendix A.4.*

The two possible cases are illustrated by Figures 1– 2. In Figure 1, the stationary state market equilibrium is intergenerationally efficient—the stationary resource stock exhibits, because of the no–arbitrage condition, a positive own rate of return (underaccumulation of the resource stock occurs). This resource stock is below the Golden Rule resource stock which coincides with the maximum sustainable yield level R_{MSY}.

The opposite case is illustrated by Figure 2 in which $R > R_{MSY}$ and hence the stationary state market equilibrium is intergenerationally inefficient. Thus, a central planner could increase welfare of the present and all future generations by a reduction in resource accumulation.

4.2 Constant unit harvest cost

In case of constant unit harvest cost, the constant harvest cost parameter λ enters both the stationary market equilibrium and the intergenerational efficiency conditions. As a consequence, a specific value of the harvest cost parameter, λ^E, determines the range of stationary market equilibria which are intergenerationally efficient or not. This is summarized in Prop. 6.

Proposition 6 (Intergenerational efficiency with constant unit harvest cost)**.** *Let* $\mathrm{g}(R) = r\left[R - R^2/R_{\max}\right]$ *and* $\mathrm{h}(R) = \lambda$*. If unit harvest cost satisfies* $0 < \lambda < \lambda^E$ *where* $\lambda^E \equiv [8\alpha - 4(1-\gamma)r] \,/\, \{r\,[2-(1-\gamma)r]\,R_{max}\}$*, then the stationary market equilibrium is intergenerationally efficient. The Golden Age applies when* $\lambda = \lambda^E$*. When* $\lambda > \lambda^E$*, the stationary market equilibrium is intergenerationally inefficient.*

Proof 6. *See Appendix A.5.*

The two cases are illustrated in Figures 3–4. In Figure 3, $\lambda < \lambda^E$ and $R < R_{max}/2 = R_{MSY}$, and hence the stationary market equilibrium is intergenerationally efficient. In contrast, in Figure 4, $\lambda > 4\alpha/(rR_{max}) > \lambda^E$, and hence the stationary market equilibrium is intergenerationally inefficient.

To understand why a stationary market solution with large harvest cost parameter λ is intergenerationally inefficient, it is useful to evaluate the consequences which a higher harvest cost parameter has for the equilibrium resource stock. The higher harvesting costs, the more costly it is to harvest, the lower is resource harvest and the use of resource harvest in commodity production. As a consequence, most of labor will be devoted to commodity production instead of harvesting. But due to decreasing productivity of labor in commodity production, output and hence welfare could be increased when more labor would be devoted to harvesting.

4.3 Inversely stock dependent harvest cost

In case of inversely stock dependent harvest cost, unit harvest cost are not constant but decrease with increasing resource stock value. In contrast to the case of constant unit harvest cost, a critical value of the harvest cost parameter λ^E cannot be stated in general except for the Golden Age case. This is summarized in Prop. 7.

Proposition 7 (Intergenerational efficiency with stock dependent harvest cost)**.** *Let* $\mathrm{g}(R) = r\left[R - R^2/R_{\max}\right]$ *and* $\mathrm{h}(R) = \lambda/R$. *For all* $0 < \lambda$ *such that* (A.5)–(A.7) *allow for* $\phi_0^R/\phi^R > 0$ *and* $\phi_0^Y/\phi^Y > 0$*, the stationary state market equilibrium is intergenerationally efficient. The Golden Age applies when* $\lambda = \lambda^E$*, where* $\lambda^E \equiv (1-\gamma)\left[2\alpha - (1-\gamma)r\right] / \{[(1-\gamma)r - \alpha][1 + \alpha - (1-\gamma)r]\}$*, and* $R^E = \alpha R_{max}/[(1-\gamma)r]$*. For* $\lambda > \alpha/r$*, the stationary market equilibrium is intergenerationally inefficient.*

Proof 7. *See Appendix A.6.*

The two cases of Prop. 7 are again illustrated in Figures 5–6. In Fig. 6, the harvest cost parameter is large ($\lambda > \alpha/r > \lambda^E$) and therefore the stationary market equilibrium is intergenerationally inefficient. For a harvest cost parameter λ such that $\phi_0^R/\phi^R > 0$ and $\phi_0^Y/\phi^Y > 0$, the stationary market equilibrium is intergenerationally efficient, as illustrated in Figure 5.

Comparing these results to the model without harvest cost, it can be concluded that under inversely stock dependent harvest cost a positive own rate of return on the resource stock, i.e. $g'(R) > 0$, is no longer necessary for intergenerational efficiency. Similarly to the case with linear harvest cost, it is also not sufficient to investigate whether the slope of resource regeneration is positive or negative because harvest costs have additional effects on both the size of the stationary state resource stock and the harvest level.

5 Discussion and conclusions

This paper compared different specifications of harvest cost in an OLG model with a renewable natural resource. The first key insight is that zero harvest costs must not be considered as a special case of constant unit harvest costs or inversely stock dependent harvest cost. This is due to the factor that labor using resource harvest is competing with commodity production for labor, which would eventually lead to a negative resource stock price. In particular, the magnitude of the harvest cost parameter relative to the growth rate of the resource stock and elasticity of resource input in commodity production decide in which range of biologically feasible resource stocks the stationary state market equilibrium occurs. We also investigate the uniqueness of stationary states and verify the general insight that, in spite of endogenous labor supply and harvest cost, the log–linear utility and Cobb-Douglas technology preclude multiple stationary state solutions.

In addition to investigating the difference to the model without harvest cost, our objective was to study the potentially different impacts of different specifications of harvest costs. Here we find that inversely stock dependent harvest cost favor the existence of a nontrivial stationary state because harvests increase with a smaller resource stock. Thus, while unit harvest costs are small for a large resource stock, they become large for a small resource stock which provides a disincentive for overexploitation of the resource stock. This effect is not present when harvest costs depend only on the harvest volume but not the stock.

Of equal importance is that the condition requiring positivity of the own rate of return on the resource stock, which ensures intergenerational efficiency of the stationary state market equilibrium in the model without harvest cost, is not required in the model with inversely stock dependent harvest cost. Consequently, a stationary state market equilibrium in the model with stock dependent harvest cost may also be intergenerationally efficient even when the own rate of return is negative.

Yet, on the other hand, the higher the harvest cost parameter the more likely a stationary state market equilibrium may eventually be intergenerationally inefficient. This is due to the fact that the higher harvesting costs, the more costly it is to harvest, the lower is resource harvest and the use of resource harvest in commodity production. As a consequence, most of labor will be devoted to commodity production instead of harvesting. But due to decreasing productivity of labor in commodity production, output and hence welfare could be increased when more labor would be devoted to harvesting.

Some directions for future research are easily identified. First, instead of linear harvest cost, a convex specification could be used. Second, the inverse impact of the resource stock could be reversed such that harvest costs increase with the resource stock, a specification suitable e.g. for species–rich ecosystems like tropical forests. Finally, also fixed costs could be considered, which may give rise to non–convexities.

A Appendix

A.1 Proof to Prop. 2

At the origin, $\text{LHS0}(0) = \text{RHS0}(0) = 0$ but by assumption $\lim_{R\to 0^+} \text{LHS0}'(R) < \lim_{R\to 0^+} \text{RHS0}'(R)$. On the other hand, $\text{LHS0}(R_{max}) = [1-(1-\gamma)\cdot r]/R_{max}$ and $\text{RHS0}(R_{max}) = 0$ and hence, by assumption of $[1-(1-\gamma)r] > 0$, $\text{LHS0}(R_{max}) > \text{RHS0}(R_{max})$. Since both $\text{LHS0}(R)$ and $\text{RHS0}(R)$ are continuous functions on $[0, R_{max}]$, the intermediate value theorem ensures the existence of a $0 < R < R_{max}$ such that $\text{LHS0}(R) = \text{RHS0}(R)$.

For the uniqueness of the stationary state solution, we need to distinguish the range of R on which $\text{RHS0}(R)$ and/or $\text{LHS0}(R)$ are monotonically increasing or decreasing. Assume first that the model parameters are such that the stationary state solution lies in $(0, R_{max}/2]$. Knowing that both $\text{LHS0}(R)$ and $\text{RHS0}(R)$ are monotonically increasing in $(0, R_{max}/2]$ and moreover that $\lim_{R\to R_{max}/2} \text{LHS0}'(R) = 1 - r(1-\gamma) > \lim_{R\to R_{max}/2} \text{RHS0}'(R) = 0$, functions $\text{RHS0}(R)$ and $\text{LHS0}(R)$ intersect once on the interval $(0, R_{max}/2]$.

If, on the other hand, the stationary state lies in $(R_{max}/2, R_{max})$, $\mathrm{RHS0}(R)$ is monotonically decreasing. If $\mathrm{LHS0}(R)$ is increasing, the intersection point with $\mathrm{RHS0}(R)$ is unique. In the opposite case of decreasing $\mathrm{LHS0}(R)$, the slope of $\mathrm{RHS0}(R)$ is larger than that of $\mathrm{LHS0}(R)$ since $\mathrm{LHS0}(R_{max}) = 1 - r(1-\gamma) > \mathrm{RHS0}(R_{max}) = 0$.

For local asymptotic stability of the stationary state solution, we have to show that $0 < \mathrm{d}R_{t+1}/\mathrm{d}R_t(R) < 1$ holds at the stationary state R which is equivalent to $\mathrm{d}R_{t+1}/\mathrm{d}R_t(R) = 1 + \mathrm{g}'(R) - \mathrm{d}X_t/\mathrm{d}R_t(R) > 0$ and $\mathrm{g}'(R) < \mathrm{d}X_t/\mathrm{d}R_t(R)$. In order to show that $1 + \mathrm{g}'(R) - \mathrm{d}X_t/\mathrm{d}R_t(R) > 0$, we rewrite (11) as $X_t = \alpha/(\alpha+\beta)\,\{R_t + \mathrm{g}(R_t) + \beta[1 + \mathrm{g}'(R_t)]R_t\}$ which yields for the first derivative $\mathrm{d}X_t/\mathrm{d}R_t(R) = \alpha/(\alpha+\beta)\,\{[1 + \mathrm{g}'(R)](1+\beta) + \beta\mathrm{g}''(R)R\}$ and hence $\mathrm{d}R_{t+1}/\mathrm{d}R_t(R) = [1 + \mathrm{g}'(R)]\beta(1-\alpha)/(\alpha+\beta) - \alpha\beta/(\alpha+\beta)\mathrm{g}''(R)R > 0$ since $\mathrm{g}''(R) > 0$.

To show that $\mathrm{g}'(R) < \mathrm{d}X_t/\mathrm{d}R_t(R)$, we have to distinguish the range of R on which $\mathrm{g}'(R) \geq 0$ from that on which $\mathrm{g}'(R) < 0$. Assume first that the model parameters are such that the stationary state solution lies in $(0, R_{max}/2]$. Then, clearly $\mathrm{g}'(R) \geq 0$ and $\mathrm{RHS0}'(R) \geq 0$. This is also true for $\mathrm{LHS0}'(R)$, because $\mathrm{LHS0}'(R) = 1 + r + [2(\gamma-2)rR]/R_{max}$ is positive for $R = R_{max}$ and therefore a fortiori also positive for smaller R. Since moreover $\mathrm{RHS0}'(R_{max}/2) < \mathrm{LHS0}'(R_{max}/2)$ and by assumption $\lim_{R\to 0^+}\mathrm{LHS0}'(R) < \lim_{R\to 0^+}\mathrm{RHS0}'(R)$, it follows from continuity of $\mathrm{LHS0}'(R)$ and $\mathrm{RHS0}'(R)$ that for $R \in (0, R_{max}/2]$ $\mathrm{LHS0}'(R) > \mathrm{RHS0}'(R) \iff \alpha/(\alpha+\beta)\,\{(1+\beta)[1+\mathrm{g}'(R)] + \beta\mathrm{g}''(R)R\} > \mathrm{g}'(R)$.

Second, assume that the stationary state $R \in (R_{max}/2, R_{max})$. Clearly, $\mathrm{g}'(R) < 0$ and $\mathrm{RHS0}'(R) < 0$. If moreover $\mathrm{LHS0}'(R) \geq 0$, the claim that $\mathrm{LHS0}'(R) > \mathrm{RHS0}'(R)$ is proven. If, on the other hand, $\mathrm{LHS0}'(R) < 0$, the (negative) slope of $\mathrm{RHS0}'(R)$ is larger than the (negative) slope of $\mathrm{LHS0}'(R)$, because for $R = R_{max}$ $\mathrm{LHS0}(R_{max}) = 1 - r(1-\gamma) > \mathrm{RHS0}(R_{max}) = 0$ and thus $\mathrm{RHS0}'(R) < \mathrm{LHS0}'(R)$ holds a fortiori for $R \in (R_{max}/2, R_{max})$. □

A.2 Proof to Prop. 3

To proof the existence and uniqueness of the stationary states, according to Prop. 1 we have to distinguish for three cases:

(i) Focusing first on the case $\lambda r R_{max} - 4\alpha < 0$, we have $\mathrm{BL}(R) > 0$ for $R \in (0, R_{max}]$. Analogously to the proof to Prop. 2 it is easy to verify that $\mathrm{LHSL}(0) = \mathrm{RHSL}(0)$ (see Fig. 3). On the other hand, when $[1-(1-\gamma)r] > 0$, $\mathrm{LHSL}(R_{max}) = [1-(1-\gamma)r]R_{max} > \mathrm{RHSL}(R_{max}) = 0$. Since both functions are continuous on $(0, R_{max}]$ and by assumption $\lim_{R\to 0^+} \mathrm{LHSL}'(R) < \lim_{R\to 0^+} \mathrm{RHSL}'(R)$, at least one stationary state solution exists.

For the uniqueness of the stationary state, we need again to distinguish the range of R on which $\mathrm{LHSL}(R)$ and $\mathrm{RHSL}(R)$ are monotonically increasing or decreasing. Assume first that the model's parameters (λ included) are such that the stationary state solution lies in $(0, R_{max}/2)$. We know from the proof of Proposition 2 that for $R \in (0, R_{max}/2)$ $\mathrm{LHSL}'(R) > 0$ and also $\mathrm{RHSL}'(R) > 0$ since $\lim_{R\to 0^+} \mathrm{RHSL}'(R) = [1-(1-\alpha)\gamma]r/\alpha > 0$ and $\lim_{R\to R_{max}/2} \mathrm{RHSL}'(R) = 0$. Since $\lim_{R\to R_{max}/2} \mathrm{LHSL}'(R) > 0$, functions $\mathrm{LHSL}(R)$ and $\mathrm{RHSL}(R)$ intersect once on the interval $(0, R_{max}/2)$.

On the other hand, for a stationary state $R \in (R_{max}/2, R_{max})$, $\mathrm{RHSL}'(R)$ is monotonically decreasing. If $\mathrm{LHSL}'(R) > 0$ then the intersection point is unique. In the opposite case of $\mathrm{LHSL}'(R) < 0$, the slope of $\mathrm{RHSL}(R)$ is larger (in absolute terms) than that of $\mathrm{LHSL}(R)$ since $\mathrm{LHSL}(R_{max}) = [1-r(1-\gamma)]R_{max} > \mathrm{RHSL}(R_{max}) = 0$.

(ii) For the case of $\lambda r R_{max} - 4\alpha = 0$, there is one pole $\hat{R} = R_{max}/2$. Since $\mathrm{g}(R)$ is maximal for $R = R_{max}/2$ and $\mathrm{BL}(R_{max}/2) = 0$, it follows that $\mathrm{BL}(R) > 0$ for all other admissible R. However, as $\lim_{R\to 0^+} \mathrm{LHSL}'(R) < \lim_{R\to 0^+} \mathrm{RHSL}'(R)$ and moreover $\lim_{R\to R_{max}/2} \mathrm{RHSL}(R) = +\infty$, $\mathrm{LHSL}(R) \neq \mathrm{RHSL}(R)$ for all R in $[0, R_{max}/2)$. To the right of the pole, i.e. $R \in (R_{max}/2, R_{max}]$, $\mathrm{RHSL}(R)$ decreases monotonically with larger R with $\lim_{R\to R_{max}/2^+} \mathrm{RHSL}(R) = +\infty$ and $\lim_{R\to R_{max}} \mathrm{RHSL}(R) = 0$. On the other hand, $\mathrm{LHSL}(R_{max}/2) = 0$ and $\mathrm{LHSL}(R_{max}) > 0$. Since both $\mathrm{LHSL}(R)$ and $\mathrm{RHSL}(R)$ are continuous functions of $R \in (R_{max}/2, R_{max}]$ an intermediate value theorem ensures a solution $\mathrm{LHSL}(R) = \mathrm{RHSL}(R)$. The solution is again unique because the slope of $\mathrm{RHSL}(R)$ is negative and the slope of $\mathrm{RHSL}(R)$ is positive or negative but in the latter case certainly smaller (in absolute terms) than that of $\mathrm{RHSL}(R)$.

(iii) If $\lambda r R_{max} - 4\alpha > 0$, two poles $\hat{R}_1$ and $\hat{R}_2$ occur (see Fig. 4). It is not difficult to see that $\mathrm{BL}(R) > 0$ for $R \in [0, \hat{R}_1) \cup (\hat{R}_2, R_{max})$ and $\mathrm{BL}(R) < 0$ for $R \in [\hat{R}_1, \hat{R}_2]$. By an analogous argument as in case (ii), it can be shown that $\mathrm{LHSL}(R) \neq \mathrm{RHSL}(R)$ for $R \in [0, \hat{R}_1)$ while there is a unique solution in $(\hat{R}_2, R_{max})$.

In order to prove local asymptotic stability of the stationary state over the interval $(0, R_{max}/2]$ and $(R_{max}/2, R_{max})$ we need to show that $0 < \mathrm{d}R_{t+1}/\mathrm{d}R_t(R) < 1 \iff 1 + \mathrm{g}'(R) - \mathrm{d}X_t/\mathrm{d}R_t(R) > 0 \wedge \mathrm{g}'(R) < \mathrm{d}X_{t+1}/X_t(R)$. Deriving again $\mathrm{d}X_{t+1}/\mathrm{d}X_t(R)$ for the case of constant unit harvest cost yields:

$$1 + \mathrm{g}'(R) - \frac{\mathrm{d}X_t}{\mathrm{d}R_t}(R) = \frac{[1 + \mathrm{g}'(R)]\alpha\beta(1-\alpha) - (\alpha - \lambda\mathrm{g}(R))^2\beta\mathrm{g}''(R)R}{\left[\alpha(\alpha+\beta) - 2\alpha(1+\beta)\lambda\mathrm{g}(R) + (1+\beta)\lambda^2\mathrm{g}(R)^2\right]}. \tag{A.1}$$

The numerator of (A.1) is certainly positive because of $\mathrm{g}''(R) < 0$ but the sign of the denominator is not obvious. But in fact, the denominator is larger than zero because it is minimal at $\mathrm{g}(R) = \alpha/\lambda$, because for this value of $\mathrm{g}(R)$ the denominator of (A.1) equals $\alpha(1+\beta)(1-\alpha) > 0$. Obviously, for all other values of $\mathrm{g}(R)$ the denominator is larger and therefore $1 + \mathrm{g}'(R) - \mathrm{d}X_t/\mathrm{d}R_t$ is larger than zero for all $R \in (0, R_{max})$.

To show that $\mathrm{g}'(R) < \mathrm{d}X_{t+1}/X_t(R)$, we need to distinguish again the range of R on which $\mathrm{g}'(R) > 0$ or $\mathrm{g}'(R) < 0$. Consider first the case in which the stationary state solution lies in $(0, R_{max}/2)$ and hence $\mathrm{g}'(R) > 0$ and $\mathrm{RHSL}'(R) > 0$ since $\lim_{R\to 0^+} \mathrm{RHSL}'(R) = (1-(1-\alpha)\gamma)r/\alpha > 0$ and $\lim_{R\to R_{max}/2} \mathrm{RHSL}'(R) = 0$ and $\mathrm{RHSL}'(R)$ is continuous. In analogy to the proof of Proposition 2 we know that $\mathrm{LHSL}'(R) > 0$ for all R in $(0, R_{max})$. Thus, since by assumption $\lim_{R\to 0^+} \mathrm{LHSL}'(R) < \lim_{R\to 0^+} \mathrm{RHSL}'(R)$ and $\mathrm{LHSL}'(R_{max}/2) > \mathrm{RHSL}'(R_{max}/2)$, it follows from continuity of $\mathrm{LHSL}'(R)$ and $\mathrm{RHSL}'(R)$ that

$$\mathrm{LHSL}'(R) = \frac{[1+\mathrm{g}'(R)](1+\beta) + \beta\mathrm{g}''(R)R}{(1+\beta)} >$$

$$\mathrm{RHSL}'(R) = \frac{\mathrm{g}'(R)\left\{\alpha(\alpha+\beta) - 2\alpha(1+\beta)\lambda\mathrm{g}(R) + (1+\beta)\lambda^2\mathrm{g}(R)^2\right\}}{(1+\beta)[\alpha - \lambda\mathrm{g}(R)]^2},$$

which equals $\mathrm{g}'(R) < \mathrm{d}X_t/\mathrm{d}R_t(R)$ for $R \in (0, R_{max})$.

Assume now that the stationary state solution lies in $(R_{max}/2, R_{max})$ and therefore $\mathrm{g}'(R) < 0$ and $\mathrm{RHSL}'(R) < 0$. If again $\mathrm{LHSL}(R) \geq 0$, the claim is proven. If not, the (negative) slope of $\mathrm{RHSL}(R)$ is larger than the (negative) slope of $\mathrm{LHSL}(R)$, i.e. $\mathrm{RHSL}'(R) < \mathrm{LHSL}(R)$ since $\mathrm{LHSL}(R_{max}) = [1 - r(1-\gamma)]R_{max} > \mathrm{RHSL}(R_{max}) = 0$. □

A.3 Proof to Prop. 4

We start again by proving the existence of a stationary state solution were we have to distinguish for the cases identified in Prop. 1. Commencing with the second case, $\lambda < \alpha/r$, it is easy to show that $\mathrm{BR}(R) > 0$ for all $R \in (0, R_{max})$ (see Fig. 5). As in the proof to Prop. 3, we can show for this case that $\mathrm{LHSR}(R) = \mathrm{LHSR}(R)$ for $0 < R < R_{max}$ if $1 - (1-\gamma)r > 0$ and moreover $\lim_{R\to 0^+} \mathrm{LHSR}'(R) < \lim_{R\to 0^+} \mathrm{RHSR}'(R)$ and hence a stationary state solution exists.

On the other hand, when $\lambda \geq \alpha/r$ either $\hat{R}_1 = \hat{R}_2 = 0$ or only the pole $\hat{R}_2 > 0$ exists. For $R \in (0, \hat{R}_2)$, $\mathrm{BR}(R) < 0$ since $\mathrm{BR}'(R) > 0$. Thus, $\mathrm{LHSR}(R)$ needs to intersect $\mathrm{RHSR}(R)$ to the right of the pole, i.e. $R \in (\hat{R}_2, R_{max})$ where $\mathrm{BR}(R) > 0$ (see Fig. 6). Since $\mathrm{LHSR}(R) > 0$ for all $R \in (\hat{R}_2, R_{max}]$, the denominator of $\mathrm{LHSR}(R)$ must be larger than zero. Hence, $\lim_{R\to\hat{R}_2^+} \mathrm{RHSR}(R) = +\infty$ while $\mathrm{LHSR}(\hat{R}_2) < \infty$. On the other hand, $\mathrm{RHSR}(R_{max}) = 0$ while $\mathrm{LHSR}(R_{max}) > 0$. As a consequence of the continuity of both $\mathrm{LHSR}(R)$ and $\mathrm{RHSR}(R)$ for $R \in (\hat{R}_2, R_{max}]$, $\mathrm{LHSR}(R) = \mathrm{RHSR}(R)$ for $\hat{R}_2 < R < R_{max}$.

The proof of the uniqueness is analogous to the proof of Prop. 3.

In order to prove local asymptotic stability of the stationary state $R \in (0, R_{max})$ we have to show again that $0 < \mathrm{d}R_{t+1}/\mathrm{d}R_t(R) < 1 \iff 1 + \mathrm{g}'(R) - \mathrm{d}X_t/\mathrm{d}R_t(R) > 0 \wedge \mathrm{g}'(R) < \mathrm{d}X_{t+1}/X_t(R)$ both for $R \in (0, R_{max}/2)$ and for $R \in (R_{max}/2, R_{max})$:

$$
\begin{aligned}
1 + \mathrm{g}'(R) - \frac{\mathrm{d}X_t}{\mathrm{d}R_t}(R) = \\
&\frac{[1+\mathrm{g}'(R)]\left\{\beta(1-2\alpha)R + \lambda R\left[1+\beta\left(1+\mathrm{g}'(R)\right)\right] - \beta\lambda\mathrm{g}(R)(3-2\alpha)\right\}}{\left\{(\alpha+\beta)R + \lambda R\left[1+\beta\left(1+\mathrm{g}'(R)\right)\right] + \lambda\mathrm{g}(R)\left\{1-2\left[1+\beta(2-\alpha)\right]\right\}\right\}} \\
&+ \frac{\beta\mathrm{g}''(R)R(\alpha R - \lambda\mathrm{g}(R))}{\left\{(\alpha+\beta)R + \lambda R\left[1+\beta\left(1+\mathrm{g}'(R)\right)\right] + \lambda\mathrm{g}(R)\left\{1-2\left[1+\beta(2-\alpha)\right]\right\}\right\}}.
\end{aligned}
\tag{A.2}
$$

The numerator of (A.2) is larger than zero for all $R \in (0, R_{max})$ because the numerator is minimal at $R = 0$ and maximal at $R = R_{max}$ and is strictly monotonically increasing on the interval $(0, R_{max})$ since the second derivative of the numerator with respect to R equals $2[1 + 2(1-\alpha)\beta]\lambda r/R_{max} > 0$. The same holds true with respect to the first bracket of the denominator since the second derivative of the expression in the bracket equals $2(1-2\alpha)\beta\lambda r/R_{max} > 0$. It remains to check the sign of $(\beta g g(R) - \alpha R - \beta\mathrm{g}''(R)R\left[\alpha R - \lambda\mathrm{g}(R)\right]$. Again this term is minimal at $R = 0$ and maximal at $R = R_{max}$

and $\lim_{R\to 0}\{(\beta g g(R) - \alpha R - \beta \mathrm{g}''(R) R\left[\alpha R - \lambda \mathrm{g}(R)\right]\} = \beta r - \alpha > 0$. Thus, both the denominator and the numerator of (A.2) are positive for all $R \in (0, R_{max})$.

To show that $\mathrm{g}'(R) < \mathrm{d}X_{t+1}/X_t(R)$, we need to distinguish again the range of R on which $\mathrm{g}'(R) > 0$ or $\mathrm{g}'(R) < 0$. Consider first the case in which the stationary state solution lies in $(0, R_{max}/2)$ and hence $\mathrm{g}'(R) > 0$. Moreover, $\mathrm{RHSR}'(R) > 0$ since $\lim_{R\to 0^+} \mathrm{RHSR}'(R) = \left[\alpha + \beta - \lambda r(1+2\beta) + \alpha\beta\lambda r\right] r/(\alpha - \lambda r) > 0$ and $\lim_{R\to R_{max}/2} \mathrm{RHSR}'(R) = (1-\alpha)^2\beta\lambda r^2/(2\alpha - \lambda r)^2 > 0$ and $\mathrm{RHSR}'(R)$ is continuous. Clearly, $\mathrm{LHSL}'(R) > 0$ for all R in $(0, R_{max})$. Thus, since by assumption $\lim_{R\to 0^+} \mathrm{LHSR}'(R) < \lim_{R\to 0^+} \mathrm{RHSR}'(R)$ and $\mathrm{LHSR}'(R_{max}/2) > \mathrm{RHSR}'(R_{max}/2)$, it follows from continuity of $\mathrm{LHSR}'(R)$ and $\mathrm{RHSL}'(R)$ that

$$\mathrm{LHSR}'(R) = [1 + \mathrm{g}'(R)](1+\beta) + \beta \mathrm{g}''(R) R > \mathrm{RHSR}'(R) = \frac{(\alpha R - \lambda \mathrm{g}(R))\left\{(\alpha+\beta) R \mathrm{g}'(R) + (\alpha+\beta)\mathrm{g}(R) - 2\lambda \mathrm{g}(R)\left[1 + \beta(2-\alpha)\right]\mathrm{g}'(R)\right\}}{\left[\alpha R - \lambda \mathrm{g}(R)\right]^2} - \frac{\left[\alpha - \lambda \mathrm{g}'(R)\right]\left\{(\alpha+\beta) R \mathrm{g}(R) - \lambda \mathrm{g}(R)^2\left[1 + \beta(2-\alpha)\right]\right\}}{\left[\alpha R - \lambda \mathrm{g}(R)\right]^2},$$

which equals $\mathrm{g}'(R) < \mathrm{d}X_t/\mathrm{d}R_t(R)$ for $R \in (0, R_{max})$.

If, on the other hand, $R \in (R_{max}/2, R_{max})$, $\mathrm{g}'(R) < 0$ and $\mathrm{RHSR}'(R) < 0$. If $\mathrm{LHSL}(R) \geq 0$ the claim is proven. If not, the (negative) slope of $\mathrm{RHSR}(R)$ is larger than the (negative) slope of $\mathrm{LHSR}(R)$, i.e. $\mathrm{RHSR}'(R) < \mathrm{LHSR}'(R)$ since $\mathrm{LHSL}(R_{max}) > \mathrm{RHSL}(R_{max}) = 0$. $\square$

A.4 Proof to Prop. 5

Setting up the Lagrangian to the optimization problem in section 4

$$\begin{aligned}\mathcal{L} = \ln\, C^1 + \beta\, \ln\, C^2 + \mu^C_{-1}\left[\ln\, C^2_0 - \ln\, \left(C^2_0\right)^{\circ}\right] + \\ \phi^Y_0\left[X^{\alpha}(1 - \mathrm{h}(R_0, X)X)^{1-\alpha} - C^1 - C^2_0\right] + \\ + \phi^Y\left[X^{\alpha}(1 - \mathrm{h}(R, X)X)^{1-\alpha} - C^1 - C^2\right] + \\ + \phi^R\left[\mathrm{g}(R) - X\right] + \phi^R_0\left[R_0 + \mathrm{g}(R_0) - R - X\right],\end{aligned}$$

yields the following first order conditions:

$$\frac{C^2}{\beta C^1} = 1 + \frac{\phi^Y_0}{\phi^Y}, \tag{A.3a}$$

$$\frac{\mu_{-1}^{C}}{C_0^2} = \phi_0^Y, \tag{A.3b}$$

$$\phi_0^Y\left\{\frac{\alpha Y_0}{X} - \frac{(1-\alpha)Y_0\mathrm{h}(R_0)}{(1-\mathrm{h}(R_0)X)}\right\} + \phi^Y\left\{\frac{\alpha Y}{X} - \frac{(1-\alpha)Y\mathrm{h}(R)}{(1-\mathrm{h}(R)X)}\right\} = \phi^R + \phi_0^R, \tag{A.3c}$$

$$\phi^R\mathrm{g}'(R) = \phi_0^R + \phi^Y(1-\alpha)X^\alpha\left[1-\mathrm{h}(R)X\right]^{-\alpha}\mathrm{h}'(R)X, \tag{A.3d}$$

$$\ln\, C_0^2 = \ln\,\left(C_0^2\right)^\circ, \tag{A.3e}$$

$$Y_0 = C^1 + C_0^2, \tag{A.3f}$$

$$Y = C^1 + C^2, \tag{A.3g}$$

$$R_0 + \mathrm{g}(R_0) = R + X, \tag{A.3h}$$

$$\mathrm{g}(R) = X. \tag{A.3i}$$

For the reference case without harvest cost, intergenerational efficiency conditions (A.3c)–(A.3d) simplify to:

$$\alpha X^{\alpha-1} = \frac{\phi^R + \phi_0^R}{\phi^Y + \phi_0^Y},$$
$$\phi^R\mathrm{g}'(R) = \phi_0^R.$$

When moreover the utility of the initially old generation is disregarded as constraint for utility maximization of the young generation in the stationary state (Diamond's (1965) 'Golden Age'), $\mu_{-1}^C = 0$ and hence $\phi_0^Y = \phi_0^R = 0$. Then, the remaining efficiency conditions collapse to:

$$\frac{C^2}{\beta C^1} = 1, \tag{A.3a'}$$

$$\alpha X^{\alpha-1} = \frac{\phi^R}{\phi^Y}, \tag{A.3c'}$$

$$\mathrm{g}'(R) = 0, \tag{A.3d'}$$

$$X^\alpha = C^1 + C^2, \tag{A.3g'}$$

$$\mathrm{g}(R) = X. \tag{A.3i'}$$

Assume first that the stationary market equilibrium is such that $\mathrm{g}'(R) > 0$. Set provisionally $q = (\phi_0^R + \phi^R)/(\phi_0^Y + \phi^Y)$ and $\mathrm{g}'(R) = \phi_0^Y/\phi^Y = \phi_0^R/\phi^R$. Then, the market equilibrium conditions evaluated at the stationary state, i.e.

$$\frac{(C^2)}{\beta\,(C^1)} = 1 + \mathrm{g}'(R), \qquad p = q\left[1 + \mathrm{g}'(R)\right], \tag{7'),(8'}$$

$$q = \alpha(X)^{1-\alpha}, \qquad w = (1-\alpha)(X)^\alpha, \tag{9'}$$

$$(X)^{\alpha} = (C^1) + (C^2), \qquad (X)^{\alpha} = (C^1) + (C_0^2), \tag{10'}$$

$$X = \mathrm{g}(R), \qquad R + X = R_0 + \mathrm{g}(R_0), \tag{6'}$$

imply (A.3a')–(A.3i').

Second, assume that μ_{-1}^{C} and $\mathrm{g}'(R) = 0$ hold in the stationary state market equilibrium. $\mathrm{g}'(R) = 0$ yields the modified stationary state market equilibrium conditions

$$\frac{(C^2)}{\beta\,(C^1)} = 1, \qquad p = q. \tag{7''),(8''}$$

But $\mu_{-1}^{C} = 0$ implies $\phi_0^Y = \phi_0^R = 0$. Hence we set $q = \phi^R/\phi^Y$, and the stationary state market equilibrium conditions, (6'), (7''), (8''), (9'), (10'), imply the the Golden Age conditions (A.3a'), (A.3c')–(A.3g'), (A.3i'). □

A.5 Proof to Prop. 6

If unit harvest costs are constant ($\mathrm{h}(R) = \lambda$), the intergenerational efficiency conditions (A.3c)– (A.3d) change to:

$$\alpha X^{\alpha-1}\left[1-\lambda X\right]^{1-\alpha} - \lambda(1-\alpha)X^{\alpha}\left[1-\lambda X\right]^{-\alpha} = \frac{\phi^R + \phi_0^R}{\phi^Y + \phi_0^Y}, \tag{A.3c''}$$

$$\phi^R \mathrm{g}'(R) = \phi_0^R, \tag{A.3d''}$$

while the other conditions are similar to the model without harvest cost.

Evaluating again the market equilibrium conditions at the stationary state gives (7') and (6'), $p = (q - w\lambda)\left[1 + \mathrm{g}'(R)\right]$, $q = \alpha(X)^{\alpha-1}[1-\lambda X]^{1-\alpha}$, $w = (1-\alpha)(X)^{\alpha}[1-\lambda X]^{-\alpha}$, $(X)^{\alpha}[1-\lambda X]^{1-\alpha} = (C^1) + (C^2)$, $(X)^{\alpha}[1-\lambda X]^{1-\alpha} = (C^1) + (C_0^2)$. Setting provisionally $q = (\phi_0^R + \phi^R)/(\phi_0^Y + \phi^Y) + \lambda w$ and again $\mathrm{g}'(R) = \phi_0^Y/\phi^Y = \phi_0^R/\phi^R$, the stationary state market equilibrium conditions imply the intergenerational efficiency conditions (A.3a)–(A.3b),(A.3c'')–(A.3d''), and (A.3e)–(A.3i).

As in the case of no–harvest cost, the stationary market equilibrium is intergenerationally efficient only if $\mathrm{g}'(R) = \phi_0^Y/\phi^Y > 0$, i.e. for $R \in (0, R_{max}/2)$. Acknowledging Prop. 3, $\lambda \geq 4\alpha/(rR_{max})$ implies inefficiency of the stationary market equilibrium since $\mathrm{g}'(R) < 0$. However, $\lambda < 4\alpha/(rR_{max})$ does not imply intergenerational efficiency of the stationary state. The upper bound on λ ensuring intergenerational efficiency can be obtained by solving LHSL$(R_{max}/2)$ = RHSL$(R_{max}/2)$. The solution is $\lambda^E \equiv$

$[8\alpha - 4(1-\gamma)r] \, / \, \{r\,[2-(1-\gamma)r]\, R_{max}\}$ which is definitely smaller than $4\alpha/(rR_{max})$. For all $\lambda \leq \lambda^E$, the stationary state market equilibrium is intergenerationally efficient (Golden Age included). □

A.6 Proof to Prop. 7

To evaluate the intergenerational efficiency of stationary state market equilibria with inversely stock dependent harvest cost, we rewrite the intergenerational efficiency conditions (A.3a), (A.3c), and (A.3d) by assuming that $(C_0^2)^\circ = (C^2)$ and $R_0 = R$.[12] As a consequence of (A.3g)–(A.3i), $\mathrm{g}(R) = X$ and $C^1 = Y - C^2$. Then, (A.3a) can be written as follows:

$$\frac{(C^2)}{\beta\left\{(\mathrm{g}(R)^\alpha\,[1-\lambda\mathrm{g}(R)/R]^{1-\alpha} - (C^2)\right\}} = 1 + \frac{\phi_0^Y}{\phi^Y}. \tag{A.5}$$

The condition for efficient harvest (A.3c) can be rewritten as:

$$\alpha\;\mathrm{g}(R)^{\alpha-1}\left[1-\lambda\frac{\mathrm{g}(R))}{R}\right]^{1-\alpha} - (1-\alpha)(\lambda/R)\mathrm{g}(R)^\alpha\left[1-\lambda\frac{\mathrm{g}(R))}{R}\right]^{-\alpha} = \frac{\left(1+\phi_0^R/\phi^R\right)}{\left(1+\phi_0^Y/\phi^Y\right)}\frac{\phi^R}{\phi^Y}, \tag{A.6}$$

with, from (A.3d),

$$\frac{\phi^R}{\phi^Y} = \frac{(1-\alpha)\mathrm{g}(R)^\alpha\,[1-\lambda\mathrm{g}(R)/R]^{-\alpha}\,\lambda\mathrm{g}(R)/(R)^2}{-\mathrm{g}'(R) + (\phi_0^R/\phi^R)}. \tag{A.7}$$

We start with the stationary market equilibrium conditions $(C^2)/(\beta(C^1)) = 1 + \mathrm{g}'(R) + w\lambda\mathrm{g}(R)/\left[(R)^2(q - w\lambda/R\right]$, $p - w\lambda\mathrm{g}(R)/(R)^2 = [1+\mathrm{g}'(R)](q - w\lambda/R)$, $q = \alpha(\mathrm{g}(R))^{\alpha-1}[1-\lambda\mathrm{g}(R)/R]^{1-\alpha}$, $w = (1-\alpha)(\mathrm{g}(R))^\alpha[1-\lambda\mathrm{g}(R)/R]^{-\alpha}$. We set provisionally $\phi_0^Y/\phi^Y = \mathrm{g}'(R) + \lambda w\mathrm{g}(R)/(R)^2(\phi_0^Y + \phi^Y)/(\phi_0^R + \phi^R)$, $q - \lambda w/R = (\phi_0^R + \phi^R)/(\phi_0^Y + \phi^Y)$, and $\phi_0^R/\phi^R = \mathrm{g}'(R) + \lambda w\mathrm{g}(R)/(R)^2(\phi^Y/\phi^R)$. Using the latter two equations together with the first order market equilibrium conditions for w and q implies the efficiency conditions (A.3a) and (A.3c).

In order to obtain λ^E and R^E, we assume the Golden Age and thus insert (A.7) into (A.6) under $\phi_0^R/\phi^R = \phi_0^Y/\phi^Y = 0$, and get after simplifying the resulting equation, $(1-\alpha)\mathrm{g}(R^E)^2\lambda^E = -\mathrm{g}'(R^E)R^E[\alpha\,R^E - \lambda^E\mathrm{g}(R^E)]$. Solving this equation for λ^E and inserting the results into

[12] This equality settings we used, although implicitly, already in the former case of constant unit cost.

the stationary state equation (12), we get $R^E = (\alpha R_{max}/[(1-\gamma)r]$. By reinserting R^E into $\lambda^E = \left[\alpha \mathrm{g}'(R^E)(R^E)^2\right] / \left\{\mathrm{g}(R^E)\left[\mathrm{g}'(R^E)R^E - (1-\alpha)\mathrm{g}(R^E)\right]\right\}$, λ^E is obtained. Note that for $\phi^R/\phi^Y > 0$ in (A.7), it is not necessary that $\mathrm{g}'(R) > 0$. □

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