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Abstract

Intra-EMU external imbalances in the pre-crisis period up to 2008 are traditionally explained by EMU-oriented factors, e.g. euro-related financial integration. Chen et al. (2013) also emphasize external trade shocks, such as the competitive challenge of emerging Asia and oil exporters to EMU-periphery's exports. Moreover, Asian-US external imbalances are attributed to financial integration between East Asia and the USA in the aftermath of the East-Asian currency crises in the late 1990s (Angeletos et al. 2011). Acknowledging these empirical facts this paper develops a Buiter (1981) three-country (EMU, Asia, US), two-region (EMU core, EMU periphery) OLG model to investigate the effects of both intra-EMU and Asian-US financial integration on intra-EMU, Asian and US external imbalances. We find that the widening of the intra-EMU external imbalances, in particular of trade imbalances, is related to the growth in Asian-US imbalances and the dynamic inefficiency of the world economy, caused by excessive saving in Asia.

Keywords: External Imbalances, European Economic and Monetary Union, Overlapping Generations, Three-Country Model

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Introduction and Motivation

The external imbalances among members of the European Economic and Monetary Union (EMU) during the pre-crisis period up to 2008 are empirically well-documented (e.g. Lane and Pels 2012). The huge external deficits in southern EMU countries (including Ireland = EMU "periphery") are traditionally explained by the intra-EMU factors: (i) financial integration and expectation of convergence within the common currency area, and (ii) "over-optimism" and excessive real appreciation in the periphery (e.g. Lane 2006, Coeurdacier and Martin 2009). While accepting these traditional explanations, Chen et al. (2013) present new stylized facts with respect to extra-EMU determinants which contributed to the evolution of intra-EMU current account imbalances. Prominent among these are the trade linkages between the EMU sub-areas and the countries outside EMU, in particular China, the CEECs and the oil exporting countries. This is true despite the whole EMU current account being roughly in balance. Periphery's current account deficit, while financed mostly by capital inflows from the core, did not increase vis-à-vis the core but vis-à-vis Asia and oil exporters. A similar effect is also true with respect to the core current account surpluses. Moreover, financial integration between Asia and the USA – albeit occurring under different institutional ramifications than those existing in the Eurozone – intensified after the East Asian currency crises in the late 1990s, as can be seen by the convergence of short-term nominal and real Asian and US interest rates. Acknowledging both the strengthened real trade linkages of EMU sub-areas vis-à-vis Asia, and the closer financial linkages among Asia and USA, it is natural to suggest that the intra-EMU external imbalances are related to the external imbalances among Asia and the USA. The main objective of this paper is to investigate whether this suggestion can be verified by use of an intertemporal current account model for EMU, Asia and the USA (see Obstfeld and Rogoff 1995 for the basic intertemporal current account model; Ca' Zorzi and Rubaszek 2012 for its empirical relevance for the Eurozone).

As is well-known, after the inception of the euro in 1999, northern and center euro countries (Austria, Belgium, Finland, France, Germany, Netherlands), in particular Germany, started to run current account surpluses, while the southern and western periphery (= PIIGS: Portugal, Ireland, Italy, Greece and Spain) accumulated huge external deficits. Moreover, there was a significant divergence in the dynamics of private debt between northern and southern countries (Pisany-Ferry 2012, Figure 4): Up to the onset of the global financial crisis southern debt boomed, mainly in order to finance housing investment, while in the core housing investment relative to GDP declined. In addition, saving rates in the periphery were significantly lower than those in the core. The mounting periphery current account deficits and core current account surpluses, where thus a logical consequence of the current account simply being the difference between national savings and investment.

Although occurring under substantially different institutional ramifications, international macroeconomic developments similar to those in Europe emerged after the East Asian currency crises in the late 1990s between Asia and the USA. While major Asian countries accumulated substantial external surpluses the US external deficit (current account and net foreign asset position) deteriorated significantly. In addition, private debt and housing investment in the USA boomed, while in Asia, housing investment as a proportion of GDP declined. Asian saving rates were also much higher than US rates resulting in US current account deficits and Asian current account surpluses.

While such casual empirical similarities in the evolution of intra-EMU and Asia-US (global) external imbalances are suggestive, it remains an open theoretical question, whether or how, divergent intra-EMU and Asia-US external imbalances can be attributed to intra-EMU and Asia-US financial integration in a dynamic general equilibrium model of the world economy. To the best of this author's knowledge, the (at least) three-country intertemporal general equilibrium model needed to address both the intra-EMU and the Asia-US external imbalances does not yet exist in the literature. What does exist are dynamic general equilibrium models which deal either with the intra-EMU or the Asian-US external imbalances. Among the former are Fagan and Gaspar (2008) and Farmer (2014), among the latter is Eugeni (2013).

Fagan and Gaspar (2008) use a two-good, two-country overlapping generations pure exchange model à la Blanchard (1985) to compare the pre-euro financial autarky steady state to euro-related financial integration between southern and northern euro countries. In view of the euro-related dynamics of housing investment in Spain and Ireland, Farmer (2014) sticks to Buiter's (1981) seminal one-good, two-country overlapping generations (OLG) model with production and capital accumulation, and finds that the financial account deficits of EMU periphery and the respective surpluses of EMU core can be traced back not only to core-periphery differences in time preference but also to differences in the production technology (capital production share), government expenditure shares and public debt to GDP ratios.

As regards the Asian-US external imbalances, Eugeni (2013) also uses a one-good, two-country OLG model in which the two countries are identical except that one country, i.e. the USA, has a pay-as-you go social security system, while the other country, i.e. China, does not. As a consequence of this institutional difference, the saving rates in the latter are significantly higher than in the former and due to the largeness of the emerging economy the world economy over-accumulates capital. The dynamic inefficiency of the world economy implies that the over-saving emerging country runs a surplus in the balance of trade. This is in line with the empirical facts for the East Asian countries.

Given the intensification of external trade following the euro launch, and the increasing financial integration between the USA and the East Asian countries, there now appears to be a

need for the simultaneous investigation of intra-EMU and Asian-US external imbalances. Moreover, since the EMU, East Asia and the USA are large open economies and are all affected by the impacts of intra-EMU and Asian-US developments on trading partners, the international interdependences among EMU, Asia and USA need to be addressed. To this end, at a minimum a three-good, three-country intertemporal equilibrium model is needed. To the best of this author's knowledge of the literature such a three-country, two-region extension of the seminal Buiter (1981) OLG model does not yet exist.

The paper thus has two main objectives: First, to present the stylized macro facts regarding current and financial account imbalances between EMU core and periphery, and between Asia and the USA, in order to motivate the model set-up. Second, to develop a three-country (EMU, Asia, USA), two-region (EMU core and periphery) OLG model in order to figure out to what extent the EMU core-periphery as well as the Asian-US external imbalances can be attributed to intra-EMU financial integration due to the common currency and/or to the financial integration occurring between the United States and East Asia in the first decade of the new millennium.

The paper is organized as follows. The next section introduces the main stylized macroeconomic facts, relating to EMU and Asia-US financial integration up to the onset of the global financial crisis in 2008. In the following section, the intertemporal OLG model under financial autarky is set up and the changes due to financial integration in both regions of the world economy are pointed out. After noting the sufficient conditions for the existence and dynamic stability of steady-state solutions under financial autarky (before euro launch and before Asian-US financial integration) we then demonstrate how the respective trade and current account imbalances can be traced back through financial integration to international differences in fundamental parameters such as saving rates, capital production shares, government expenditure quotas and public debt to GDP ratios. Concluding remarks in the final section summarize the key results of the modeling approach.

Stylized Macroeconomic Facts: Financial Autarky versus Financial Integration

In order to illuminate the design of the three-good, three-country and two-region OLG model, some of the relevant stylized facts are now presented in this section. These relate to the macroeconomic performance of the EMU members, Asia and the USA, and the evolution of the current account and the net foreign asset positions in the EMU, Asia and the USA before the launch of the euro in 1999 and up to 2008. Following Fagan and Gaspar (2008, p. 9), the EMU countries are separated into two groups based on the differences in short-term real interest rates in the late 1990s, i.e. before the euro launch. The first group, usually denoted as the "core" countries, comprises the low interest rate countries Austria, Belgium, France, Germany and the

Netherlands.[1] The second group, denoted as "periphery" or converging countries, consists of countries which had relatively high interest rates before the introduction of the euro (see Fig. 1).

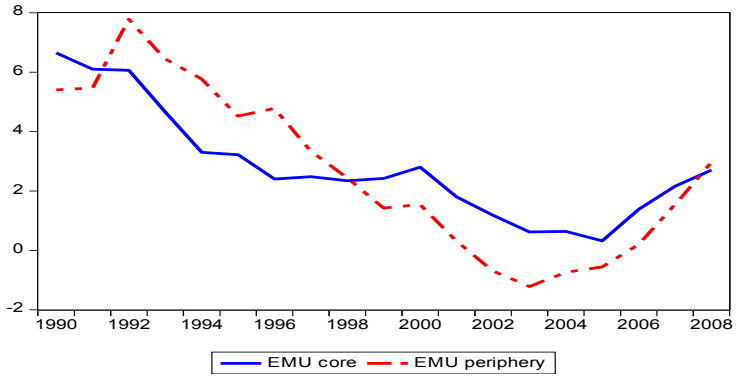

Fig. 1 Real short-term interest rates in EMU core and periphery 1990-2008. Source: AMECO

Figure 1 reveals that in contrast to the pre-EMU situation (before 1999), EMU periphery's higher real interest rates decreased towards lower rates in EMU core.[2]

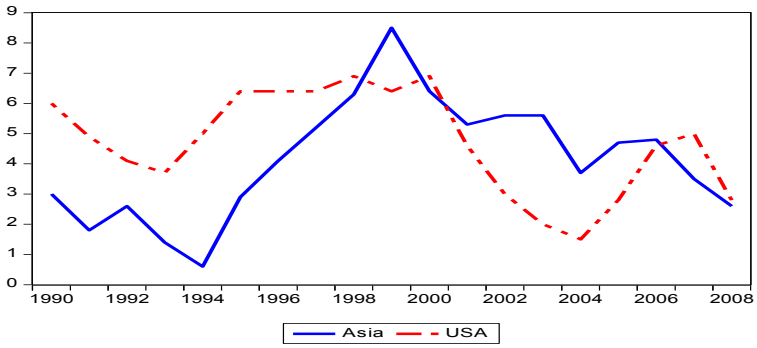

Fig. 2 Real short-term interest rates in USA and Asia 1990-2008. Source: World Bank: World Development Indicators

Figure 2 shows a similar convergence of higher US short term real interest rates in the late 1990s towards the lower Asian rates, following the 1990s East-Asian currency crises (= at the time of euro inception).

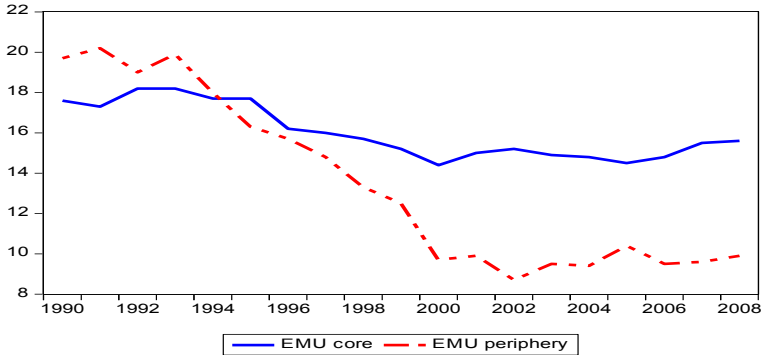

Fig. 3 Personal saving rates in EMU core and periphery 1995-2008. Source: AMECO.

[1] Nowadays Finland is included within core countries. In line with Fagan and Gaspar (2008) we exclude Finland from core countries since in the 1990s the Finnish economy was distorted by special factors after the collapse of the Soviet Union.

[2] Remaining differences in the real interest rates are due to inflation rate differences across EMU core and periphery.

Regarding differences in economic fundamentals, Figure 3 portrays the existence of a substantially lower personal saving ratio (= gross household savings as percent of gross disposable income) in the EMU periphery than in the core. Similarly, Figure 4 reveals that the US personal saving rate is persistently, substantially lower than the Asian rate, both in the 1990s and 2000s.

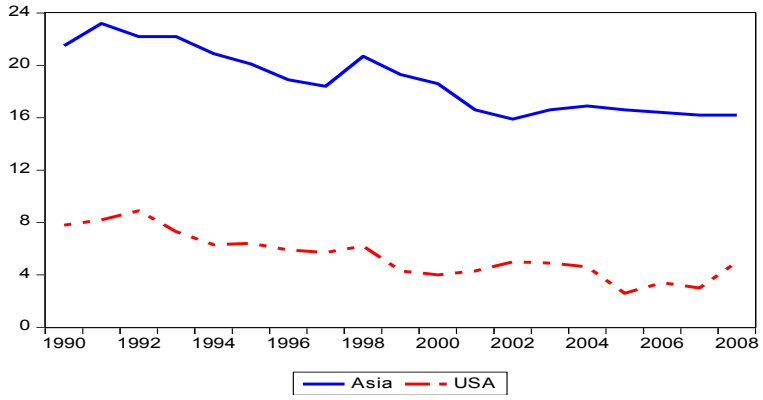

Fig. 4 Asian and US personal saving rates 1990-2008. Source: Source: China Statistical Yearbook Database, CEIC, AMECO, Economic Statistics System, Bank of Korea, FRED.

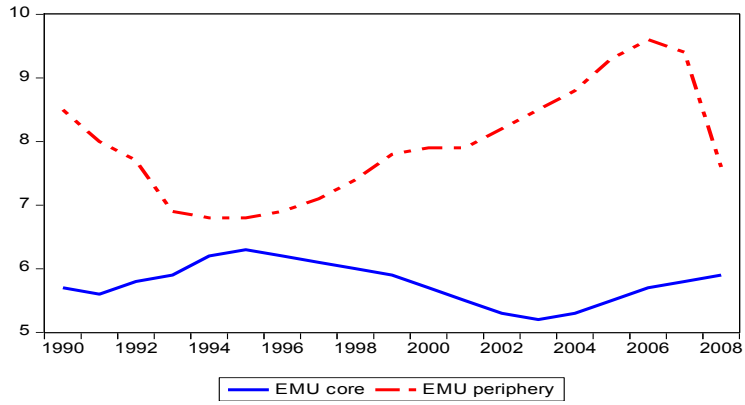

Fig. 5 Housing investment (as percent of GDP) in EMU periphery and core 1990-2008. Source: AMECO

Figures 5 and 6 portray housing investment (as percent of GDP) in EMU periphery and core, and in Asia and the USA, respectively. While housing investment rose significantly in the EMU periphery and in the USA, it declined in the EMU core and in Asia.

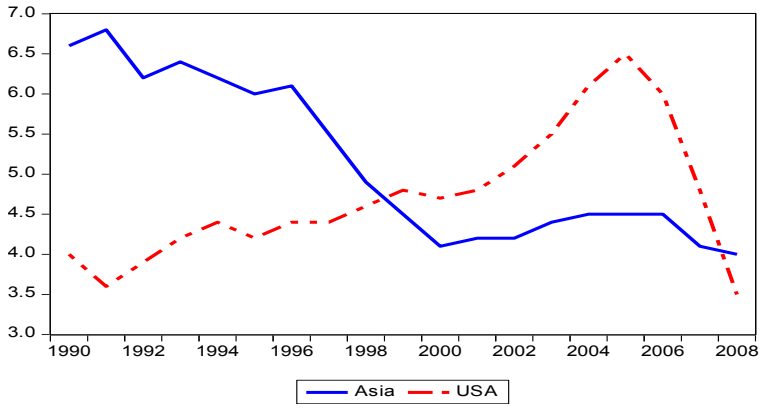

Fig. 6 Housing investment (as percent of GDP) in Asia and in USA 1990-2008. Source: BEA National Economic Accounts

Starting from a significantly lower personal saving ratio in EMU periphery relative to the core, housing investment expenditures in the periphery experienced a boom, while housing investment

in the core countries declined. Through the sharp increase in private domestic expenditures in the periphery, and the muted response of output (Fagan and Gaspar 2008), external balances in the periphery significantly deteriorated (see Fig. 7).

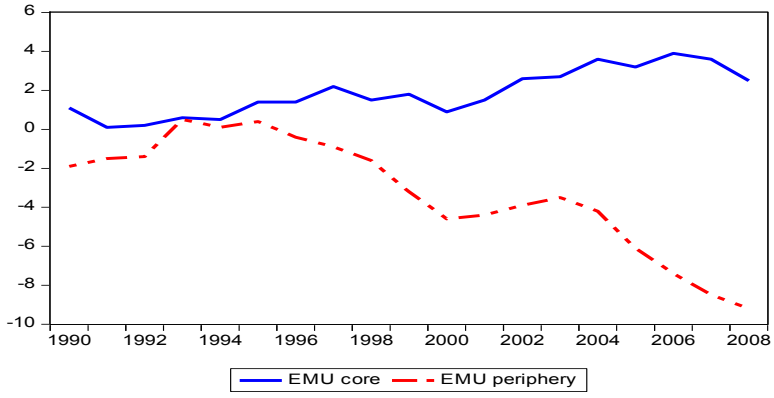

Fig. 7 Current account balances (as percent of GDP) in EMU periphery and core 1990-2008. Source: updated and extended version of a dataset constructed by Lane and Milesi-Ferretti (2007)

Not surprisingly, EMU periphery's current account deficits led to the accumulation of a significant net foreign debtor position. See Figure 8 below.

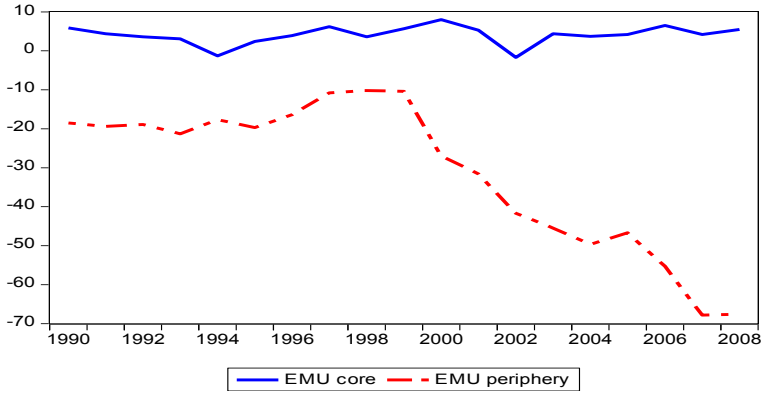

Fig. 8 Net foreign assets (as percent of GDP) in EMU core and periphery 1990-2008. Source: updated and extended version of a dataset constructed by Lane and Milesi-Ferretti (2007)

The differences with respect to the evolution of current account and net foreign asset positions in Asia and the USA are again similar to those in Europe. See figures 9 and 10.

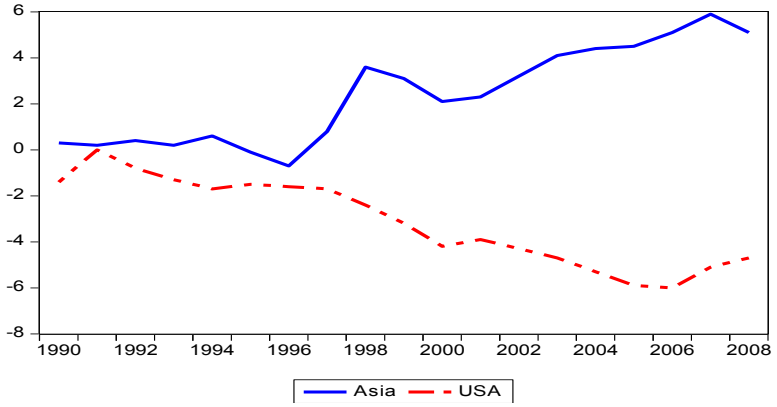

Fig. 9 Current account to GDP ratios of Asia and USA 1990-2008. Source: updated and extended version of a dataset constructed by Lane and Milesi-Ferretti (2007)

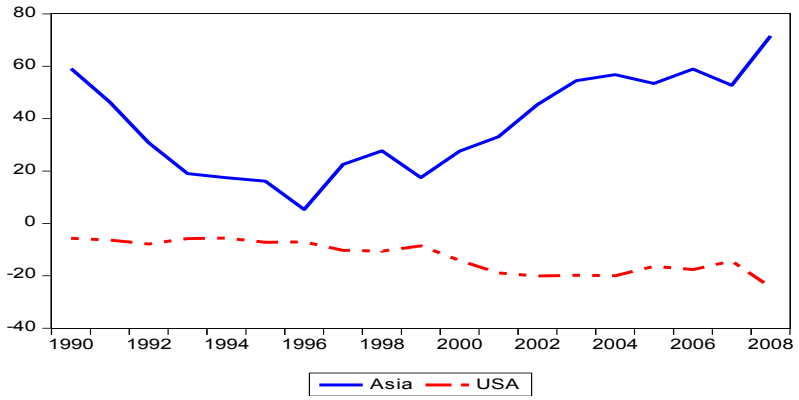

Fig. 10 Net foreign asset positions (in percent to GDP) of Asia and USA 1990-2008. Source: updated and extended version of a dataset constructed by Lane and Milesi-Ferretti (2007)

Basic Model

Consider an infinite-horizon model economy consisting of three areas ("countries") of the world economy, namely (i) the EMU, comprising two regions, named North (indexed by *N*) representing EMU's core, and South (indexed by *S*), representing EMU's periphery countries, (ii) the countries characterized by a current account surplus outside the EMU (indexed by *A*) representing Asia and oil-exporting countries, and (iii) the current-account deficit countries (indexed by *U*) representing mainly the USA. In each country one commodity, representing the aggregate of thousands of goods and services is produced. This can be used for the purpose of consumption as well as for investment. The EMU specializes completely in the production of good X, "Asia" in the production of good Y, and "USA" in the production of good Z. Perfectly competitive firms in EMU's South and North, in Asia and in the USA employ in every period $t = 1,2,...$ labor services $N_t^i,\ i = S,N,A,U$ and capital services $K_t^i,\ i = S,N,A,U$, using the Cobb-Douglas (CD) production function $M^i(a_t N_t^i)^{1-\alpha^i}(K_t^i)^{\alpha^i},\ i = S,N,A,U$, to produce southern (northern) EMU aggregate output $X_t^S(X_t^N)$, Asia's aggregate output Y_t and US aggregate output Z_t where $M^i > 0,\ i = S,N,A,U$ denote total factor productivity in EMU's South (North), in Asia and in the USA, respectively. a_t is the common labor productivity and $0 < \alpha^i < 1,\ i = S,N,A,U$ with $\alpha^U \approx \alpha^N < \alpha^S < \alpha^A$ are the capital production shares in EMU South, EMU North, Asia and in the USA.

One-period profit maximization by firms in EMU's South (North), in Asia and in the USA implies the following FOCs:

$$w_t^i = (1-\alpha^i)M^i a_t (K_t^i / a_t N_t^i)^{\alpha^i}, i = S,N,A,U, \tag{1}$$

$$q_t^i = \alpha^i M^i (K_t^i / a_t N_t^i)^{\alpha^i - 1},\ i = S,N,A,U, \tag{2}$$

whereby $w_t^i,\ i = S,N,A,U$ denotes the real wage rate in each region and each country. $q_t^i,\ i = S,N,A,U$ denotes real unit capital user costs in each region and each country $i = S,N,A,U$.

As usual in a Diamond (1965) type OLG framework, two generations of homogeneous individuals overlap in each period t. At date t, a new generation of size L_t^i enters the economy of country (region) $i = S, N, A, U$. For simplicity we assume that the population growth factors of all countries (regions) are identical and equal to G^L. In view of the empirically rather similar GDP growth rates in southern and northern EMU countries (Fagan and Gaspar 2008) we assume that the respective growth factors of labor productivities G^{a^S} and G^{a^N} are equal in EMU's South and North, an assumption which also applies rather well to the USA, but rather less so to current-account surplus countries like China, India and other Asian countries. However, taking account of the catch-up growth component in emerging countries' GDP growth rates the simplifying assumption $G^{a^S} = G^{a^N} = G^{a^U} = G^{a^A} = G^a$ seems acceptable. This implies that the natural growth factor $G^n = G^a G^L$ is the same in all countries.

Each generation lives for two periods, working during the first when young, and retiring in the second when old. The choice variables of each generation, when young, are denoted by superscript 1, and, when old, they are denoted by superscript 2. For each member of the generation entering the economy in period t, the supply of labor to firms is wage-inelastic since households attribute no value to leisure.

In order to describe the optimization problems of households more specifically the institutional framework regarding international transactions across the three countries and across EMU core and periphery is now addressed. Regarding the three countries, we assume that each country has its own currency and that before the inception of the EMU, the southern and northern EMU member countries had their own currency, too. To mimic the period before the introduction of the common currency in 1999 we follow Gourinchas and Jeanne (2006) as well as Fagan and Gaspar (2008), and assume that before 1999, EMU's South and North were financially autarkic. In contrast to the de-facto financial relationships between subsequent EMU countries, Asia and the USA which existed before euro inception, we also assume financial autarky for Asia and the USA in the pre-euro period. In contrast to financial autarky, we do, however, allow for trade relations between later EMU, Asia and US during the pre-euro period, albeit on a moderate and balanced scale, thus mimicking the fact of mainly Japanese trade linkages vis-à-vis later EMU countries and the US. China and India did not play any important role in international trade during the pre-euro period.

Complete nominal, and to a lesser extent, real interest convergence across EMU's South and North after the euro launch signifies financial integration across EMU's South and North. This stylized fact is portrayed in our intertemporal equilibrium model in line with Fagan and Gaspar (2008) as an equality of real interest rates of southern and northern EMU countries along the intertemporal equilibrium path. While by no means as complete as that within EMU, there is also – as Fig. 2 above demonstrates –some real interest convergence or financial integration across

Asia and USA in the early 2000s. We take this stylized fact as support for our rather strong modeling assumption that after the euro launch an uncovered parity condition, in terms of real interest rates, holds across both Asia and the USA. In line with the empirical fact (Chen et al. 2013) that internal investors from outside EMU invested their wealth in northern EMU financial assets we also assume an uncovered real interest parity condition between USA and EMU. In other words: after euro inception financial integration prevails worldwide but not as strictly as within the EMU.

In order to work out the consequences of intra-EMU, Asian and US financial integration and the trade developments of EMU vis-à-vis non-EMU countries as clearly as possible, the optimization problems of (younger) households and firms as well as the market clearing conditions are now described separately for the cases of financial autarky and intra-EMU, Asian and US financial integration.

Pre-Euro and Asian-US Financial Autarky

In order to facilitate the modeling of the pre-euro situation as financial autarky, we first recall that southern EMU real interest rates were sizeably larger than the corresponding northern rates. Second, in the 1990s EMU South (with the exception of Portugal) did not run large current account deficits (as ratio of GDP). Hence, our modeling assumes that in the pre-euro period both the current account and the net foreign asset position of EMU South and North were zero. In contrast, in the 1990s Asia (including oil exporters) ran current account surpluses (as percent of GDP) roughly equivalent in size to the current account deficit of the USA (Engler 2009, p. 2). However, since at this time the US net foreign asset position was only moderately negative and China and other emerging Asian countries did not contribute much to the imbalance, we assume that the USA and Asia were financially autarkic, just as the later EMU South and North were. Third, in contrast to the tremendous post-crisis accumulation of public debt, particularly in southern EMU, in Japan and in the USA, in the 1990s and 2000s (up to 2008), the average debt to GDP ratios for EMU periphery, EMU core and the USA remained constant over time or even receded slightly. In Asia (with the exception of Japan) public debt to GDP ratios decreased, and remained far below the EMU and US ratios. We also assume that the (un-weighted) average of government debt to GDP ratios in Asia (including Japan) remains constant over time. Additionally, as Figure 3 above shows, after 1993 the personal saving rate in EMU South was lower than in EMU North. From Figure 4 we know that Asia's personal saving rate is significantly higher than the corresponding northern EMU rate, while the US personal saving rate is slightly below the southern EMU personal saving rate (see Figures 3 and 4).

In line with Chen et al. (2013) we accept the stylized fact of merchandise trade between EMU's North and South being relatively modest after euro launch, and we assume for the sake of simplicity that the same composite commodity is produced in North and South. Thus, while

younger households in EMU South (North) cannot choose between consumption of the domestic and of the northern (southern) commodity they can buy Asian and US goods in addition to the domestically produced good even before the euro launch.

Against this empirical background of stylized facts and simplifying modeling assumptions the intertemporal utility maximization problem in later EMU's South before euro inception (= financial autarky) reads as follows:

$$max \rightarrow \zeta^x \ln x_t^{S,1} + \zeta^y \ln y_t^{S,1} + \zeta^z \ln z_t^{S,1} + \beta^S (\zeta^x \ln x_{t+1}^{S,2} + \zeta^y \ln y_{t+1}^{S,2} + \zeta^z \ln z_{t+1}^{S,2})$$

s. t.:

$$(i)\ \ x_t^{S,1} + (1/e_t^A)\, y_t^{S,1} + (1/e_t^U)\, z_t^{S,1} + s_t^S = w_t^S (1-\tau_t^S),\ \text{with}\ s_t^S \equiv K_{t+1}^{S,S}/L_t + B_{t+1}^{S,S}/L_t,$$

$$(ii)\ \ x_{t+1}^{S,2} + (1/e_{t+1}^A) y_{t+1}^{S,2} + (1/e_{t+1}^U) z_{t+1}^{S,2} = q_{t+1}^S (K_{t+1}^{S,S}/L_t) + \left(1 + i_{t+1}^S\right)(B_{t+1}^{S,S}/L_t).$$

Here $0 < \beta^S \leq 1$ denotes the time discount factor of (later) EMU's southern younger generation, $\zeta^k, k = x, y, z$ with $\zeta^x + \zeta^y + \zeta^z = 1$ represents the utility elasticity of the consumption of good k, $x_t^{S,1}$ is the consumption per capita of the commodity produced in EMU's South acquired at unit relative price, $y_t^{S,1}$ is South's consumption of the Asian good bought at the relative price of $1/e_t^A$, and $z_t^{S,1}$ is southern consumption of the US good acquired at the relative price of $1/e_t^U$. e_t^A denotes the units of the Asian good per unit of EMU good (= EMU terms of trade vis-à-vis Asia), while e_t^U portrays the units of the US good per unit of EMU good (=EMU terms of trade vis-à-vis USA). s_t^S is South's per capita savings, τ_t^S denotes region South's flat wage tax rate, $x_{t+1}^{S,2}$ ($y_{t+1}^{S,2}, z_{t+1}^{S,2}$) is old-age consumption per capita of the commodity produced in South (Asia, USA), $K_{t+1}^{S,S}/L_t^S$ is real capital produced in EMU South which South's younger household wants to hold at the beginning of the retirement period, $B_{t+1}^{S,S}/L_t^S$ stands for EMU South government bonds which South's younger household wants to hold at the beginning of its retirement period and i_{t+1}^S denotes the real interest rate on southern EMU government bonds. In line with pre-crisis empirical reality, the southern EMU young household invests its savings only in domestic real capital and government bonds. Constraint (i) depicts the working period budget constraint while constraint (ii) represents the retirement period budget constraint.

Analogously, the intertemporal utility maximization problem of the typical northern EMU household reads as follows:

$$\text{Max} \rightarrow \zeta^x \ln x_t^{N,1} + \zeta^y \ln y_t^{N,1} + \zeta^z \ln z_t^{N,1} + \beta^N (\zeta^x \ln x_{t+1}^{N,2} + \zeta^y \ln y_{t+1}^{N,2} + \zeta^z \ln z_{t+1}^{N,2})$$

s.t.:

$$(i)\ x_t^{N,1} + (y_t^{N,1}/e_t^A) + (z_t^{N,1}/e_t^U) + s_t^N = w_t^N (1-\tau_t^N),\ s_t^N \equiv (K_{t+1}^{N,N}/L_t^N) + (B_{t+1}^{N,N}/L_t^N,$$

$$(ii)\, x_{t+1}^{N,2} + (y_{t+1}^{N,2}/e_{t+1}^A) + (z_{t+1}^{N,2}/e_{t+1}^U) = q_{t+1}^N (K_{t+1}^{N,N}/L_t^N) + \left(1 + i_{t+1}^N\right)(B_{t+1}^{N,N}/L_t^N).$$

Here, $x_t^{N,1}$ ($y_t^{N,1}, z_t^{N,1}$) stands for the purchases of later EMU (Asian, US) goods by EMU North young household, with $s_t^N, \tau_t^N, K_{t+1}^{N,N}, B_{t+1}^{N,N}$ being interpreted in a similar fashion to that stated for the corresponding variables in EMU South.

The typical Asian young household solves the following optimization problem:

$$\text{Max} \rightarrow \zeta^x \ln x_t^{A,1} + \zeta^y \ln y_t^{A,1} + \zeta^z \ln z_t^{A,1} + \beta^A(\zeta^x \ln x_{t+1}^{A,2} + \zeta^y \ln y_{t+1}^{A,2} + \zeta^z \ln z_{t+1}^{A,2})$$

s.t. :

$$(i)\, e_t^A\, x_t^{A,1} + y_t^{A,1} + (e_t^A / e_t^U) z_t^{A,1} + s_t^A = w_t^A(1-\tau_t^A),\ s_t^A \equiv (K_{t+1}^{A,A} / L_t^A) + (B_{t+1}^{A,A} / L_t^A),$$

$$(ii)\, e_{t+1}^A x_{t+1}^{A,2} + y_{t+2}^{A,2} + (e_{t+1}^A / e_{t+1}^U) z_{t+1}^{A,2} = q_{t+1}^A (K_{t+1}^{A,A} / L_t^A) + \left(1 + i_{t+1}^A\right)(B_{t+1}^{A,A} / L_t^A).$$

Here, $x_t^{A,1}$ stands for the purchases (= consumption) of later EMU goods by the Asian young household at the relative price of e_t^A, while the purchase of the US product by the Asian young household occurs at the relative price e_t^A / e_t^U, i.e. units of the Asian product per unit of the US good. All other variables may be interpreted similarly to those in EMU South's young household optimization problem.

Finally, the typical US young household faces the following optimization problem:

$$\text{Max} \rightarrow \zeta^x \ln x_t^{U,1} + \zeta^y \ln y_t^{U,1} + \zeta^z \ln z_t^{U,1} + \beta^U(\zeta^x \ln x_{t+1}^{U,2} + \zeta^y \ln y_{t+1}^{U,2} + \zeta^z \ln z_{t+1}^{U,2})$$

s.t. :

$$(i)\, e_t^U\, x_t^{U,1} + (e_t^U / e_t^A) y_t^{U,1} + z_t^{U,1} + s_t^U = w_t^U(1-\tau_t^U),\ s_t^U \equiv (K_{t+1}^{U,U} / L_t^U) + (B_{t+1}^{U,U} / L_t^U),$$

$$(ii)\, e_{t+1}^U\, x_{t+1}^{U,2} + (e_{t+1}^U / e_{t+1}^A) y_{t+1}^{U,2} + z_{t+1}^{U,2} = q_{t+1}^U (K_{t+1}^{U,U} / L_t^U) + (1 + i_{t+1}^U)(B_{t+1}^{U,U} / L_t^U).$$

Here $x_t^{U,1}$ stands for US young household's purchases of the EMU product while e_t^U / e_t^A now indicates the units of the US product per unit of the Asian product which equals the relative price of US consumption for the Asian good, $y_t^{U,1}$.

The government of each country (region) $i = S, N, A, U$ taxes labor income and uses the revenues from additional borrowing to finance the interest costs on existing government debt and government expenditures. The government budget constraint of country (region) i reads as follows:

$$B_{t+1}^i - B_t^i + \tau_t^i w_t^i L_t = i_t^i B_t^i + \Gamma_t^i,\ i = S, N, A, U,\ t = 0,1,2,\ldots, \tag{3}$$

where Γ_t^i denotes real government expenditures and B_t^i is the level of real government debt in country (region) $i = S, N, A, U$ at the beginning of period t. In line with Diamond (1965), we assume that government expenditures are unproductive.

In addition to the restrictions imposed by household and firm optimization and by the above government budget constraints, markets for labor have to clear in all countries (regions) and in all periods.

$$N_t^i = L_t^i,\ i = S, N, A, U,\ t = 0,1,2,\ldots \tag{4}$$

Since the asset markets are competitive, transaction and adjustment costs do not occur, no risk (aversion) prevails, the following no-arbitrage condition (= national Fisher equation) holds in all countries (regions):

$$1 + i_{t+1}^i = q_{t+1}^i + 1 - \delta,\ i = S, N, A, U,\ t = 0,1,2,\ldots, \tag{5}$$

whereby $0 < \delta \leq 1$ depicts the common fixed depreciation rate of private capital (period by period) in country (region) *i*.

The asset market clearing conditions in all countries (regions) read as follows:

$$L_t^i s_t^i = K_{t+1}^i + B_{t+1}^i,\ i = S,N,A,U,\ t = 0,1,2,\ldots, \tag{6}$$

$$B_t^i = B_t^{i,i}, K_{t+1}^i = K_{t+1}^{i,i},\ i = S,N,A,U,\ t = 0,1,2,\ldots \tag{7}$$

Finally, the product markets in EMU, Asia and US clear for all $t = 0,1,2,\ldots$ according to the following conditions:

$$\begin{aligned} X_t^S + X_t^N = {} & L_t x_t^{S,1} + L_{t-1} x_t^{S,2} + \Gamma_t^S + K_{t+1}^S + L_t x_t^{N,1} + L_{t-1} x_t^{N,2} + \Gamma_t^N + K_{t+1}^N + L_t^A x_t^{A,1} + L_{t-1}^A x_t^{A,2} \\ & + L_t^U x_t^{U,1} + L_{t-1}^U x_t^{U,2}, \end{aligned} \tag{8}$$

$$Y_t = L_t^A y_t^{A,1} + L_{t-1}^A y_t^{A,2} + \Gamma_t^A + K_{t+1}^A + L_t y_t^{S,1} + L_{t-1} y_t^{S,2} + L_t y_t^{N,1} + L_{t-1} y_t^{N,2} + L_t^U y_t^{U,1} + L_{t-1}^U y_t^{U,2}, \tag{9}$$

$$Z_t = L_t^U z_t^{U,1} + L_{t-1}^U z_t^{U,2} + \Gamma_t^U + K_{t+1}^U + L_t z_t^{S,1} + L_{t-1} z_t^{S,2} + L_t z_t^{N,1} + L_{t-1} z_t^{N,2} + L_t^A z_t^{A,1} + L_{t-1}^A z_t^{A,2}. \tag{10}$$

In order to be able to model the fact of time-stationarity of country (region) *i's* public debt to GDP ratios between 1999 and 2008 we transform total outstanding government debt in country (region) *i's* government budget constraint into debt to GDP ratios. This is achieved by dividing both sides of (3) by X_t for $i = S,N$, by Y_t for $i = A$, by Z_t for $i = U$ and by defining the debt to GDP ratios as $b_t^i = B_t^i / X_t^i,\ i = S,N$, $b_t^A = B_t^A / Y_t$, $b_t^U = B_t^U / Z_t$ we obtain for country i:

$$G_t^{X,i} b_{t+1}^i = (1 + i_t^i) b_t^i + \gamma_t^i - \tau_t^i (1 - \alpha^i),\ G_t^{X,i} \equiv X_{t+1}^i / X_t^i,\ \gamma_t^i \equiv \Gamma_t^i / X_t^i,\ w_t^i L_t / X_t^i = 1 - \alpha^i, i = S,N, \tag{11}$$

$$G_t^Y b_{t+1}^A = (1 + i_t^A) b_t^A + \gamma_t^A - \tau_t^A (1 - \alpha^A),\ G_t^Y \equiv Y_{t+1} / Y_t,\ \gamma_t^A \equiv \Gamma_t^A / Y_t,\ w_t^A L_t^A / Y_t = 1 - \alpha^A, \tag{12}$$

$$G_t^Z b_{t+1}^U = (1 + i_t^U) b_t^U + \gamma_t^U - \tau_t^U (1 - \alpha^U),\ G_t^Z \equiv Z_{t+1} / Z_t,\ \gamma_t^U \equiv \Gamma_t^U / Z_t,\ w_t^U L_t^U / Z_t = 1 - \alpha^U. \tag{13}$$

Dividing the asset market clearing condition (6) on both sides by $X_t^i, i = S,N$, and using the definition of the capital output ratio $v_t^i \equiv K_t^i / X_t^i,\ i = S,N$, (6) can be rewritten as follows:

$$G_t^{X,i} b_{t+1}^i + G_t^{X,i} v_{t+1}^i = L_t s_t^i / X_t^i = \sigma^i (1 - \alpha^i)(1 - \tau_t^i),\ \sigma^i \equiv \beta^i / (1 + \beta^i),\ i = S,N. \tag{14}$$

In view of the C-D production function, and noting $G_t^{X,i} = (K_{t+1}^i)^{\alpha^i} (a_{t+1} L_{t+1})^{1-\alpha^i} / (K_t^i)^{\alpha^i} (a_t L_t)^{1-\alpha^i}$ $= (a_{t+1} L_{t+1}) / (a_t L_t) (K_{t+1}^i / a_{t+1} L_{t+1})^{\alpha^i} / (K_t^i / a_t L_t)^{\alpha^i}, i = S,N$, it turns out that $G_t^{X,i} = G^n \left(v_{t+1}^i / v_t^i \right)^{\alpha^i / (1-\alpha^i)}$.

Acknowledging the empirical fact that the pre-crisis public debt to GDP ratios in all countries remained roughly constant over time we assume time-stationary public debt to GDP ratios

$$B_t^i / X_t^i = B_{t+1}^i / X_{t+1}^i = b^i, b^i > 0,\ i = S,N, B_t^A / Y_t = B_{t+1}^A / Y_{t+1} = b^A, b^A > 0, B_t^U / Z_t = B_{t+1}^U / Z_{t+1} = b^U, b^U > 0. \tag{15}$$

Moreover, we assume time-stationary government expenditure shares:

$$\gamma_t^i = \gamma_{t+1}^i = \gamma^i, \forall t,\ 0 < \gamma^i < 1,\ i = S,N,A,U. \tag{16}$$

The government budget constraints (11-13) together with (15) and (16) yield $1 - \tau_t^i$ as follows:

$$\begin{aligned} & 1 - \tau_t^i = \frac{1 - \alpha^i - \gamma^i}{1 - \alpha^i} + \frac{b^i}{1 - \alpha^i} \left[G_t^{X,i} - \left(1 + i_t^i \right) \right],\ i = S,N, 1 - \tau_t^A = \frac{1 - \alpha^A - \gamma^A}{1 - \alpha^A} + \frac{b^A}{1 - \alpha^A} \left[G_t^Y - \left(1 + i_t^A \right) \right], \\ & 1 - \tau_t^U = \frac{1 - \alpha^U - \gamma^U}{1 - \alpha^U} + \frac{b^U}{1 - \alpha^U} \left[G_t^Z - \left(1 + i_t^U \right) \right]. \end{aligned} \tag{17}$$

Using the Cobb-Douglas production function it is easily seen that $K_t^i / X_t^i \equiv v_t^i = (1/M^i)[K_t^i/(a_t N_t^i)]^{1-\alpha^i}, i = S,N, K_t^A / Y_t^A \equiv v_t^A = (1/M^A)[K_t^A/(a_t N_t^A)]^{1-\alpha^A}$, $K_t^U / Y_t^U \equiv v_t^U = (1/M^U)[K_t^U/(a_t N_t^U)]^{1-\alpha^U}$.

Thus, the FOC for profit maximizing capital service input (2) can be equivalently written as follows:

$$\alpha^i / v_t^i = q_t^i = i_t^i + \delta,\ i = S,N,A,U\,. \tag{18}$$

In order to simplify the algebra, we assume $\delta = 1$. Then, acknowledging (18) in (17) and considering $G_t^{X,i} = G^n\left(v_{t+1}^i / v_t^i\right)^{\frac{\alpha^i}{1-\alpha^i}}$, $i = S,N$ ($G_t^Y = G^n\left(v_{t+1}^A / v_t^A\right)^{\frac{\alpha^A}{1-\alpha^A}}$, $G_t^Z = G^n\left(v_{t+1}^U / v_t^U\right)^{\frac{\alpha^U}{1-\alpha^U}}$) yields:

$$(1-\tau_t^i)(1-\alpha^i) = 1-\alpha^i-\gamma + b^i G^n (v_{t+1}^i / v_t^i)^{\alpha^i/(1-\alpha^i)} - \alpha^i b^i / v_t^i,\ i = S,N,A,U. \tag{19}$$

The intertemporal equilibrium dynamics of the capital-output ratio in all countries (regions) is obtained by inserting (19) into (14):

$$(v_{t+1}^i)^{1/(1-\alpha^i)} + b^i(1-\sigma^i)(v_{t+1}^i)^{\alpha^i/(1-\alpha^i)} = (\sigma^i/G^n)[1-\alpha^i-\gamma^i-(\alpha^i b^i)/v_t^i](v_t^i)^{\alpha^i/(1-\alpha^i)},\ i = S,N,A,U\,. \tag{20}$$

As usual, a steady-state intertemporal equilibrium is defined as a fixed point of the difference equation in (20): $v_{t+1}^i = v_t^i = v^i,\ i = S,N,A,U$. Proposition 1 provides the existence conditions.

Proposition 1 (Existence of steady state solutions in country (region) i)

Suppose that $0 < b^i \le \bar{b}^i < \beta^i\left(1-\alpha^i-\gamma^i\right)/G^n$, $i = S,N,A,U$ *while* $\bar{b}^i$ *solves* $\beta^i\left(1-\alpha^i-\gamma^i\right) - G^n\bar{b}^i = 2\sqrt{\alpha^i\beta^i\left(1+\beta^i\right)G^n\bar{b}^i}$. *Then, there are exactly two strictly positive steady state solutions*:

$$_1v_2^i = (2G^n)^{-1}\{(1-\alpha^i-\gamma^i)\sigma^i - (1-\sigma^i)b^iG^n \mp \sqrt{\quad{}^i-\gamma^i)\sigma^i-(1-\sigma^i)b^iG^n]^2 - 4\alpha^ib^iG^n\sigma^i}\}. \tag{21}$$

Proof. See Farmer (2013, p. 11)■

Since there are two steady-state solutions (local) dynamic stability also needs to be investigated. This is done in proposition 2.

Proposition 2 (Dynamic stability of steady state solutions in country (region) i)

Suppose that $0 < b^i < \bar{b}^i,\ i = S,N,A,U$. *Then, the steady-state solution* v_1^i *in* (21) *is asymptotically unstable while the steady-state solution* v_2^i *in* (21) *is asymptotically stable.*

Proof. See Farmer (2013, p. 11). ■

Knowing that the larger steady state solution in (21) is asymptotically stable we use it to attribute the empirically observed pre-euro North-South, Asian and US differences with respect to the real interest rates to EMU North-South, Asian and US differences regarding fundamentals, including private saving rates, government expenditure ratios, capital production shares and government debt to GDP ratios. To this end, we first try to find out how the fundamental parameters impact the steady-state value of the capital-output ratio in (21). Second, we need

information about the magnitudes of the saving rates, capital production shares, government expenditure quotas and public debt to GDP ratios in pre-EMU South and North, in Asia and in USA. Comparing pairwise the right hand sides of (21) we are led to the following proposition.

Proposition 3 (Properties of financial autarky parameter sets)

Suppose that $b^i < \bar{b}^i$, $i = S, N, A, U$ *and for simplicity* $b^S = b^N = b^A = b^U$. *If* $\alpha^S > \alpha^N$, $\gamma^S \geq \gamma^N$ *and* $\sigma^S < \sigma^N$, *then* $v_2^S < v_2^N$ *implies* $i^S > i^N$. *Moreover, if* $\alpha^A > \alpha^U$, $\gamma^A \geq \gamma^U$ *and* $\sigma^U < \sigma^A$, *then* $v_2^U < v_2^A$ *implies* $i^U > i^A$.

Proof. See Farmer (2013, p. 12) ■

The second step is to ensure that the assumptions behind proposition 3 are empirically warranted with respect to conditions prevailing in the three countries and two regions in the late 1990s. Both $\alpha^S > \alpha^N$ and $\alpha^A > \alpha^U$ are empirically warranted since capital production shares are larger in less developed countries and both the southern EMU countries and Asian countries are less developed (lower GDP per capita) than the northern EMU countries and the USA. China, itself among the Asian countries, is a prominent empirical example for the fact that the capital production share is higher in emerging than in more advanced countries (see Bai and Quian (2010) on the high Chinese capital production share of nearly 50%, and Caselli and Feyrer (2007) on the much lower US capital production share of 30%).[3] In view of the empirical evidence provided by Figures 3 and 4 above it is natural to assume that $\sigma^S < \sigma^N$ and $\sigma^U < \sigma^A$, i.e. that the saving rate of southern EMU is less than that of northern EMU and that the US saving rate is smaller than the Asian saving rate.

Proposition 3 says that the relatively high capital production share and the low saving rate in EMU South imply under financial autarky that the steady-state capital output ratio in EMU South is lower than in EMU North, and is associated with a higher real interest in EMU South than in EMU North. Analogously, the significantly lower US saving rate compared to the Asian saving rate implies, in spite of a higher Asian capital production share, a lower capital output ratio and a higher real interest rate in autarky. These claims are intuitively plausible. A low saving rate implies for a given capital output ratio low savings thus driving the capital output ratio down to ensure asset market clearing. The capital output ratio is also depressed by a relatively high capital income share since this implies a relatively low labor income share associated with low per capita savings. Due to decreasing marginal productivity of capital the lower capital output ratio is associated with a higher interest rate.

[3] The opposite holds with respect to the government expenditure quota and public debt to GDP ratios: less developed countries exhibit lower expenditure quotas and debt to GDP ratios than highly developed countries. While for this case proposition 3 is not true in general, it can be shown numerically that the implication in proposition 3 remains true even for smaller government expenditures and debt to GDP ratios in less developed countries.

Not surprisingly, under financial autarky the ratio of the net foreign asset position to GDP in country $i = S, N, A, U$ defined as $\phi^i = \sigma^i[1-\alpha^i-\gamma^i-(\alpha^i b^i)/v^i] - G^n[v^i + b^i(1-\sigma^i)]$ is zero. Using the budget constraints of young and old households plus the zero-profit and market clearing conditions one can show that in steady state the current account to GDP ratio denoted as ca^i, $i = S, N, A, U$ and the trade balance to GDP ratio denoted as $tb^i, i = S, N, A, U$ are related to the respective net foreign asset to GDP ratio as follows: $ca^i = [(G^n - 1)/G^n]\phi^i$ and $tb^i = \{[G^n - (1+i)]/G^n\}\phi^i$. Clearly, both the current account and the trade balance to GDP ratio are zero in all countries (regions). Note, however, that balanced trade between EMU, Asia and USA does not mean no trade at all. On the other hand, zero net foreign asset positions mean that no international borrowing and lending takes place in spite of the interest rate differential across countries. Obviously, the costs associated with shifting capital from the low-yielding EMU North and Asia, to the profitable EMU South and USA, are prohibitively large. When modeling the advent of the common currency in Europe and the financial integration between Asia and USA we assume that these international capital mobility costs, i.e. the costs associated with exchange rate fluctuations in pre-euro countries, are completely removed over night, while the structural parameters of all economies remain as assumed in proposition 3.

International Equilibrium under intra-EMU and Asian-US Financial Integration

To mimic the financial integration arising through the set-up of the EMU and the Asian-US financial integration we assume in line with stylized facts (Chen et al. 2013) that northern EMU invests its savings in southern physical capital[4] and government bonds, that Asia buys US government bonds, and that the USA purchases northern EMU government bonds without incurring any transaction costs. However, also in line with empirical data, we assume that the southern EMU young household buys neither northern real capital, nor northern government bonds nor Asian or US assets.

Thus, the intertemporal optimization problem of the southern young household under financial integration is the same as under financial autarky. Analogously, the intertemporal utility maximization problem of the typical northern EMU household under financial integration is essentially similar to that under financial autarky with the exception of the investment of northern savings per capita and the old-age budget constraint which read now as follows:

$$\begin{aligned} & s_t^N \equiv \frac{K_{t+1}^{N,N}}{L_t} + \frac{K_{t+1}^{S,N}}{L_t} + \frac{B_{t+1}^{N,N}}{L_t} + \frac{B_{t+1}^{S,N}}{L_t}, \\ & (ii)\, x_{t+1}^{N,2} + \frac{y_{t+1}^{N,2}}{e_{t+1}^{A}} + \frac{z_{t+1}^{N,2}}{e_{t+1}^{U}} = q_{t+1}^{N}\left(\frac{K_{t+1}^{N,N}}{L_t}\right) + q_{t+1}^{S}\left(\frac{K_{t+1}^{S,N}}{L_t}\right) + \left(1+i_{t+1}^{N}\right)\left(\frac{B_{t+1}^{N,N}}{L_t}\right) + \left(1+i_{t+1}^{S}\right)\left(\frac{B_{t+1}^{S,N}}{L_t}\right). \end{aligned} \tag{22}$$

[4] To mimic the facts presented in Figures 5 and 6 above we assume that physical capital is mainly accumulated by housing investment.

Here, $K_{t+1}^{S,N}/L_t$ and $B_{t+1}^{S,N}/L_t$ denote the respective stocks of southern real capital and government bonds which the northern EMU young household wants to hold at the beginning of period $t+1$. Since physical capital and government bonds in each EMU region are perfectly substitutable, and since within the monetary union both assets can be assumed to be perfectly mobile across South and North, the following international Fisher equation (= real international interest parity condition) holds in addition to the national Fisher equations (5):

$$1+i_{t+1}^{S}=1+i_{t+1}^{N}. \tag{23}$$

The typical Asian young household solves under financial integration essentially the same problem as under financial autarky with the exception of the use of per-capita savings and the old-age budget constraint which read now as follows:

$$\begin{aligned} & s_t^A \equiv \frac{K_{t+1}^{A,A}}{L_t^A}+\frac{B_{t+1}^{A,A}}{L_t^A}+\frac{e_t^A}{e_t^U}\frac{B_{t+1}^{U,A}}{L_t^A}, \\ & (ii)\, e_{t+1}^A x_{t+1}^{A,2}+y_{t+2}^{A,2}+\frac{e_{t+1}^A z_{t+1}^{A,2}}{e_{t+1}^U}=q_{t+1}^A\frac{K_{t+1}^{A,A}}{L_t^A}+\left(1+i_{t+1}^A\right)\left(\frac{B_{t+1}^{A,A}}{L_t^A}\right)+\left(1+i_{t+1}^U\right)\frac{e_{t+1}^A}{e_{t+1}^U}\left(\frac{B_{t+1}^{U,A}}{L_t^A}\right). \end{aligned} \tag{24}$$

Here $B_{t+1}^{U,A}/L_t^A$ denotes the stock of US government bonds which the Asian young household wants to hold at the beginning of period $t+1$. In line with pre-crisis reality the Asian young household does not hold EMU government bonds.

Analogously, savings per capita and the old-age budget constraint of the typical US young household are as follows:

$$\begin{aligned} & s_t^U \equiv \frac{K_{t+1}^{U,U}}{L_t^U}+\frac{B_{t+1}^{U,U}}{L_t^U}+e_t^U\frac{B_{t+1}^{N,U}}{L_t^U}, \\ & (ii)\, e_{t+1}^U x_{t+1}^{U,2}+\frac{e_{t+1}^U y_{t+1}^{U,2}}{e_{t+1}^A}+z_{t+1}^{U,2}=q_{t+1}^U\frac{K_{t+1}^{U,U}}{L_t^U}+\left(1+i_{t+1}^U\right)\left(\frac{B_{t+1}^{U,U}}{L_t^U}\right)+\left(1+i_{t+1}^N\right)e_{t+1}^U\left(\frac{B_{t+1}^{N,U}}{L_t^U}\right). \end{aligned} \tag{25}$$

Again in line with pre-crisis empirical reality, the US young household does hold only northern EMU government bonds.

In order to ensure arbitrage-free terms of trade, the following international real interest parity conditions in addition to (23) ought to hold:

$$1+i_{t+1}^A=\frac{e_{t+1}^A}{e_t^A}(1+i_{t+1}^N),\ \forall t=0,1,2,\ldots, \tag{26}$$

$$1+i_{t+1}^U=\frac{e_{t+1}^U}{e_t^U}(1+i_{t+1}^N),\ \forall t=0,1,2,\ldots. \tag{27}$$

The markets for southern and northern EMU and Asian and US real capital clear according to:

$$K_{t+1}^S=K_{t+1}^{S,S}+K_{t+1}^{S,N},\ K_{t+1}^N=K_{t+1}^{N,N},K_{t+1}^A=K_{t+1}^{A,A},K_{t+1}^U=K_{t+1}^{U,U},\ t=0,1,2\ldots. \tag{28}$$

The markets for southern and northern EMU, Asian and US government bonds clear according to:

$$B_{t+1}^S=B_{t+1}^{S,S}+B_{t+1}^{S,N},\ B_{t+1}^N=B_{t+1}^{N,N}+B_{t+1}^{N,U},B_{t+1}^A=B_{t+1}^{A,A},B_{t+1}^U=B_{t+1}^{U,U}+B_{t+1}^{U,A},\ t=0,1,2\ldots. \tag{29}$$

The international real interest parity conditions (26), (27) ensure that the worldwide amount of savings equals the worldwide supply of assets from southern and northern EMU, Asia and the US:

$$L_t s_t^S + L_t s_t^N + \frac{L_t^A s_t^A}{e_t^A} + \frac{L_t^U s_t^U}{e_t^U} = K_{t+1}^S + K_{t+1}^N + B_{t+1}^S + B_{t+1}^N + \frac{K_{t+1}^A + B_{t+1}^A}{e_t^A} + \frac{K_{t+1}^U + B_{t+1}^U}{e_t^U},\ t = 0,1,2,\ldots . \quad (30)$$

Finally, the product markets in EMU, Asia and US clear for all $t = 0,1,2,\ldots$ according to the following conditions:

$$\begin{aligned} X_t^S + X_t^N = {} & L_t x_t^{S,1} + L_{t-1} x_t^{S,2} + \Gamma_t^S + K_{t+1}^S + L_t x_t^{N,1} + L_{t-1} x_t^{N,2} + \Gamma_t^N + K_{t+1}^N + L_t^A x_t^{A,1} + L_{t-1}^A x_t^{A,2} \\ & + L_t^U x_t^{U,1} + L_{t-1}^U x_t^{U,2}, \end{aligned} \quad (31)$$

$$Y_t = L_t^A y_t^{A,1} + L_{t-1}^A y_t^{A,2} + \Gamma_t^A + K_{t+1}^A + L_t y_t^{S,1} + L_{t-1} y_t^{S,2} + L_t y_t^{N,1} + L_{t-1} y_t^{N,2} + L_t^U y_t^{U,1} + L_{t-1}^U y_t^{U,2}, \quad (32)$$

$$Z_t = L_t^U z_t^{U,1} + L_{t-1}^U z_t^{U,2} + \Gamma_t^U + K_{t+1}^U + L_t z_t^{S,1} + L_{t-1} z_t^{S,2} + L_t z_t^{N,1} + L_{t-1} z_t^{N,2} + L_t^A z_t^{A,1} + L_{t-1}^A z_t^{A,2}. \quad (33)$$

Having described the optimization problems of households and firms as well as the market clearing conditions, the intertemporal equilibrium dynamics can now be derived.

From (18) and from the international Fisher equations (23), (26) and (27) the following relationships between southern EMU, Asian, US and northern EMU capital output ratios and the terms of trade result:

$$v_{t+1}^S = \frac{\alpha^S}{\alpha^N} v_{t+1}^N,\ t = 0,1,2,\ldots , \quad (34)$$

$$e_{t+1}^A = e_t^A \frac{\alpha^A}{\alpha^N} \frac{v_{t+1}^N}{v_{t+1}^A},\ t = 0,1,2,\ldots , \quad (35)$$

$$e_{t+1}^U = e_t^U \frac{\alpha^U}{\alpha^N} \frac{v_{t+1}^N}{v_{t+1}^U},\ t = 0,1,2,\ldots . \quad (36)$$

Dividing (30) on both sides by X_t^N and introducing the definitions of the capital output ratios as well as the debt output ratios, the asset market clearing condition (30) can be rewritten as follows:

$$\begin{aligned} \frac{L_t s_t^S}{X_t^S}\frac{X_t^S}{X_t^N} + \frac{+L_t s_t^N}{X_t^N} + \frac{L_t^A s_t^A}{e_t^A Y_t}\frac{Y_t}{X_t^N} + \frac{L_t^U s_t^U}{e_t^U Z_t}\frac{Z_t}{X_t^N} = {} & (v_{t+1}^S + b_{t+1}^S) G_t^{X^S} \frac{X_t^S}{X_t^N} + (v_{t+1}^N + b_{t+1}^N) G_t^{X^N} \\ & + \frac{v_{t+1}^A + b_{t+1}^A}{e_t^A} G_t^Y \frac{Y_t}{X_t^N} + \frac{v_{t+1}^U + b_{t+1}^U}{e_t^U} G_t^Z \frac{Z_t}{X_t^N},\ t = 0,1,2,\ldots . \end{aligned} \quad (37)$$

Using the production functions, the ratios of southern EMU's, Asia's and US's to northern EMU's GDP turn out to be as follows:

$$\frac{X_t^S}{X_t^N} = \frac{L_t^S}{L_t^N} \frac{(M^S)^{1/(1-\alpha^N)}}{(M^N)^{1/(1-\alpha^N)}} \left(\frac{\alpha^S}{\alpha^N}\right)^{\alpha^S/(1-\alpha^S)} \left(v_t^N\right)^{(\alpha^S-\alpha^N)/(1-\alpha^S)(1-\alpha^N)}, \quad (38)$$

$$\frac{Y_t}{X_t^N} = \frac{L_t^A}{L_t^N} \frac{(M^A)^{1/(1-\alpha^A)}}{(M^N)^{1/(1-\alpha^N)}} (v_t^A)^{\alpha^A/(1-\alpha^A)} (v_t^N)^{\alpha^N/(\alpha^N-1)}, \quad (39)$$

$$\frac{Z_t}{X_t^N}=\frac{L_t^U}{L_t^N}\frac{(M^U)^{1/(1-\alpha^U)}}{(M^N)^{1/(1-\alpha^N)}}(v_t^U)^{\alpha^U/(1-\alpha^U)}(v_t^N)^{\alpha^N/(\alpha^N-1)}. \tag{40}$$

Acknowledging (37)-(40), the definitions of the GDP growth rates, the optimal savings functions resulting from household's utility maximization problems and (19) in (37) yield:

$$\{\sigma^S[1-\alpha^S-\gamma^S-\frac{\alpha^S b^S}{v_t^S}]-G^n\left(\frac{v_{t+1}^S}{v_t^S}\right)^{\frac{\alpha^S}{1-\alpha^S}}[v_{t+1}^S+b^S(1-\sigma^S)]\}\frac{L_t^S}{L_t^N}\frac{(M^S)^{\frac{1}{1-\alpha^S}}}{(M^N)^{\frac{1}{1-\alpha^N}}}\left(\frac{\alpha^S}{\alpha^N}\right)^{\frac{\alpha^S}{1-\alpha^S}}\left(v_t^N\right)^{\frac{(\alpha^S-\alpha^N)}{(1-\alpha^S)(1-\alpha^N)}}$$

$$+\sigma^N[1-\alpha^N-\gamma^N-\frac{\alpha^N b^N}{v_t^N}]-G^n\left(\frac{v_{t+1}^N}{v_t^N}\right)^{\frac{\alpha^N}{1-\alpha^N}}[v_{t+1}^N+b^N(1-\sigma^N)]=-\frac{1}{e_t^A}\{\sigma^A[1-\alpha^A-\gamma^A-\frac{\alpha^A b^A}{v_t^A}]$$

$$-G^n\left(\frac{v_{t+1}^A}{v_t^A}\right)^{\frac{\alpha^A}{1-\alpha^A}}[v_{t+1}^A+b^A(1-\sigma^A)]\}\frac{L_t^A}{L_t^N}\frac{(M^A)^{1/(1-\alpha^A)}}{(M^N)^{1/(1-\alpha^N)}}(v_t^A)^{\frac{\alpha^A}{1-\alpha^A}}(v_t^N)^{\frac{\alpha^N}{(\alpha^N-1)}}-\frac{1}{e_t^U}\{\sigma^U[1-\alpha^U-\gamma^U-\frac{\alpha^U b^U}{v_t^U}]$$

$$-G^n\left(\frac{v_{t+1}^U}{v_t^U}\right)^{\frac{\alpha^U}{1-\alpha^U}}[v_{t+1}^U+b^U(1-\sigma^U)]\}\frac{L_t^U}{L_t^N}\frac{(M^U)^{1/(1-\alpha^U)}}{(M^N)^{1/(1-\alpha^N)}}(v_t^U)^{\frac{\alpha^U}{1-\alpha^U}}(v_t^N)^{\frac{\alpha^N}{(\alpha^N-1)}}. \tag{41}$$

Equation (41) represents the fourth equation of the intertemporal equilibrium dynamics of the capital-output ratios, $v_t^S, v_t^N, v_t^A, v_t^U$, and the terms of trade e_t^A and e_t^U of the three-good, three-country OLG model under financial integration.

The two remaining dynamic equation we obtain by dividing Asia's good respective the US good market clearing condition (32) respective (33) by the combined EMU good market clearing condition (31):

$$\frac{(1-\gamma^A)Y_t-K_{t+1}^A}{(1-\gamma^S)X_t^S-K_{t+1}^S+(1-\gamma^N)X_t^N-K_{t+1}^N}=\frac{L_t^A y_t^{A,1}+L_{t-1}^A y_t^{A,2}+L_t^S y_t^{S,1}+L_{t-1}^S y_t^{S,2}+L_t^N y_t^{N,1}+L_{t-1}^N y_t^{N,2}}{L_t^S x_t^{S,1}+L_{t-1}^S x_t^{S,2}+L_t^N x_t^{N,1}+L_{t-1}^N x_t^{N,2}+L_t^A x_t^{A,1}+L_{t-1}^A x_t^{A,2}}$$
$$+\frac{+L_t^U y_t^{U,1}+L_{t-1}^U y_t^{U,2}}{+L_t^U x_t^{U,1}+L_{t-1}^U x_t^{U,2}}, \tag{42}$$

$$\frac{(1-\gamma^U)Z_t-K_{t+1}^U}{(1-\gamma^S)X_t^S-K_{t+1}^S+(1-\gamma^N)X_t^N-K_{t+1}^N}=\frac{L_t^U z_t^{U,1}+L_{t-1}^U z_t^{U,2}+L_t^S z_t^{S,1}+L_{t-1}^S z_t^{S,2}+L_t^N z_t^{N,1}+L_{t-1}^N z_t^{N,2}}{L_t^S x_t^{S,1}+L_{t-1}^S x_t^{S,2}+L_t^N x_t^{N,1}+L_{t-1}^N x_t^{N,2}+L_t^A x_t^{A,1}+L_{t-1}^A x_t^{A,2}}$$
$$+\frac{+L_t^A z_t^{A,1}+L_{t-1}^A y_t^{A,2}}{+L_t^U x_t^{U,1}+L_{t-1}^U x_t^{U,2}}. \tag{43}$$

Upon inserting the utility maximizing consumption functions resulting from household's utility maximization problems into the right hand side of equation (42) and (43), it fortunately turns out that (42) respective (43) can be rewritten as follows:

$$\frac{(1-\gamma^A)Y_t-K_{t+1}^A}{(1-\gamma^S)X_t^S-K_{t+1}^S+(1-\gamma^N)X_t^N-K_{t+1}^N}=\frac{\zeta^y}{\zeta^x}e_t^A, \tag{44}$$

$$\frac{(1-\gamma^U)Z_t-K_{t+1}^U}{(1-\gamma^S)X_t^S-K_{t+1}^S+(1-\gamma^N)X_t^N-K_{t+1}^N}=\frac{\zeta^z}{\zeta^x}e_t^U. \tag{45}$$

Dividing the numerator on both sides of (44) by Y_t and the denominator on both sides of (44) by X_t^N and taking account of (38) and (39), we obtain the fifth equation of the intertemporal equilibrium dynamics:

$$1-\gamma^A-G^n\left(\frac{v_{t+1}^A}{v_t^A}\right)^{\frac{\alpha^A}{1-\alpha^A}}v_{t+1}^A=\frac{\zeta^y}{\zeta^x}e_t^A\frac{L_t^N}{L_t^A}\frac{(M^N)^{1/(1-\alpha^N)}}{(M^A)^{1/(1-\alpha^A)}}(v_t^N)^{\frac{\alpha^N}{1-\alpha^N}}(v_t^A)^{\frac{\alpha^A}{(\alpha^A-1)}}\{[1-\gamma^S-G^n\left(\frac{v_{t+1}^S}{v_t^S}\right)^{\frac{\alpha^S}{1-\alpha^S}}v_{t+1}^S]\times$$
$$\times\frac{L_t^S}{L_t^N}\frac{(M^S)^{\frac{1}{1-\alpha^S}}}{(M^N)^{\frac{1}{1-\alpha^N}}}\left(\frac{\alpha^S}{\alpha^N}\right)^{\frac{\alpha^S}{1-\alpha^S}}\left(v_t^N\right)^{\frac{(\alpha^S-\alpha^N)}{(1-\alpha^S)(1-\alpha^N)}}+1-\gamma^N-G^n\left(\frac{v_{t+1}^N}{v_t^N}\right)^{\frac{\alpha^N}{1-\alpha^N}}v_{t+1}^N\}. \quad (46)$$

Dividing the numerator on both sides of (45) by Z_t and the denominator on both sides of (45) by X_t^N and taking account of (38) and (40), we obtain the sixth equation of the intertemporal equilibrium dynamics:

$$1-\gamma^U-G^n\left(\frac{v_{t+1}^U}{v_t^U}\right)^{\frac{\alpha^U}{1-\alpha^U}}v_{t+1}^U=\frac{\zeta^z}{\zeta^x}e_t^U\frac{L_t^N}{L_t^U}\frac{(M^N)^{1/(1-\alpha^N)}}{(M^U)^{1/(1-\alpha^U)}}(v_t^N)^{\frac{\alpha^N}{1-\alpha^N}}(v_t^U)^{\frac{\alpha^U}{(\alpha^U-1)}}\{[1-\gamma^S-G^n\left(\frac{v_{t+1}^S}{v_t^S}\right)^{\frac{\alpha^S}{1-\alpha^S}}v_{t+1}^S]\times$$
$$\times\frac{L_t^S}{L_t^N}\frac{(M^S)^{\frac{1}{1-\alpha^S}}}{(M^N)^{\frac{1}{1-\alpha^N}}}\left(\frac{\alpha^S}{\alpha^N}\right)^{\frac{\alpha^S}{1-\alpha^S}}\left(v_t^N\right)^{\frac{(\alpha^S-\alpha^N)}{(1-\alpha^S)(1-\alpha^N)}}+1-\gamma^N-G^n\left(\frac{v_{t+1}^N}{v_t^N}\right)^{\frac{\alpha^N}{1-\alpha^N}}v_{t+1}^N\}. \quad (47)$$

It turns out that the intertemporal equilibrium dynamics consists of six non-linear first-order difference equations which can be used to determine the following six dynamic variables: $v_t^S, v_t^N, v_t^A, v_t^U,\ e_t^A, e_t^U$.

In a steady state with $v_{t+1}^S=v_t^S=v^S, v_{t+1}^N=v_t^N=v^N, v_{t+1}^A=v_t^A=v^A, v_{t+1}^U=v_t^U=v^U,\ e_{t+1}^A=e_t^A=e^A$ and $e_{t+1}^U=e_t^U=e^U$, the system of first-order difference equations collapses on the following system of steady state equations:

$$v^S=(\alpha^S/\alpha^N)v^N, \quad (48)$$

$$v^A=(\alpha^A/\alpha^N)v^N, \quad (49)$$

$$v^U=(\alpha^U/\alpha^N)v^N, \quad (50)$$

$$\phi^S\xi^{S,N}(v^N)^{\chi^{S,N}}+\phi^N=-(e^A)^{-1}\phi^A\xi^{A,N}(v^N)^{\chi^{A,N}}-(e^U)^{-1}\phi^U\xi^{U,N}(v^N)^{\chi^{U,N}}, \quad (51)$$

$$(1-\gamma^A-G^nv^A)\xi^{A,N}(v^N)^{\chi^{A,N}}=e^A(\zeta^y/\zeta^x)[(1-\gamma^S-G^nv^S)\xi^{S,N}(v^N)^{\chi^{S,N}}+1-\gamma^N-G^nv^N], \quad (52)$$

$$(1-\gamma^U-G^nv^U)\xi^{U,N}(v^N)^{\chi^{U,N}}=e^U(\zeta^z/\zeta^x)[(1-\gamma^S-G^nv^S)\xi^{S,N}(v^N)^{\chi^{S,N}}+1-\gamma^N-G^nv^N]. \quad (53)$$

Inserting (48)-(50) into (51)-(53), then solving (53) for e^U and inserting the result into (51), we obtain two equations of e^A as function of v^N. Proposition 4 tells us under which conditions non-trivial steady state solutions exist.

Proposition 4 (Existence of non-trivial steady states under EMU financial integration)
Let $\boldsymbol{\omega}=(\alpha^S,\alpha^N,\alpha^A,\alpha^U,\beta^S,\beta^N,\beta^A,\beta^U,\gamma^S,\gamma^N,\gamma^A,\gamma^U,\zeta^x,\zeta^y,L^S,L^N,L^A,L^U,M^S,M^N,M^A,M^U,$ $b^S,b^N,b^A,b^U,G^n)$ *be the parameter vector and* $\boldsymbol{\Omega}=[0,1]^{14}\times\Re_+^{13}$ *be the parameter space in the*

EMU-Asia-US steady-state market equilibrium under financial integration. For any admissible parameter combination, $\boldsymbol{\omega} \in \boldsymbol{\Omega}$*, there exist some* $\bar{b}^S \in (0,\infty)$*,* $\bar{b}^N \in (0,\infty)$*,* $\bar{b}^A \in (0,\infty)$ *and* $\bar{b}^U \in (0,\infty)$ *such that for* $b^S \in (0,\bar{b}^S)$*,* $b^N \in (0,\bar{b}^N)$*,* $b^A \in (0,\bar{b}^A)$ *and* $b^U \in (0,\bar{b}^U)$*, there are two nontrivial steady state solutions* $(e_L^A, e_L^U, v_L^S, v_L^N, v_L^A, v_L^U) >> 0$ *and* $(e_H^A, e_H^U, v_H^S, v_H^N, v_H^A, v_H^U) >> 0$*. For* $b^S = b^N = b^A = b^U = 0$*, there is only one non-trivial steady state.*

Proof. The proof can be given as yet only numerically, however, for a broad set of admissible and empirically plausible set of structural and policy parameters.

Since there are in general two steady state solutions, we had to investigate the local dynamic stability of each steady state by calculating the eigenvalues of the Jacobian matrix of the intertemporal equilibrium dynamics (34)-(36), (41) and (46)-(47) in a small neighborhood of the steady states. Due to the analytical complexity of the six-dimensional dynamical system we are not able to prove dynamic stability in general but can only show saddle-point stability for the larger steady state solution for certain numerically specified parameter sets.

Proposition 5 (Saddle-point stability of the larger steady state solution in a numerically specified model economy)

Suppose that the conditions in proposition 4 hold. Moreover, assume that the following numerical values are attributed to model parameters mentioned in proposition 4: $G^n = 2$, $\zeta^x = 1/2$, $\zeta^y = 1/4$, $\zeta^z = 1/4$, $\beta^s = 0.45, \beta^N = 0.5, \beta^A = 0.6, \beta^U = 0.42, \alpha^S = 0.17, \alpha^N = 0.16,$ $\alpha^A = 0.18, \alpha^U = 0.16$, $\gamma^S = 0.17, \gamma^N = 0.2, \gamma^A = 0.18, \gamma^U = 0.15, M^S = 1$, $M^N = 1.5, M^A = 1, M^U = 1.5, L^S = L^N = 85, L^A = 1300, L^U = 1350, b^S = 0.027, b^N = 0.023, b^A = 0.015, b^U = 0.025$. *Given this parameter set, the steady state* $(e_H^A, e_H^U, v_H^S, v_H^N, v_H^A, v_H^U)$ *is saddle-point stable while* $(e_L^A, e_L^U, v_L^S, v_L^N, v_L^A, v_L^U)$ *is saddle-point unstable.*

Financially Integrated versus Financially Autarkic Steady State

Knowing that the larger steady state solution is dynamically stable, proposition 4 below can then be used to provide an answer to the main question whether financial integration (i.e. the convergence of northern and southern EMU real interest rates and the presence of real open interest parity conditions across EMU, Asia and USA), contributes to the divergence of southern and northern EMU, Asia's and US trade balances, current account balances and net foreign asset positions.

Proposition 6 (Trade balance, current account and net foreign asset position effects of EMU and Asian-US financial integration)

Suppose that the assumptions of proposition 3 hold, i.e. southern EMU financial autarky (FA) interest rate, $(i^S)^{FA}$*, is larger than northern EMU financial autarky interest rate,* $(i^N)^{FA}$*, and Asian financial autarky interest rate,* $(i^A)^{FA}$ *is larger than US autarky interest rate* $(i^U)^{FA}$*. Suppose,*

moreover, that the structural and policy parameters of the three-country-three-good OLG model economy are such that in the financially integrated (FI) steady state the natural growth factor G^n *is larger than the common real interest rate,* i^{FI}*. Then, the southern EMU and US trade balance, current account and net foreign asset position to GDP ratios are negative, while the respective northern EMU and Asian ratios become larger than zero, i.e.* $(tb^S)^{FI} < 0, (ca^S)^{FI} < 0, (\phi^S)^{FI} < 0$ *and* $(tb^U)^{FI} < 0, (ca^U)^{FI} < 0, (\phi^U)^{FI} < 0$ *while* $(tb^N)^{FI} > 0, (ca^N)^{FI} > 0, (\phi^N)^{FI} > 0$ *and* $(tb^A)^{FI} > 0, (ca^A)^{FI} > 0, (\phi^A)^{FI} > 0$.

Proof. By assumption, we have $(i^S)^{FA} > (i^N)^{FA}$ and $(i^U)^{FA} > (i^A)^{FA}$. Thus, $1+(i^S)^{FA} = \alpha^S/(v^S)^{FA} > 1+(i^N)^{FA} = \alpha^N/(v^N)^{FA}$ and $1+(i^U)^{FA} = \alpha^U/(v^U)^{FA} > 1+(i^A)^{FA} = \alpha^A/(v^A)^{FA}$. Financial integration means that the difference between southern and northern EMU, and that between Asian and US autarky interest rates diminishes as the southern EMU and the US interest rate decline, and as the northern EMU and Asian interest rate rise. Due to the decreasing marginal productivity of capital, the decline in southern EMU and US interest rate is associated with a rise in southern EMU and US capital output ratios, and with a fall in northern EMU and Asian ratios. Recalling the definition of country's i net foreign asset position in steady state as $\phi^i(v^i) \equiv \sigma^i(1-\alpha^i-\gamma^i-\alpha^i b^i/v^i) - G^n[v^i + b^i(1-\sigma^i)]$, differentiation of ϕ^i with respect to v^i yields $\phi^{i\prime}(v^i) = \alpha^i\sigma^i b^i/(v^i)^2 - G^n$. From the proof of proposition 2 in Farmer (2013, p. 11) we know that there is a small neighborhood for country i's autarky steady state with the larger capital output ratio, v_2^i, in which $\phi^{i\prime}\left((v_2^i)^{FA}\right) < 0$ holds. Hence, country i's net foreign asset position deteriorates with a rising capital output ratio, and improves with a declining capital output ratio. Since at the autarky value of v_2^i country i's net foreign asset position is zero, and since the net foreign asset position of country *i* declines with rising capital output ratio at $(v_2^S)^{FI}$ and at $(v_2^U)^{FI}$, the southern EMU and the US net foreign asset position is smaller than zero, i.e. $\phi^S\left((v_2^S)^{FI}\right) < 0$ and $\phi^U\left((v_2^U)^{FI}\right) < 0$. At the same time, at $(v_2^N)^{FI}$ and $(v_2^A)^{FI}$, the northern EMU and the Asian net foreign asset position must be larger than zero, i.e. $\phi^N\left((v_2^N)^{FI}\right) > 0$ and $\phi^A\left((v_2^A)^{FI}\right) > 0$. Due to the steady state relations of the trade balance and the current account balance with respect to the net foreign asset position, i. e. $(tb^i)^{FI} = \{[G^n - (1+i^{FI})]/G^n\}(\phi^i)^{FI}$ and $(ca^i)^{FI} = [(G^n - 1)/G^N](\phi^i)^{FI}$, and given the assumption of dynamic inefficiency ($G^n > 1+i^{FI}$), southern EMU and US trade and current balances become negative, while northern EMU and Asian trade and current account balances become positive. ■

A numerical illustration

Before concluding, the numerical specification of the basic model reported above to verify dynamic stability of the larger steady state solution is used to illustrate both the explanatory power and the limitations of our three-good, three-country OLG model. The parameter values are chosen such that the model is able to roughly reproduce the main stylized facts listed above under financial autarky, and to accord with the assumptions in the propositions above. Table 1 reports the steady state values of main endogenous variables in EMU South, EMU North, in Asia and in USA under financial autarky.

Table 1 Main endogenous variables in EMU South, EMU North, in Asia and in USA (calculated on a yearly basis) under *financial autarky*

	Capital Output ratio	Real interest rate (in %)	EMU terms of trade relative to	Ratio of Current Account to GDP	Ratio of Net Foreign Assets to GDP
EMU South	1.85	3.37		0	0
EMU North	2.10	2.6		0	0
Asia	2.65	2.15	10.94	0	0
USA	1.92	2.98	21.16	0	0

Source: Own calculations

Note that the yearly real interest rate in EMU South, North, in Asia and in USA exhibited in Table 1 is not too far from the real interest rates in the late 1990s portrayed in Fig. 1 and Fig. 2. In contrast, the saving rates implied by the parameter set above are significantly higher than the saving rates portrayed in Figures 3-5 whereby the difference between model result and empirical data is smallest for the Asian saving rates. Finally, the assumption of financial autarky which implies zero net foreign asset and current account to GDP ratios for EMU South, EMU North, Asia and USA is roughly in accordance with the empirical data for the late 1990s as can be seen from Figures 8 - 11.

Table 2 Main endogenous variables in EMU South, EMU North, in Asia and in USA (calculated on a yearly basis) under *financial integration*

	Capital Output ratio	Real interest rate (in %)	Current Account to GDP (%)	Net Foreign Assets to GDP (%)	Net exports to GDP ratio (%)	EMU Terms of trade vis-à-vis
EMU South	2.19	2.68	-2.34	-60.37	-0.00076	
EMU North	2.06	2.68	+0.25	+6.60	+ 0.000082	
Asia	2.32	2.68	+2.38	+61.32	+ 0.00077	11.1
USA	2.06	2.68	-1.02	-26.57	-0.00033	21.13

Source: Own calculation

The comparison of the results in Table 2 to those in Table 1 reveals both satisfactory and non-satisfactory facts. Regarding the satisfactory results we observe: First, through financial integration (equalization of real interest rates across EMU, Asia and USA) the capital output ratio both in EMU South and US increases while it decreases in EMU North and in Asia in line with empirical observations. Second, the common real interest rate is significantly lower than the autarky real interest rate in EMU South and in US and higher than the autarky real interest rate in EMU North and Asia, also in line with empirical facts. Third, the net foreign asset to GDP ratios exhibited in the fifth column of Table 2 feature rather close to those portrayed in Figures 9 and 11. It is also worth mentioning that the huge intra-EMU imbalance with respect to the net foreign asset to GDP ratio can be much better reproduced by our three-country than by Farmer's (2014) two-country model.

Regarding the non-satisfactory results, we observe from Table 2 that the current account to GDP ratios feature too small in comparison to the empirical facts portrayed in Fig. 8 and 10. This is particularly true with respect to the US current account to GDP ratio. It seems to be that the current account to GDP ratio is mainly driven by the interest payments on net foreign assets and not so much by merchandise trade imbalances. Indeed, this suggestion is confirmed by looking at the net exports to GDP ratios reported in the sixth column of Table 2. Obviously, the merchandise trade imbalances generated by our three-country model are much too small to contribute satisfactorily to a theoretical explanation of the empirically observed current account imbalances.

Summary and Conclusion

This paper explores, within a three-good, three-country OLG model with production, capital accumulation and public debt, the convergence of high real interest rates towards lower rates and the emergence of external imbalances between both EMU core and periphery, and between Asia and the US, for the period between after the inception of the common currency and the onset of the global financial crisis in 2008. It also models the pre-euro situation and the Asian-US financial relations in the 1990s in terms of financial autarky, and the intra-EMU and the Asian-US financial linkages in 2000s in terms of financial integration, characterized by convergence of real interest rates across EMU, Asia and the USA.

At steady states under conditions of financial autarky, a lower saving rate, a higher capital production share and a higher debt to GDP ratio in EMU South were shown to comply with the empirically discrepancy between southern (high) and northern (low) EMU real interest rates and zero external imbalances across EMU South and EMU North before the advent of the common currency. Similarly, a much lower US saving rate compared to the Asian saving rate, together with a relatively higher US public debt to GDP ratio was associated with a higher US real interest rate compared to Asia, in spite of the relatively higher Asian capital production share prevailing before Asian-US financial integration beginning in 2000s.

After the inception of the common currency, free real capital mobility among EMU South and North led northern EMU households facing initially relatively high southern EMU interest rates to invest their wealth in southern EMU housing and residential objects. Free financial capital mobility after the East-Asian currency crises led Asian households facing initially relatively high US interest rates to buy US government bonds. As a consequence, southern EMU and US interest rates fell, and northern EMU and Asian interest rates rose. Simultaneously, southern EMU and US capital output ratios increased, while northern EMU and Asia's capital output ratios decreased inducing a negative net foreign asset position in (relatively low saving) southern EMU and in the USA, and a positive net foreign asset position in (relatively high saving) northern EMU and Asia. Associated with the southern EMU and US net foreign debtor position, the southern EMU and US

current account became negative due to the higher interest payments needed on net foreign debts while the opposite held true with respect to the northern EMU and Asia's current account surplus. In addition, southern EMU and US current account deficits also occurred due to trade balance deficits as a result of increased (net) imports from Asia, while northern EMU and the Asian trade balances turned positive as a result of increased net exports to Asia and to the USA, which conforms to the new stylized facts put forward by Chen et al. (2013). However, southern EMU and US trade balance deficits vis-à-vis net foreign debts, and northern EMU and Asia's trade balance surpluses vis-à-vis net foreign credits can only emerge under conditions of dynamic inefficiency in the world economy.[5] In our large, open model economy dynamic inefficiency does not result from the assumptions employed, but from the impact of high saving rates in Asia and in northern EMU in comparison to the low saving rates in US and southern EMU economies, together with the impact of the further international differences in capital production shares, government expenditure and public debt ratios as stated in proposition 3. High Asian, in particular, Chinese household saving rates was traced back to the missing pay-as-you go pension system (Eugeni 2013), but – whether traced back or simply assumed – excessively high Asian saving rates together with the other international differences in structural and policy parameters, led via the dynamic inefficiency of the whole economic system, to the empirically observed trade balance surpluses of northern EMU and of Asia, as well as to the trade balance deficits of southern EMU and of the USA. Thus, once the intensified trade linkages of EMU South and North vis-à-vis Asia after the euro launch are taken into account, and once the high Asian saving rates following from the East-Asian currency crises are considered, we conclude that a more correct modeling of pre-crisis intra-EMU external imbalances triggered by euro-related financial integration requires that the financial integration between Asia and the USA also be taken into account.

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[5] Under dynamic efficiency, net foreign debtor countries ought to run trade balance surpluses while net foreign creditors need to switch to trade balance deficits.

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